Optics measurement algorithms for the proton energy frontier

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Outline

- Motivation
- β-function from BPM turn-by-turn data
- Improvements
    - Error analysis
    - Model accuracy
    - Hardware/Software

Motivation

Motivation

- Tight tolerances for β-beat due to
  - Available mechanical aperture

„For the LHC the total tolerance for the β-beat has been specified as 20 %.“

–LHC Design Report I

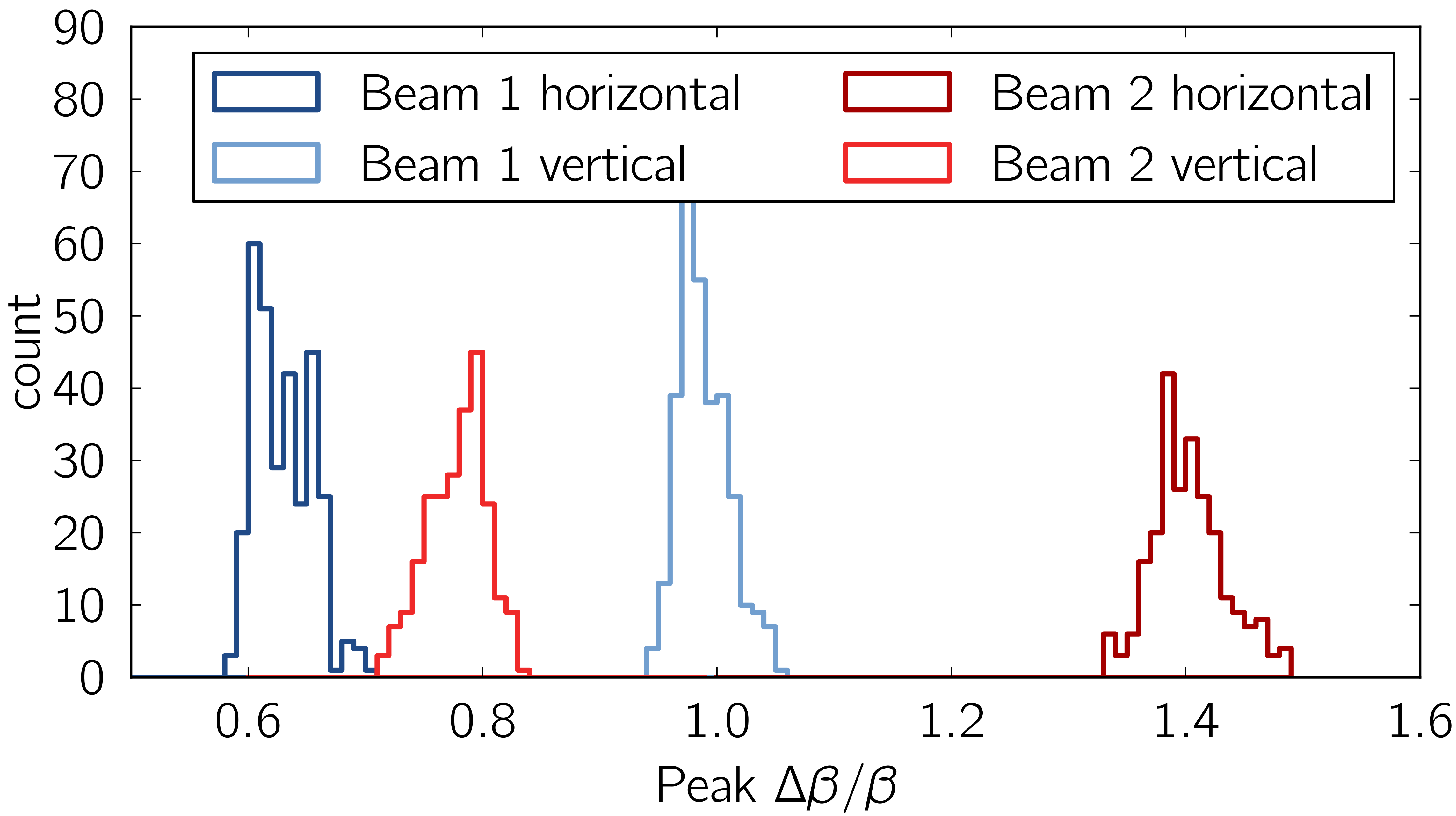
- In 2012 a β-beat of up to 100% was observed before local corrections

Motivation

- Run II at 6.5 TeV —> allows for smaller beta*
- —> enhances optics errors of triplet magnets
- More quadrupole magnets in saturation regime
- Broken MQT magnet (tune trim)

Motivation

- Extrapolating to 40cm beta* at 6.5 TeV



Motivation

- Higher damage potential at 6.5 TeV

➡ Limits maximum excitation amplitude and total beam charge

➡ Reduced signal to noise ratio for optics measurement

Motivation

- In 2012 insufficient resolution to measure:
  - IP beta-functions
  - Beta-functions during the ramp for emittance studies

Measurement method

Optics measurement

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- Measurement of BPM turn-by-turn data x_i

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- Harmonic analysis $C(w) = \sum_{i=0}^{N-1} x_i \cos(w\,i),\, S(w) = \sum_{i=0}^{N-1} x_i \sin(w\,i)$

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➡ Phase advance of betatron oscillation between BPM

$$\phi(w) = -\arctan\left(\frac{S(w)}{C(w)}\right)$$

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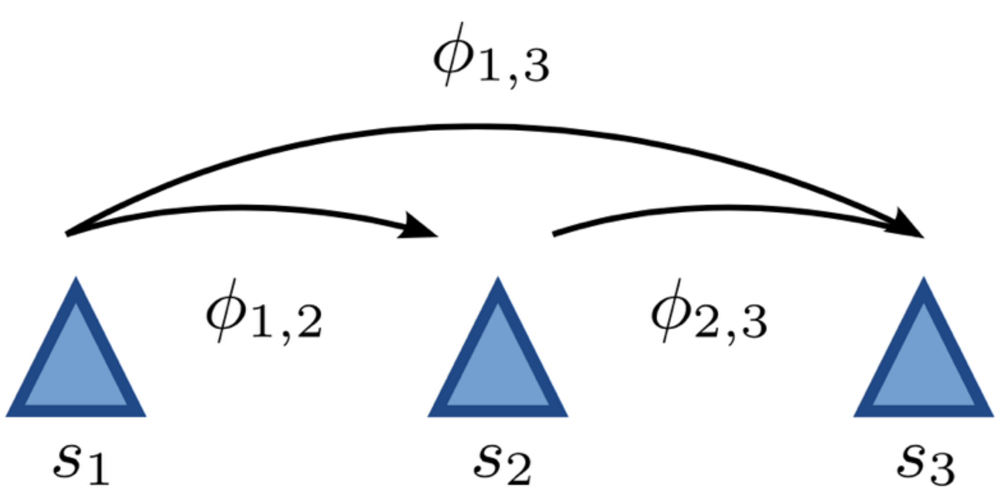

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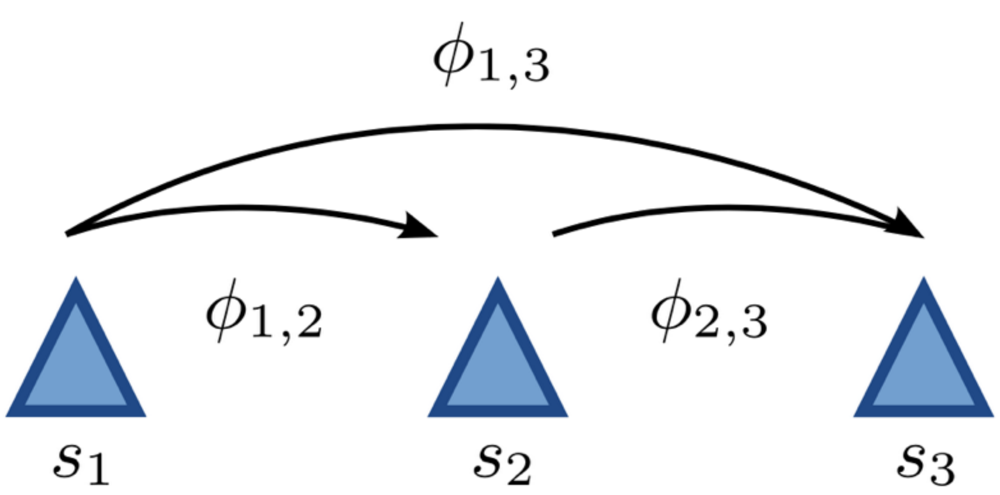

$$\beta_i = \frac{\epsilon_{ijk}\cot(\phi_{i,j}) + \epsilon_{ikj}\cot(\phi_{i,k})}{\epsilon_{ijk}\frac{M_{11(i,j)}}{M_{12(i,j)}} + \epsilon_{ikj}\frac{M_{11(i,k)}}{M_{12(i,k)}}}$$

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- Optimum phase advances

$$\phi_{i,j} = \frac{\pi}{4} + n_1\frac{\pi}{2},$$
$$\phi_{i,k} = \frac{\pi}{4} + (2n_2 + 1 - n_1)\frac{\pi}{2},$$

$$n_1, n_2 \in \mathbb{Z}.$$

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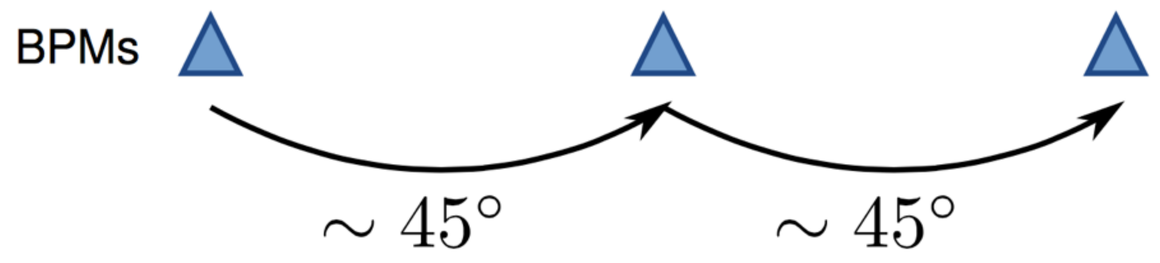
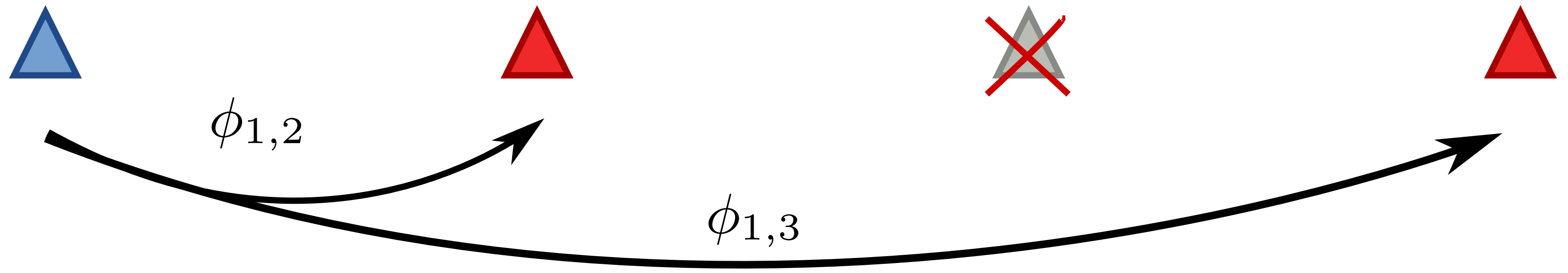
- Multiples of π should be avoided as the cotangent becomes infinite

Situation in the arcs



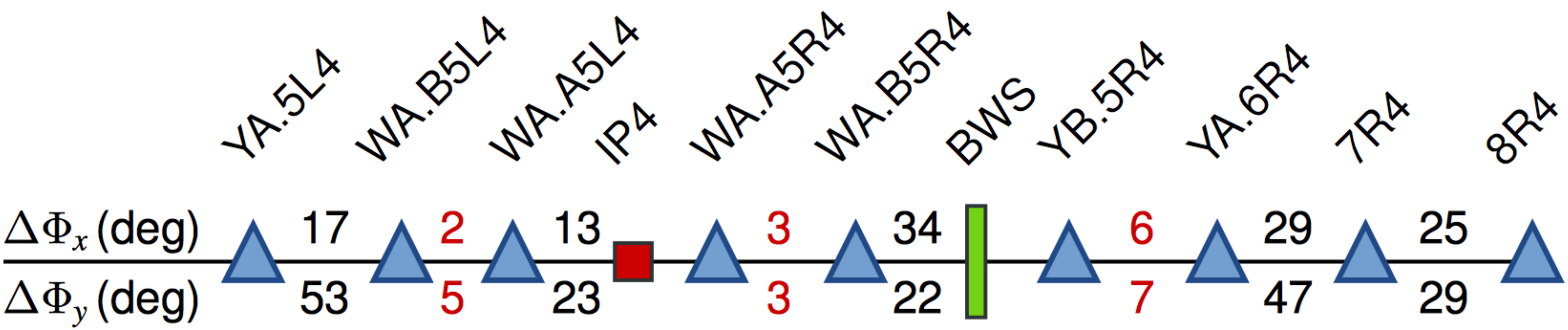
- Phase advances between consecutive BPMs not always suited for measurement
- Previous implementation used only **neighboring BPMs**
- Optimum if probed BPM in the middle
- If probed BPM right/left of other BPMs the optimum is to skip one BPM



Situation in the interaction regions (IRs)

- Phase advances are irregular and can be very small
- Using neighboring BPM will result in large uncertainties

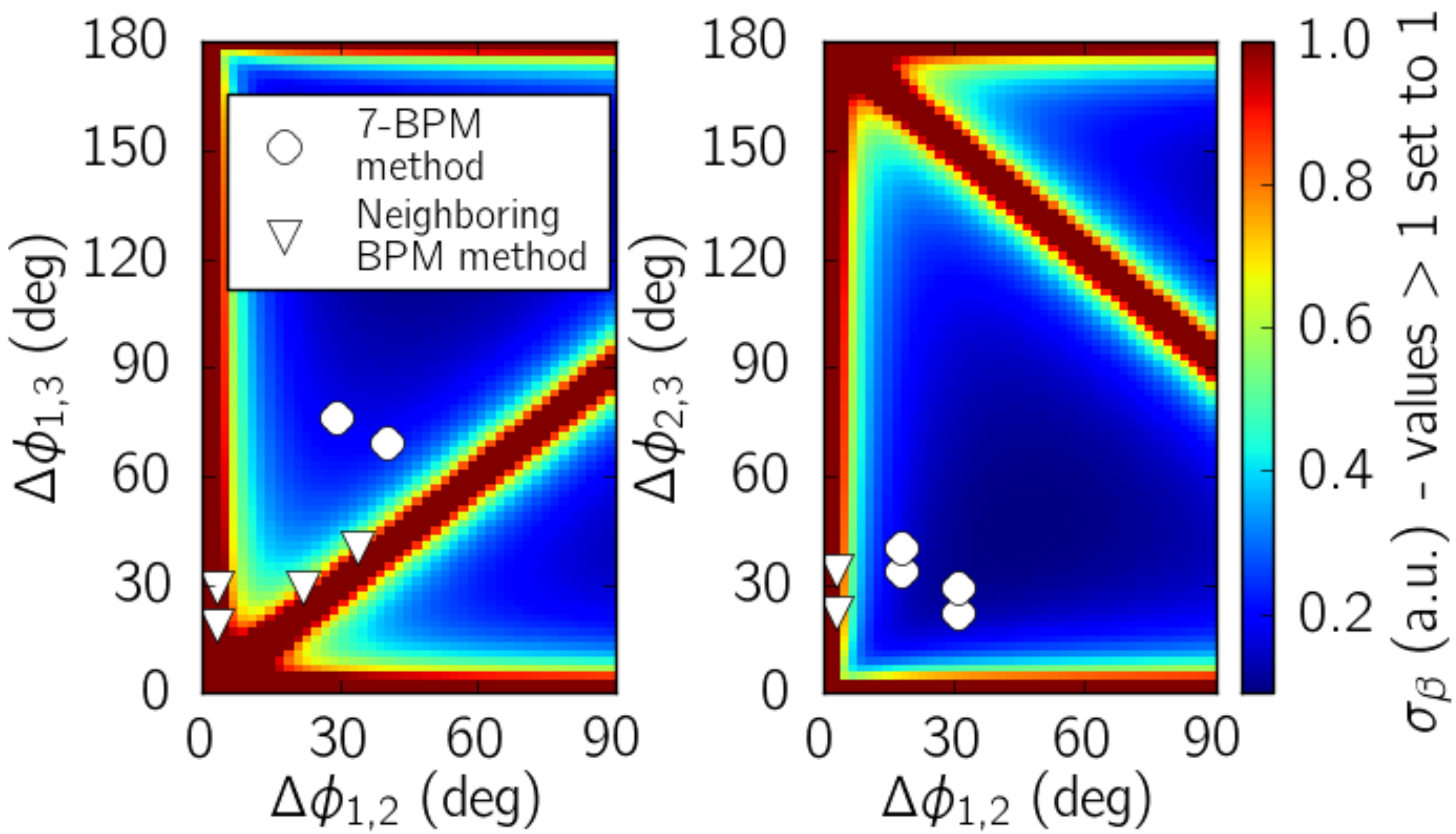
Sketch of phase advances in IR4



Improvements

Improvements

- Increase the range from which BPM combinations are chosen
- Choose BPM combinations with good phase advances



Improvements

- Consider more β_i from different BPM combinations
- Minimize S (least squares)

$$S(\beta) = \sum_{i=1}^{N}\sum_{j=1}^{N}(\beta_i - \beta)V_{ij}^{-1}(\beta_j - \beta)$$

$$\beta = \sum_{i=1}^{N} w_i \beta_i \qquad\qquad w_i = \frac{\sum_{k=1}^{N} V_{ik}^{-1}}{\sum_{k=1}^{N}\sum_{j=1}^{N} V_{jk}^{-1}}.$$

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Problem: Good knowledge of V_{ij}

Statistical error

- Uncertainty of the phase advance derived as standard deviation of n measurement files

$$\sigma_{\phi_{i,j}} = t(n)\sqrt{\frac{1}{n-1}\sum_{k=1}^{n}\left(\overline{\phi_{i,j}} - \phi_{i,j,(k)}\right)^2}$$

- $t(n)$ is a correction for small sample size from Student t-distribution

- Amount of measurements is always limited due to beam time

Number of measurements	$t(n)$
2	1.84
3	1.32
4	1.20
5	1.15
10	1.06

Statistical error

- All phase advances that share one BPM are correlated

$$\rho(\phi_{i,j}, \phi_{i,k}) = \frac{\partial \phi_{i,j}}{\partial \phi_i} \frac{\partial \phi_{i,k}}{\partial \phi_i} \frac{\sigma_{\phi_i}^2}{\sigma_{\phi_{i,j}} \sigma_{\phi_{i,k}}}.$$

- Uncertainty of phase advance from standard deviation of all measurement files

- Not possible for single phase uncertainty since the value is arbitrary and may vary from measurement to measurement

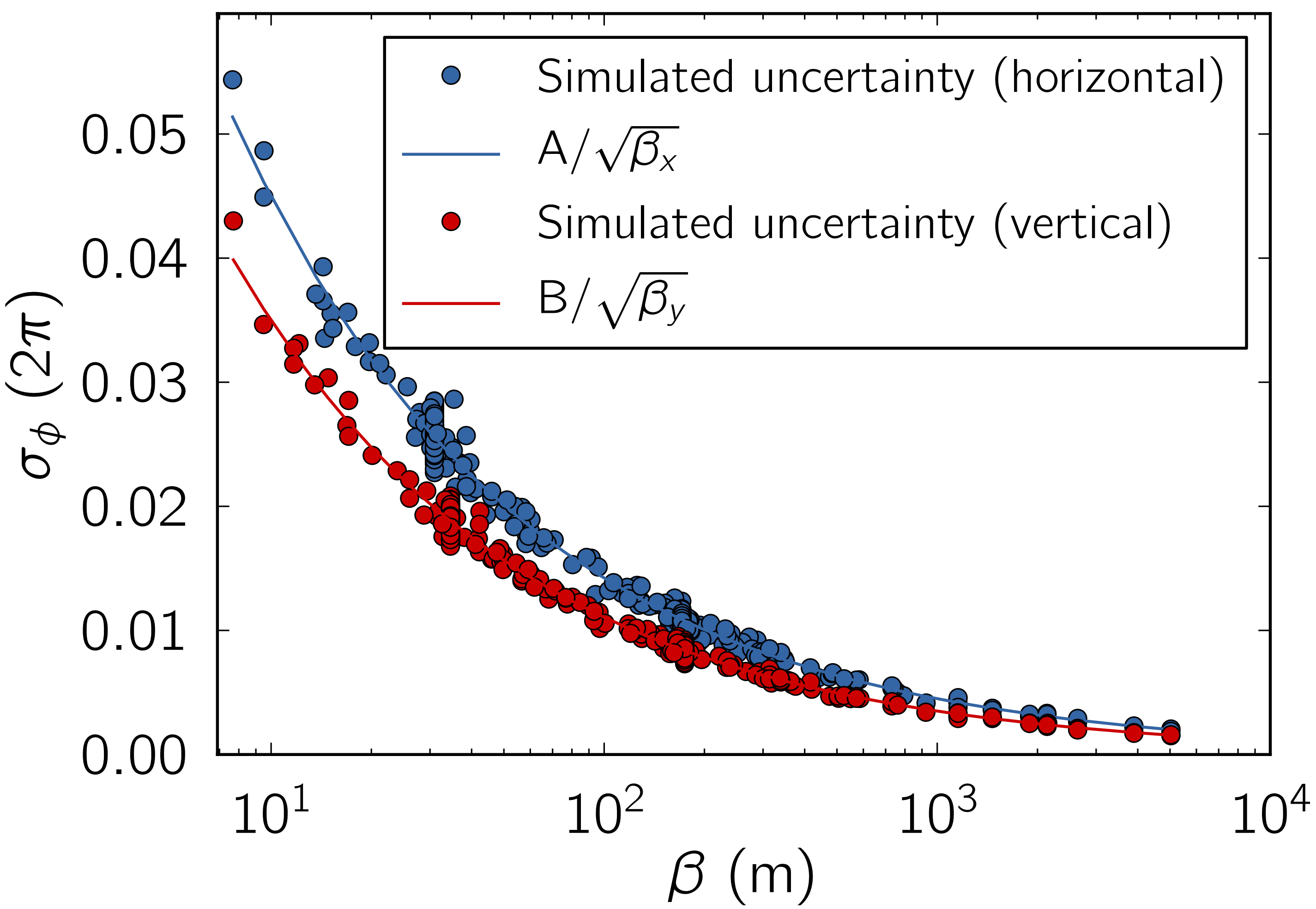
Statistical error

- Phase uncertainty depends on beta-function

$$\sigma_\phi \sim \beta^{-\frac{1}{2}}$$

- Approximate single phase uncertainty as

$$\sigma^2_{\phi_{i,j}} = \sigma^2_{\phi_i}\left(1 + \frac{\beta_i}{\beta_j}\right)$$



Statistical error

- For a probed BPM with ϕ_1 the covariance matrix is

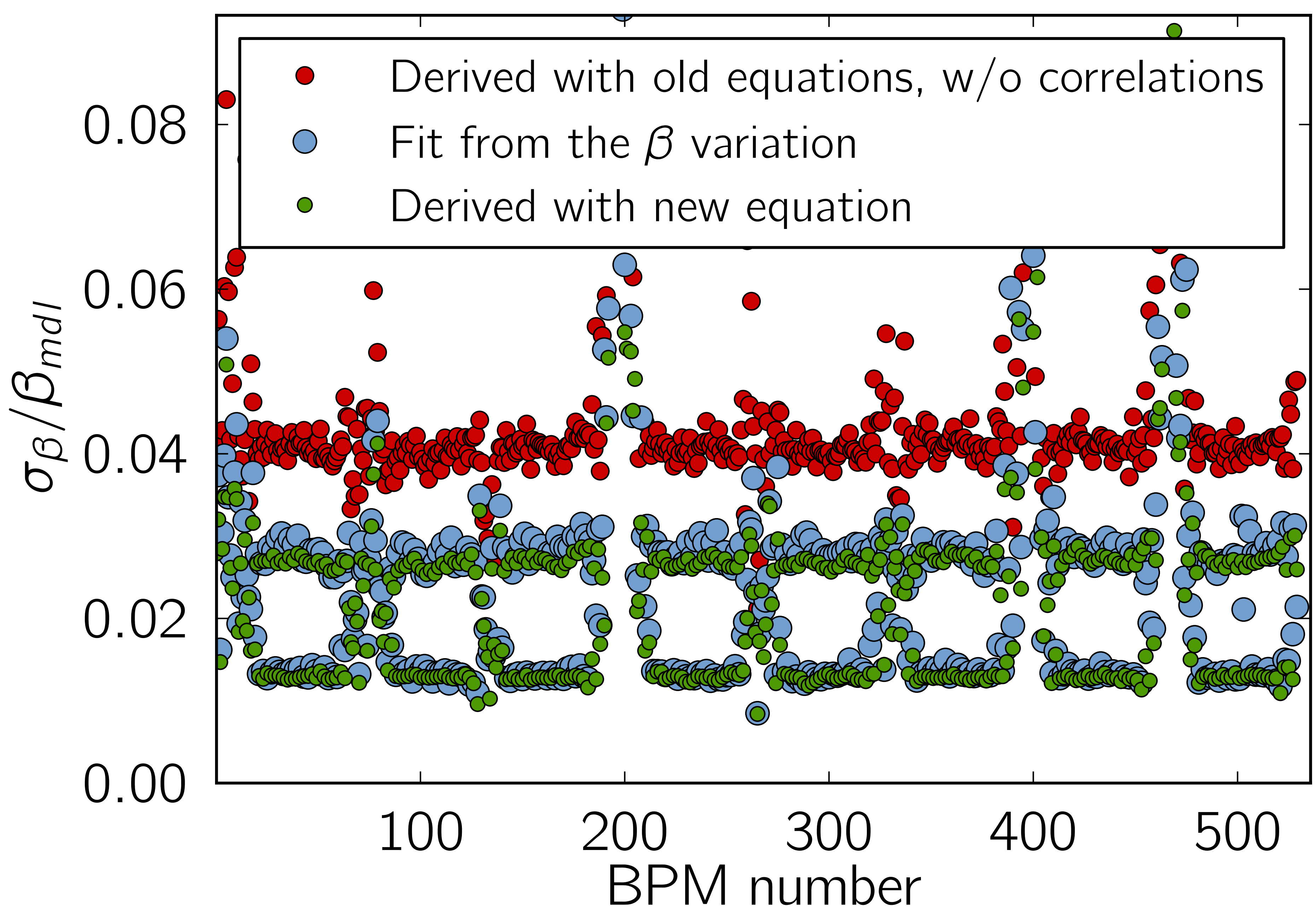
$$C_{i-1,j-1} = \rho(\phi_{1,i}, \phi_{1,j})\sigma_{\phi_{1,i}}\sigma_{\phi_{1,j}},$$
$$i \geq 2, j \geq 2$$

- This can be transformed to a covariance matrix for the different beta-functions

$$T = \begin{pmatrix} \frac{\partial \beta_1}{\partial \phi_{1,2}} & \cdots & \frac{\partial \beta_N}{\partial \phi_{1,2}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \beta_1}{\partial \phi_{1,n}} & \cdots & \frac{\partial \beta_N}{\partial \phi_{1,n}} \end{pmatrix} \qquad V_{stat} = T^T C T$$

Statistical error

- Test of error bars in a simulation show good agreement

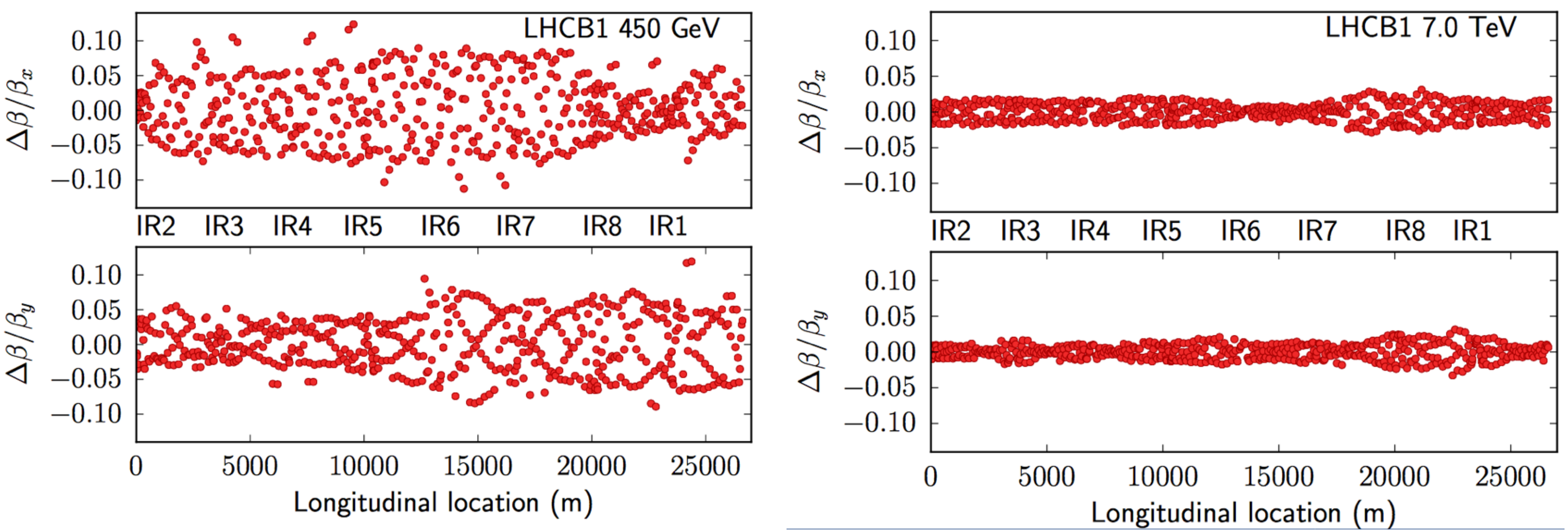


Systematic errors

$$\beta_i = \frac{\epsilon_{ijk}\cot(\phi_{i,j}) + \epsilon_{ikj}\cot(\phi_{i,k})}{\epsilon_{ijk}\frac{M_{11(i,j)}}{M_{12(i,j)}} + \epsilon_{ikj}\frac{M_{11(i,k)}}{M_{12(i,k)}}}$$

- Improve the accuracy of the optics model

➡ Include measured dipole b2 errors



Systematic errors

$$\beta_i = \frac{\epsilon_{ijk}\cot(\phi_{i,j}) + \epsilon_{ikj}\cot(\phi_{i,k})}{\mathbf{\epsilon_{ijk}\frac{M_{11(i,j)}}{M_{12(i,j)}} + \epsilon_{ikj}\frac{M_{11(i,k)}}{M_{12(i,k)}}}}$$

- We consider the following perturbations of the optics model
  - Uncertainty of dipole b2 errors
  - Quadrupole gradient uncertainty
  - Longitudinal displacement of quadrupoles
  - Transverse displacement of sextupoles

Systematic errors

- Monte-Carlo Simulation using MADX for deriving the covariance matrix

➡ V_{syst}

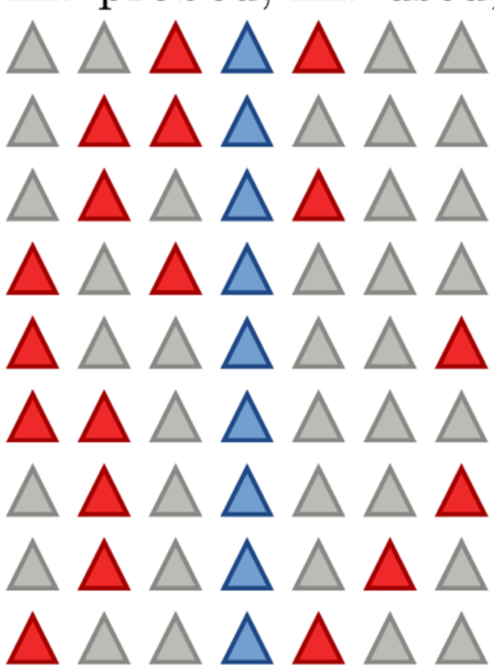
Systematic errors

- Monte-Carlo Simulation using MADX for deriving the covariance matrix

➡ V_{syst}

BPM combination

△: probed, △: used, △: unused



BPM combination	Systematic error (%)
	0.3
	0.4
	1.0
	1.1
	1.4
	1.7
	1.8
	7.1
	7.9

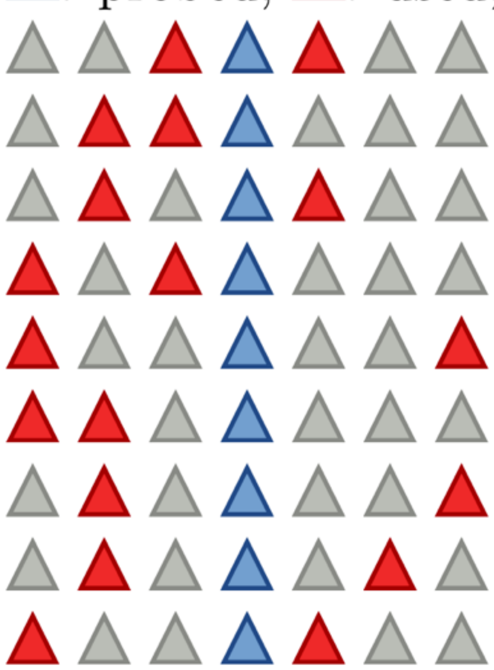
Systematic errors

- Monte-Carlo Simulation using MADX for deriving the covariance matrix

➡ V_{syst}

BPM combination — **Systematic error (%)**

△: probed, ▲: used, △: unused



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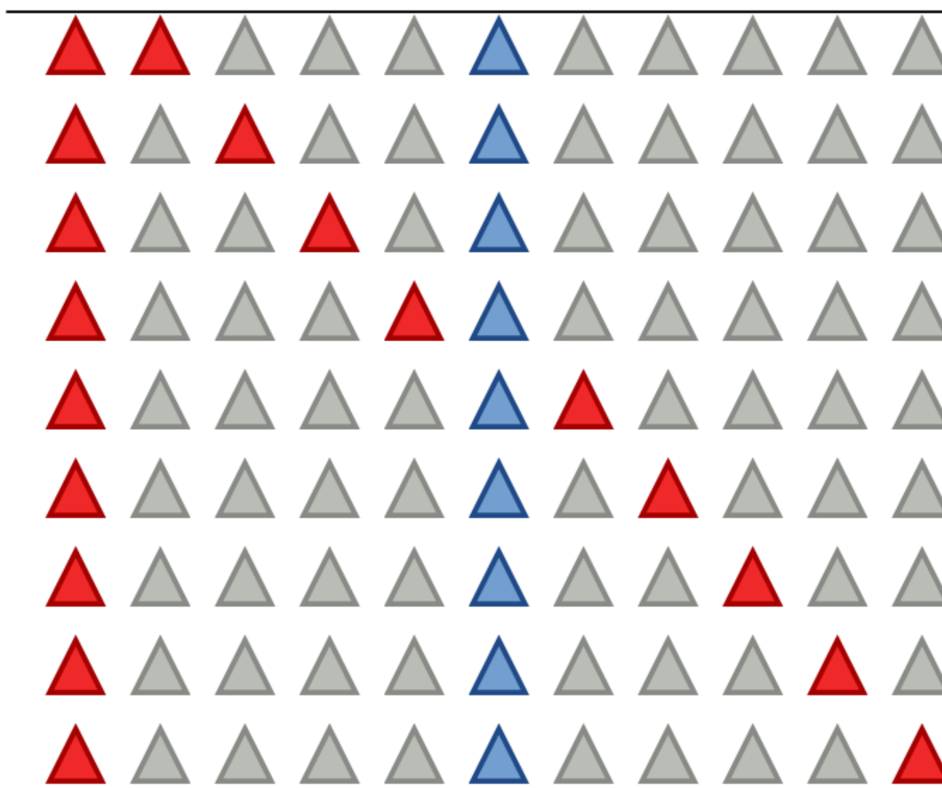


22.3
1.3
1.9
6.1
1.0
3.0
4.5
5.2
1.6

Systematic errors

- Final covariance matrix $V_{ij} = V_{ij,stat} + V_{ij,syst}$

$$S(\beta) = \sum_{i=1}^{N}\sum_{j=1}^{N}(\beta_i - \beta)V_{ij}^{-1}(\beta_j - \beta)$$

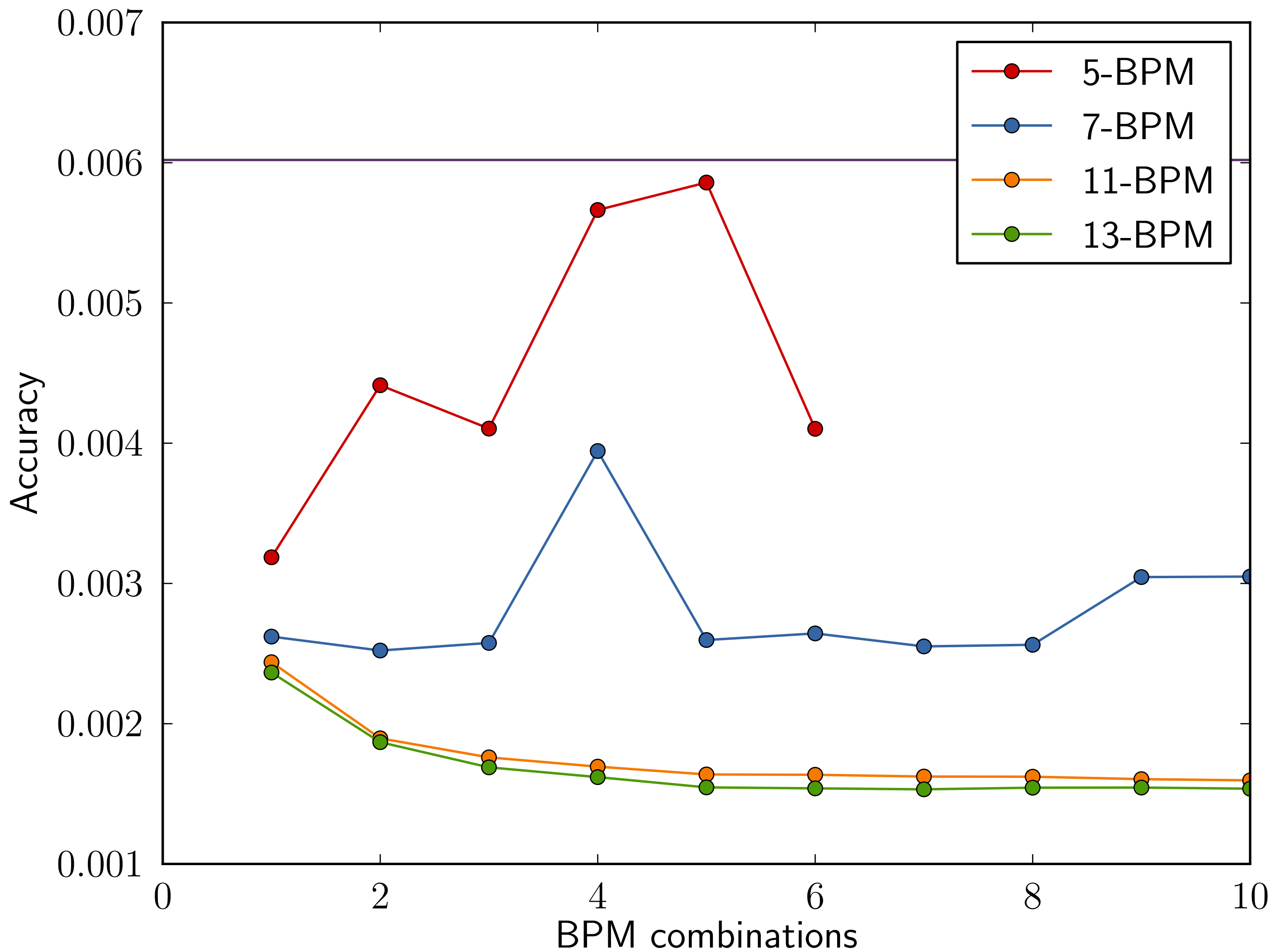
$$\beta = \sum_{i=1}^{N} w_i \beta_i \qquad w_i = \frac{\sum_{k=1}^{N} V_{ik}^{-1}}{\sum_{k=1}^{N}\sum_{j=1}^{N} V_{jk}^{-1}}.$$

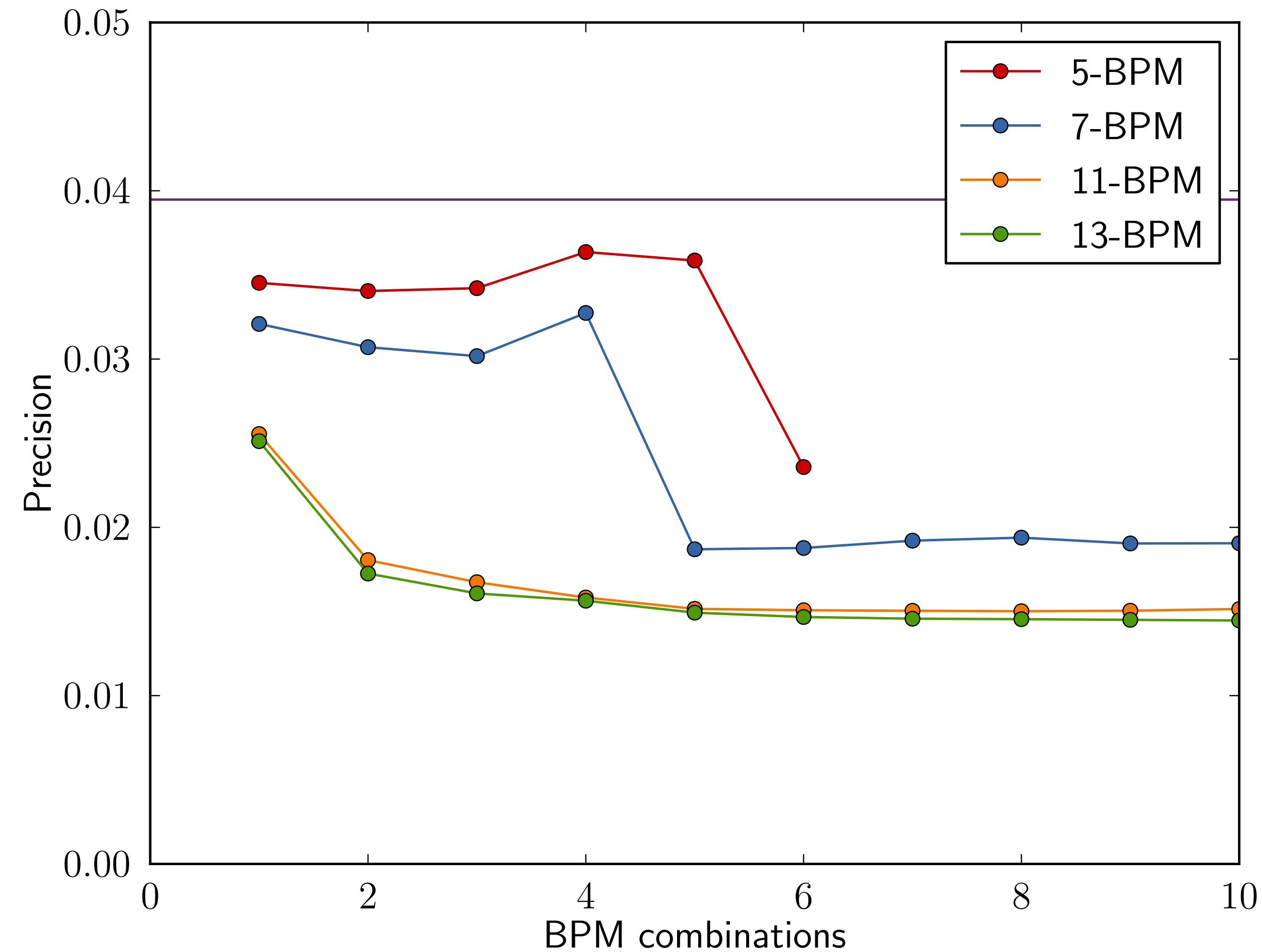
- Computation of systematic covariance matrix time consuming for large ranges of BPMs

- How many BPM combinations should be regarded?

Uncertainty from simulated measurement

- Simulation of optics measurement under realistic conditions
- Scan of using different amount of BPM combinations which are chosen from different range of BPMs
- Accuracy: average relative shift from true value
- Precision: average relative spread





Hardware / Software

- Precision of phase advance depend on length of turn-by-turn data

1. AC-dipole —> increase excitation time
2. BPM —> adapt software for longer acquisition time

➡ Around factor 3 longer (approx. 6000 turns)

- Improved non-linear calibration of BPMs expected

A. Nosych, ’Geometrical non-linearity correction procedure of LHC beam position monitors’

Beta-function during the ramp

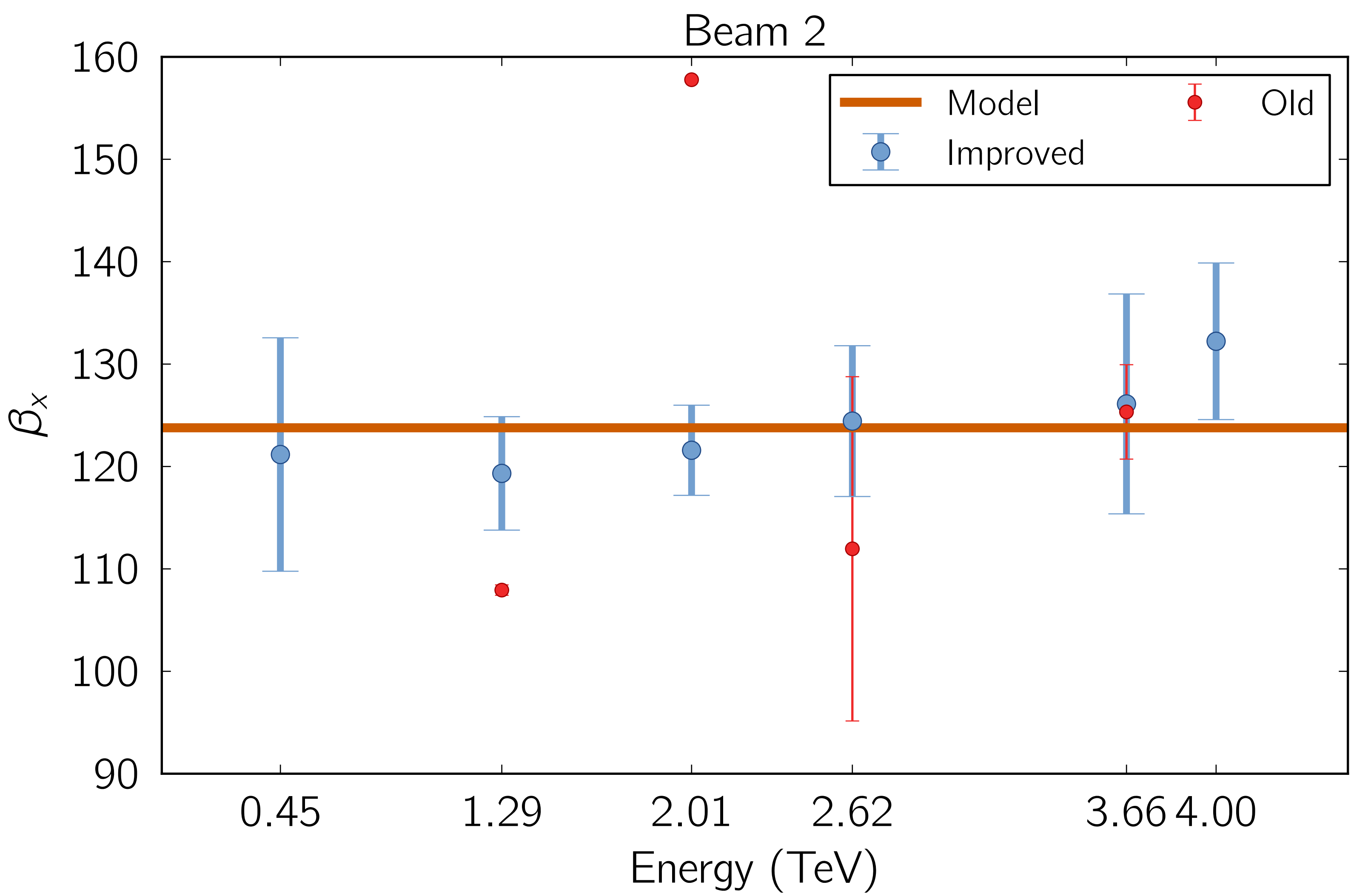
- Propagation to beam wire scanner
- New analytic equations for error propagation

$$\sigma_{\beta_s}^2 = \left(\beta_s \sin(2\phi)\frac{\alpha_0}{\beta_0} + \beta_s \cos(2\phi)\frac{1}{\beta_0}\right)^2 \sigma_{\beta_0}^2 + \left(\beta_s \sin(2\phi)\right)^2 \sigma_{\alpha_0}^2$$

Beta-function during the ramp

- Propagation to beam wire scanner
- New analytic equations for error propagation

$$\sigma^2_{\beta_s} = \left(\beta_s \sin(2\phi)\frac{\alpha_0}{\beta_0} + \beta_s \cos(2\phi)\frac{1}{\beta_0}\right)^2 \sigma^2_{\beta_0} + \left(\beta_s \sin(2\phi)\right)^2 \sigma^2_{\alpha_0}$$



Summary

- LHC run at 6.5 TeV requires more precise optics measurements and corrections
- Full covariance matrix for beta-measurement

➡ Increased resolution when combining more data

- Re-analyzing 2012 data gives better resolution for
  - Beta-function at IP
  - Beta-function during the ramp

Outlook

- Optics commissioning at 6.5 TeV is near
- Systematic errors
  - BPM displacements not yet regarded
  - For calculation of local corrections
  - For propagation of optical parameters to elements

Thank you for your attention!