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Effects of Quad Fringe Fields and Magnetic Interference

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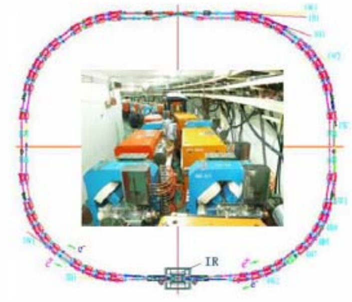
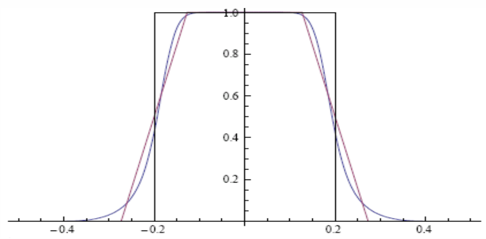
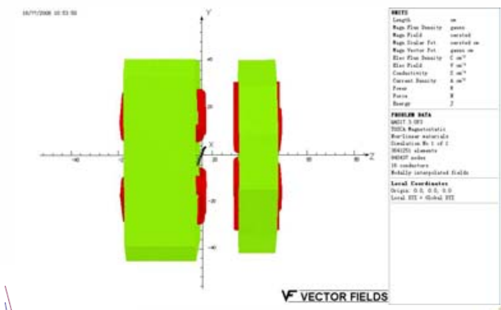
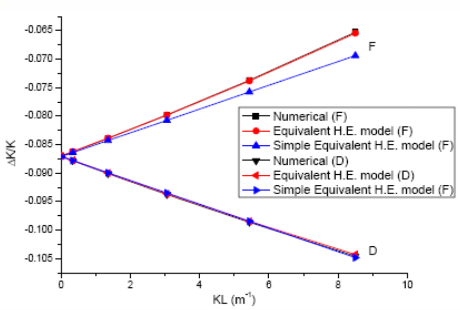
Outline

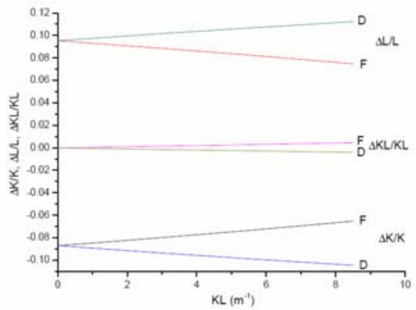
- Introduction
- Estimation of tune shift
  - Quad fringe fields
  - Magnetic interference
- Linear fringe map
- Equivalent hard edge model
- Numerical test
- Summary

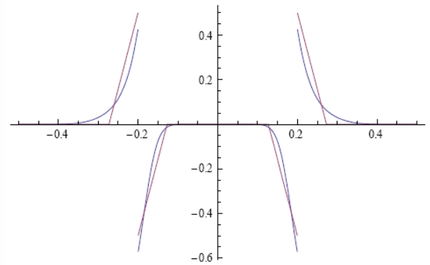












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The BEPCII

- An upgrade project of the Beijing Electron Positron Collider (BEPC)
- A factory-like **double-ring** collider@τ-Charm energy region
- Providing beams to both high energy physics experiments and synchrotron radiation users
- Constructed in the same tunnel for BEPC
- Keeping all previous beamlines unchanged

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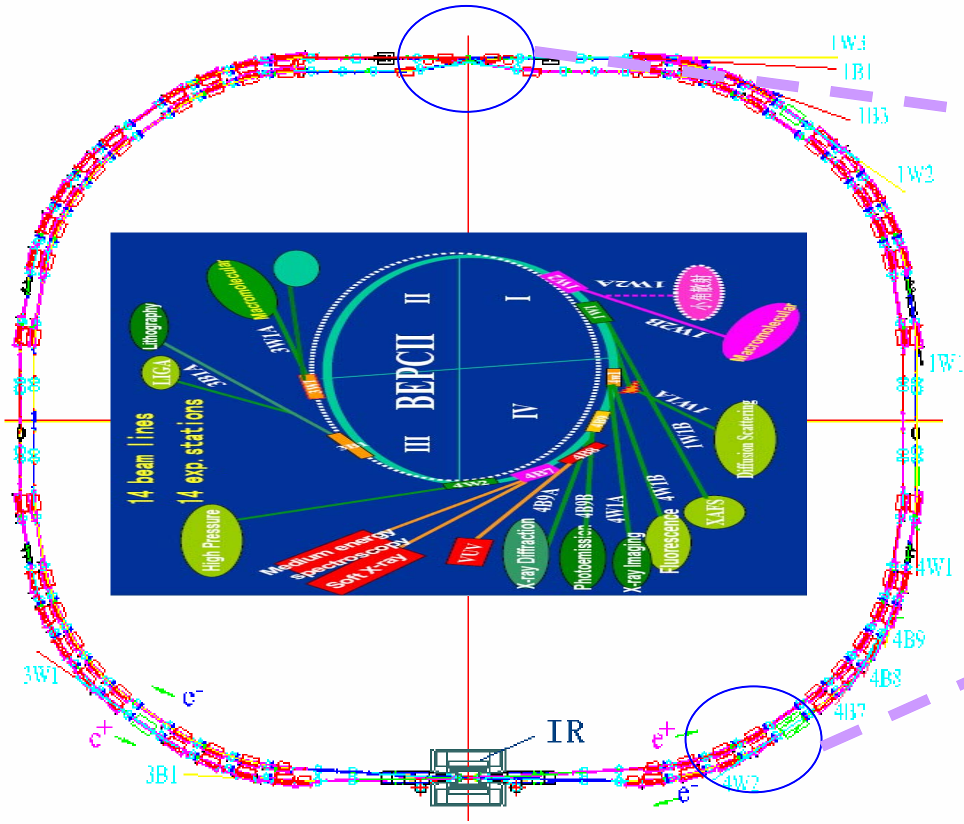
"Three" rings in the same tunnel



BSR: Bepcii Synchrotron Ring
BER: Bepcii Electron Ring
BPR: Bepcii Positron Ring

Main parameters of BEPCII

Parameters	Unit	Colliding mode	SR mode
Operation energy (E)	GeV	1.0–2.1	2.5
Injection energy (E_{inj})	GeV	1.55–1.89	1.89
Circumference (C)	m	237.53	241.13
β^*-function at IP (β_x^* / β_y^*)	cm	100/1.5	
Tunes ($\nu_x/\nu_y/\nu_s$)		6.53/5.58/0.034	7.28/5.18/0.036
Hor. natural emittance (ε_{x0})	mm·mr	0.14 @1.89 GeV	0.12
Damping time ($\tau_x/\tau_y/\tau_e$)		25/25/12.5 @1.89 GeV	12/12/6
RF frequency (f_{rf})	MHz	499.8	499.8
RF voltage per ring (V_{rf})	MV	1.5	1.5~3.0
Bunch number (N_b)		93	
Bunch spacing	m	2.4	
Beam current	mA	910 @1.89 GeV	250
Bunch length (cm) σ_l	cm	~1.5	
Impedance $\|Z/n\|_0$	Ω	~ 0.2	
Crossing angle	mrad	±11	
beam-beam parameter		0.04/0.04	
Beam lifetime	hrs.	3.0	15
luminosity@1.89 GeV	$10^{33}cm^{-2}s^{-1}$	1	

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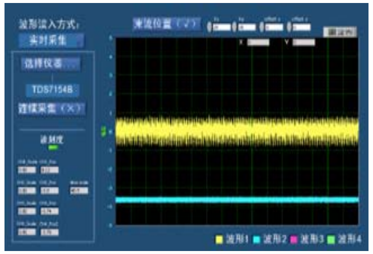
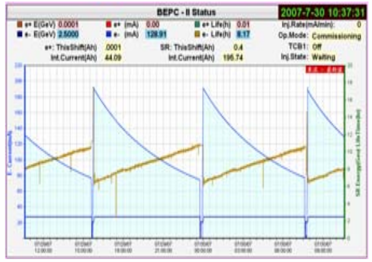
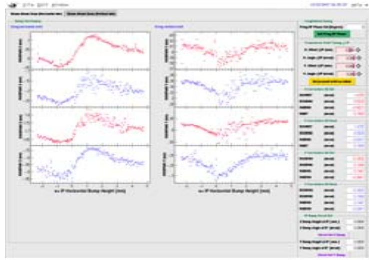
Milestones

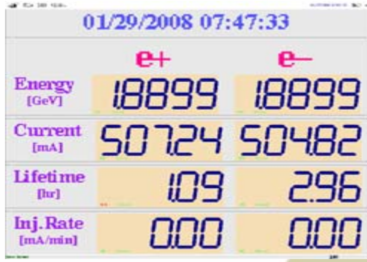


- Mar. 2006: Ring installation started
- Nov. 12, 2006: Commissioning started
- Nov. 18, 2006: Beam accumulated in BSR
- Dec. 25, 2006: Beam provided to SR users
- Feb. 09, 2007: e- beam stored in BER
- Mar. 04, 2007: e+ beam stored in BPR
- Mar. 25, 2007: First collision observed
- Jun. 15, 2007: Second SR run
- Jan. 29, 2008: 500mA*500mA collision

  Luminosity exceeded $1 \times 10^{32} cm^{-2}s^{-1}$







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Critical success factors in the commissioning (personal viewpoints)

- Motivated team
  - Good preparation and management
  - Team work spirit
- Components with very high quality assurance
  - Magnets
  - Power supply
  - Beam instrumentation: BPM, BLM, ...
  - Feedback system
- Control and diagnostics tools with high efficiency
  - BBA and COD correction
  - Optics correction based on response matrix
  - Injection control
  - Collision tuning

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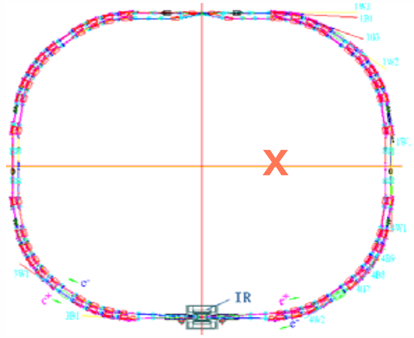
Challenges (personal viewpoints)



- **Complicated design scheme**
  - **Non-symmetric lattice for collision**
  - **Mirror symmetric lattice for SR**
- **Very tight schedule for installation and commissioning**
- **Commissioning**
  - **Hardware fault detection with beam, such as Magnet, BPM, etc.**
  - **Improvised softwares**
  - **Ramping with wigglers (E=1.89GeV->2.5GeV)**
  - **Sensitivity of the lattice to imperfections**
  - **Beam instabilities and intensity limitations**
  - **Lifetime**

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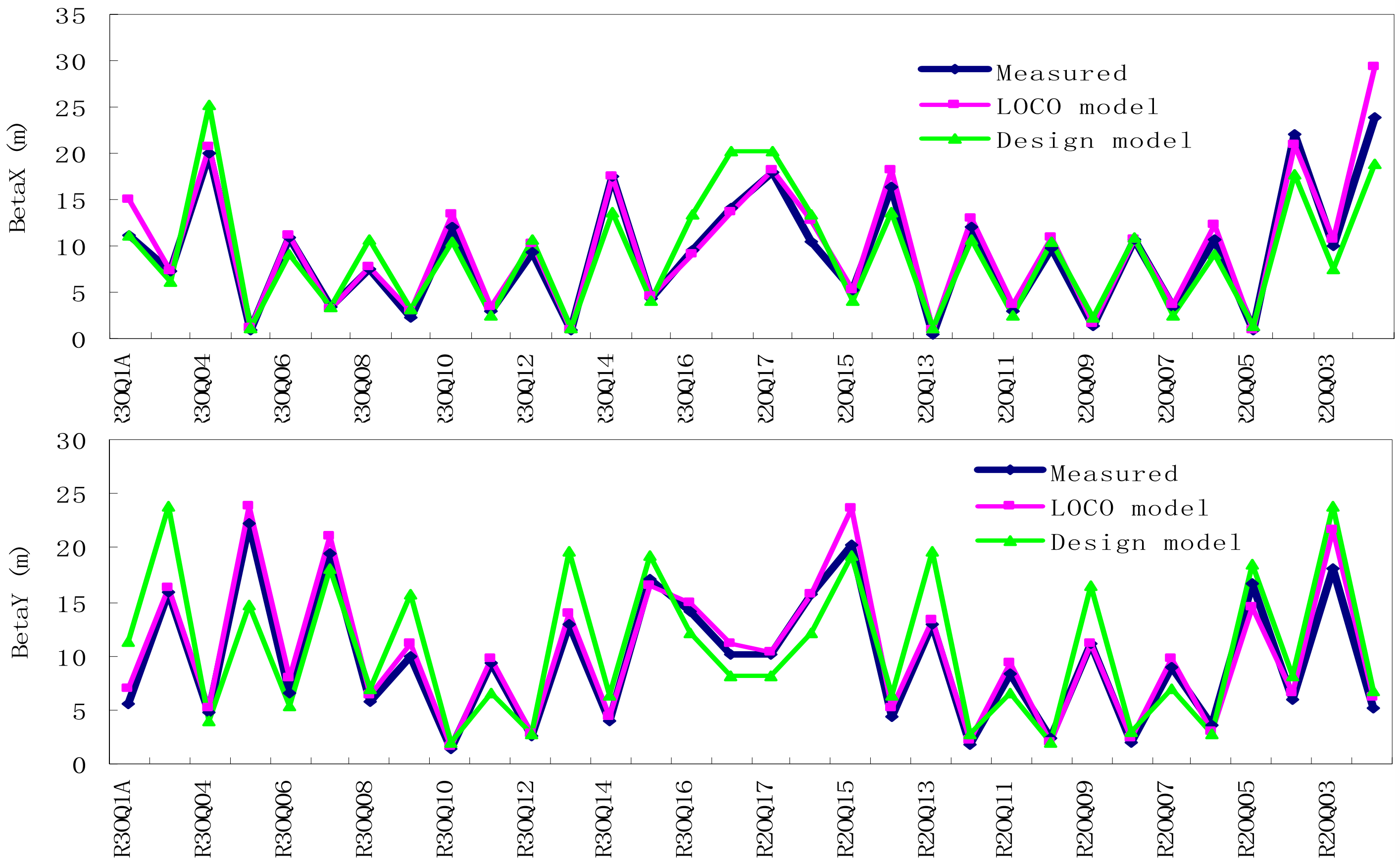
Difference between design and measured optics



Y.Y. Wei

Beta function of BSR (half of the mirror symmetric ring)
LOCO model: fitting model based on response matrix measurement
LOCO: Linear Optics from Closed Orbits

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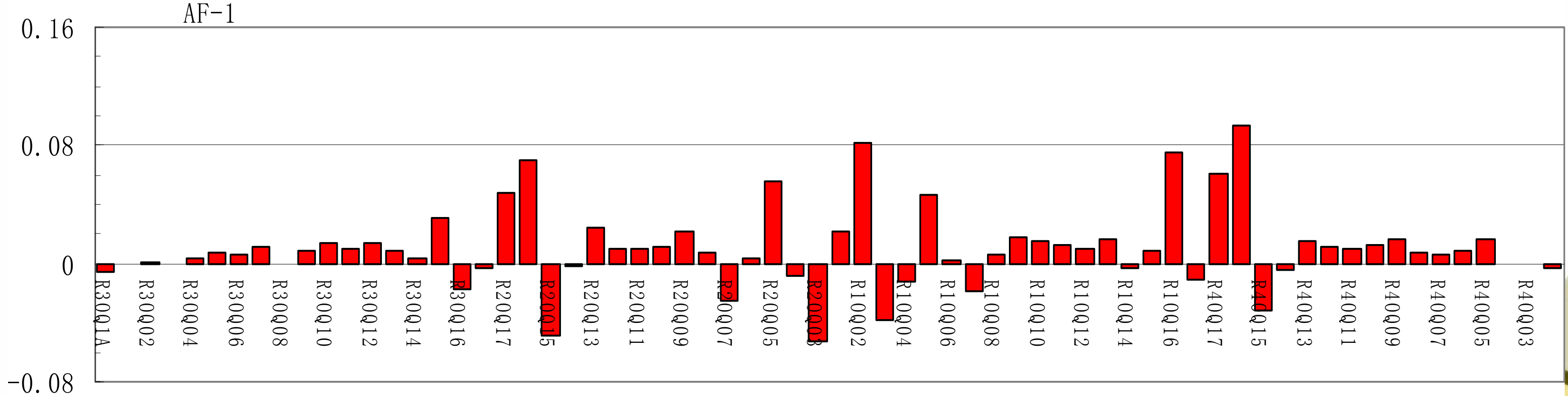
Difference between design and measured optics (cont)

Quadrupole fudge factor AF:
The change of quadrupole strengths to restore the optics

$$\Delta K_1 = K_1 \Delta AF \qquad \Delta AF = AF - 1$$

SR optics BSR_07jan01:
Nominal tunes: (7.27, 5.37) **Measured: (7.205, 5.281)**
Negative tune shift: (-0.065, -0.09)



BSR fudge factors

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Fudge factors indicate an overall positive shift of quad strengths

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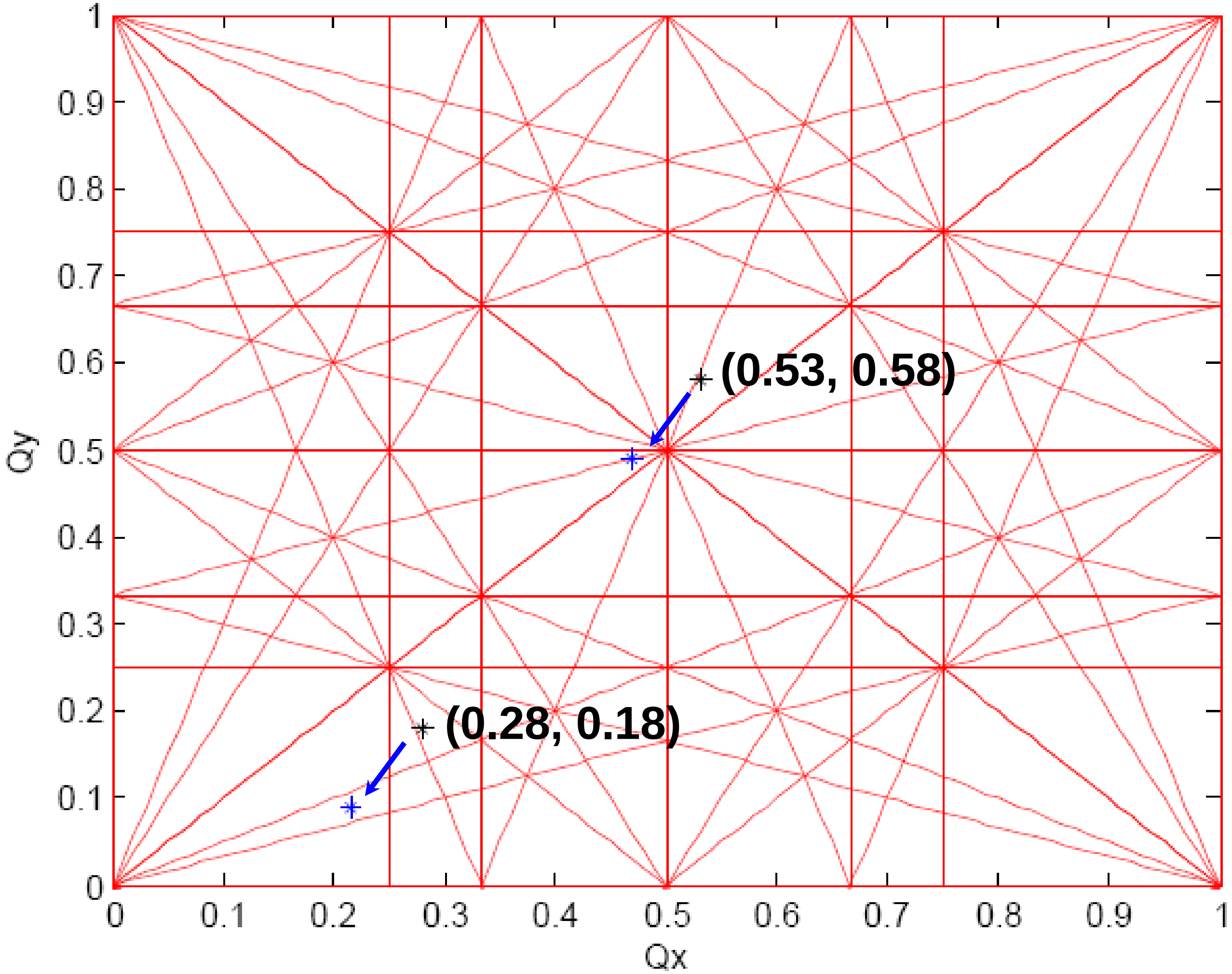
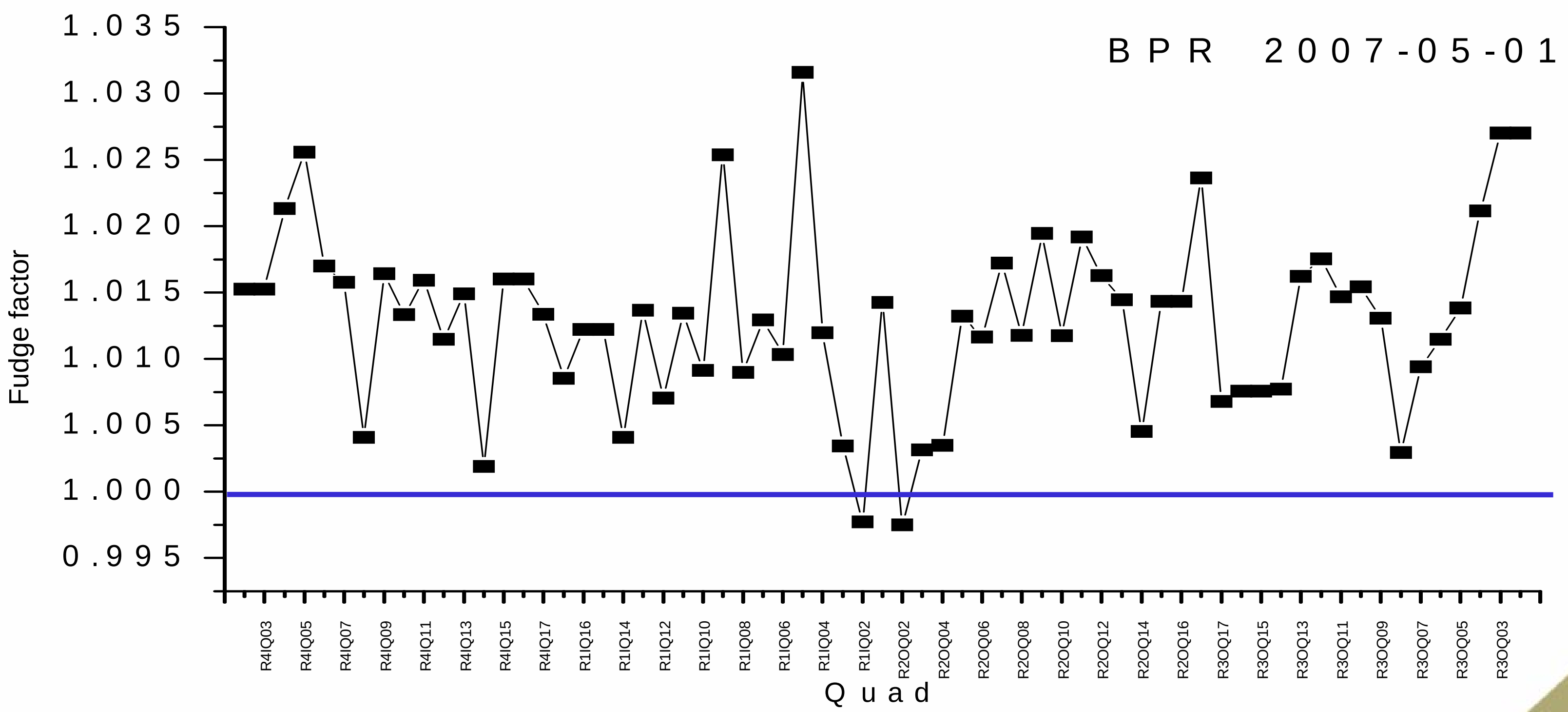
Tuning BSR @ nominal tune (7.28, 5.18) ...
Tuning BER/BPR @ nominal tune (6.53, 5.58)...



Resonances up to 4th order

Difference between design and measured optics (cont)

BPR AF: 1.01~1.02



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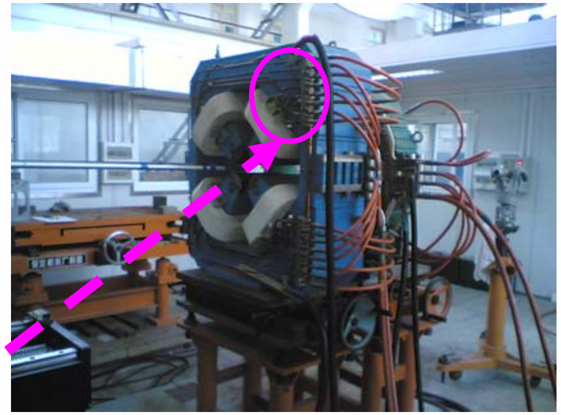
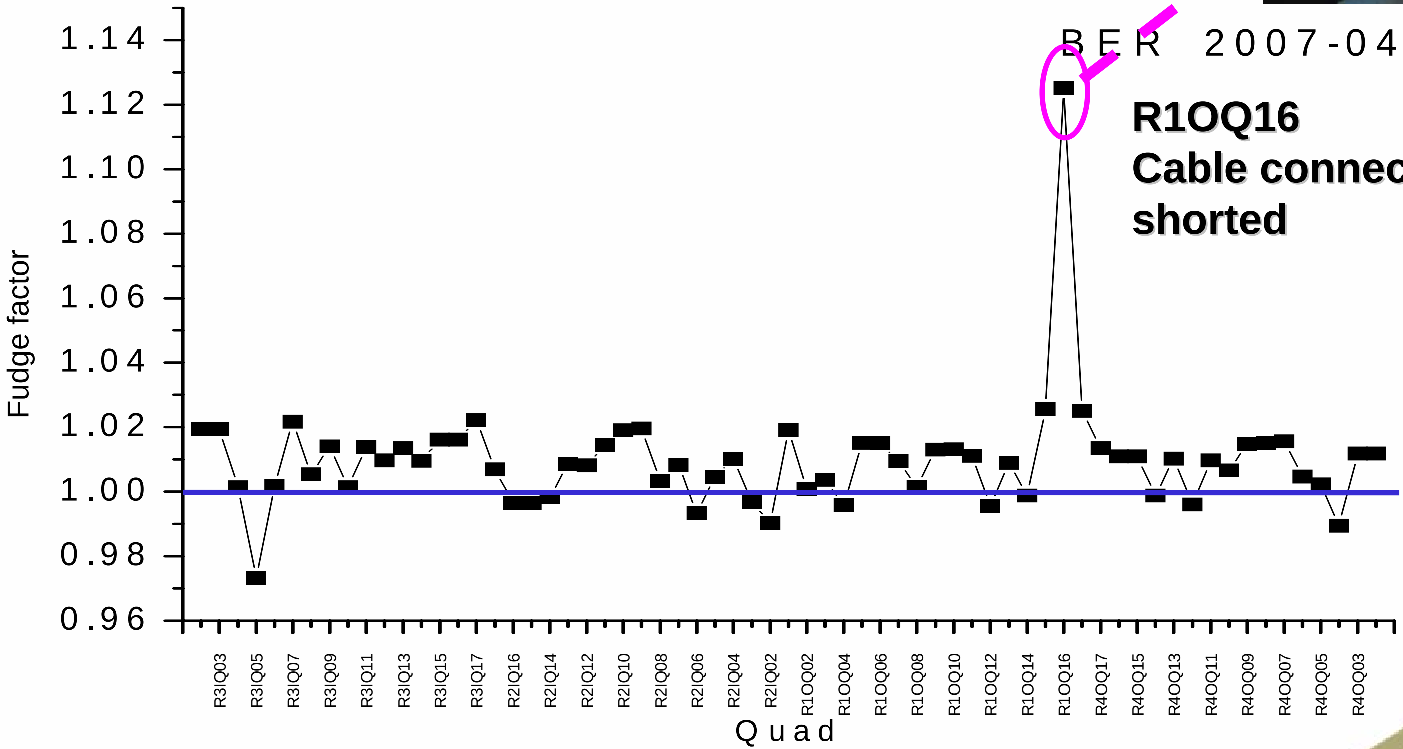
Difference between design and measured optics (cont)



BER AF: 1.01~1.02



R1OQ16
Cable connecting to one pole shorted

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Why negative tune shifts and large fudge factors of 1.01~1.02?

- Fudge factors: to compensate gradient errors from
  - Magnet alignment (random)
  - Magnetic measurement (X)
  - Faulty powering (X)
  - Fringe fields
  - Magnetic interference
- Fringe fields and magnetic interference are reasonable candidates
  - Fringe fields neglected in the design stage but important for small rings
  - Short distances between quads and sexts due to limited spaces

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Quadrupole modeling

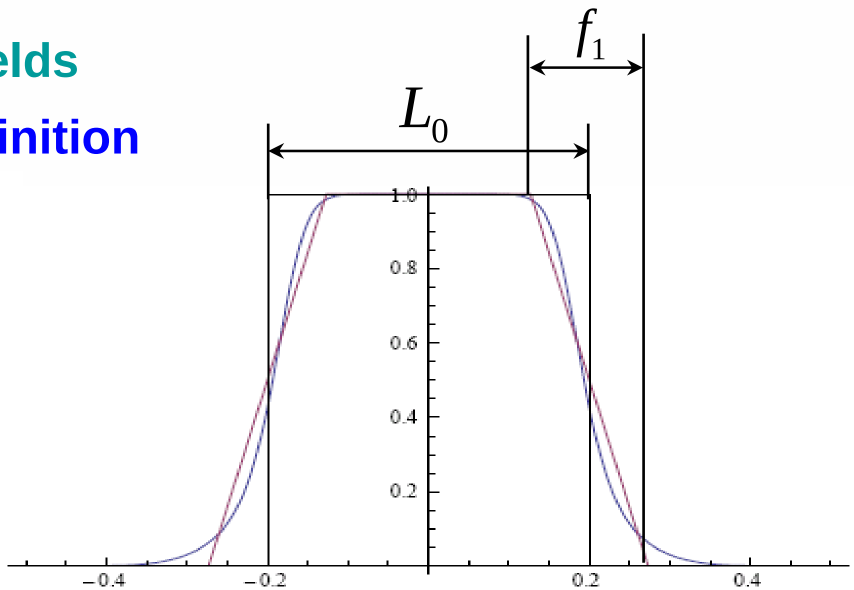
- **Hard edge approximation: simple but unphysical**

$$L_0 = \frac{1}{G_0}\int_{-\infty}^{\infty} G(s)ds$$

- **Trapezoidal fringe model**
  - **Approximation of fringe fields**
  - **Fringe extension: SAD definition**

$$f_1 = \sqrt{24\left|\int_0^{\infty} \frac{\tilde{G}(s)}{G_0}(s-s_0)ds\right|}$$

$$\tilde{G}(s) = \begin{cases} G(s) - G_0 & 0 < s < s_0 \\ G(s) & s \geq s_0 \end{cases}$$



SAD: http://acc-physics.kek.jp/sad

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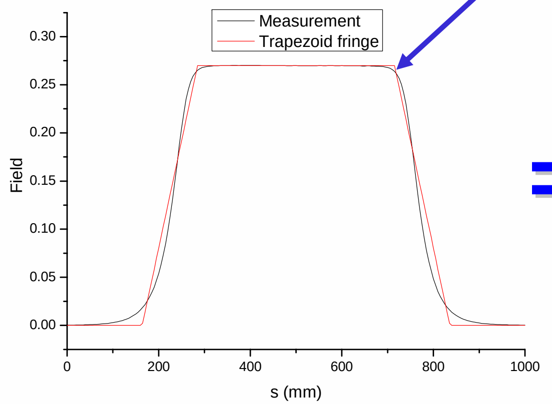
Linear magnet imperfections

$$K(s) = K_0(s) + k(s)$$



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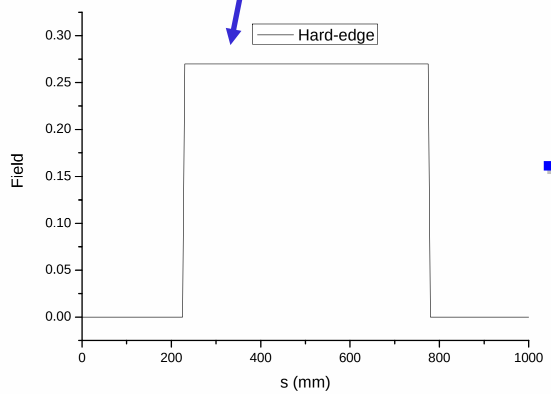

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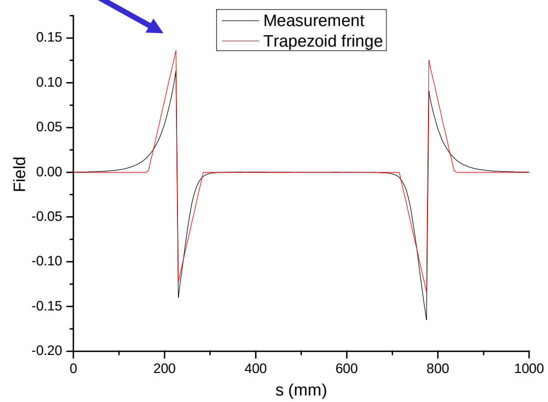

Real magnet **Ideal magnet** **Gradient errors**

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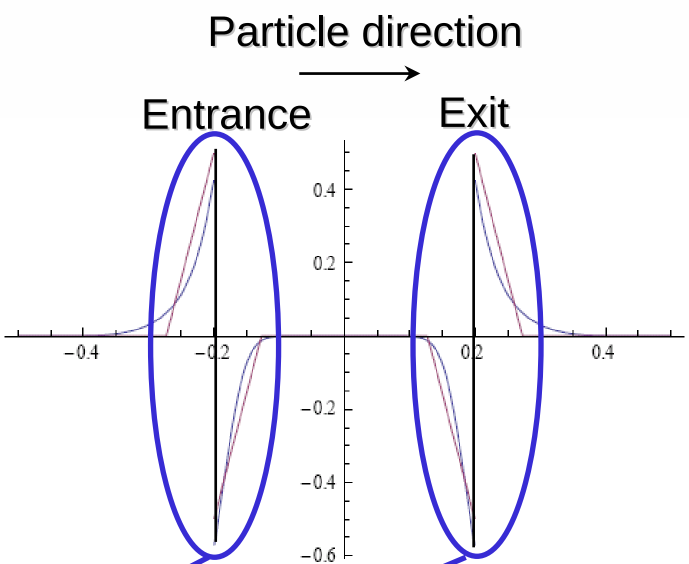
Linear magnet imperfections (cont)

- **The theory**

$$\frac{d^2u}{ds^2} \pm [K_0(s) + k(s)]u = 0$$

$$\Delta\nu = \frac{1}{4\pi}\oint \beta(s)k(s)ds$$

Beta function measurement and manipulation



Gradient error

- **Transformation of Courant-Snyder parameters**

$$\beta(s) = \beta(s_0) - 2\alpha(s_0)(s - s_0) + \frac{1+\alpha^2(s_0)}{\beta(s_0)}(s - s_0)^2$$

$$\Delta\nu = \Delta\nu_{in} + \Delta\nu_{out} = \frac{1}{48\pi}K_0(\alpha_{in} - \alpha_{out})f_1^2$$

Linear magnet imperfections (cont)

- Some conclusions
  - Quad fringe fields do lead to tune shift and beta-beating
  - Tune shift is always negative
  - Tune shift is proportional to quad focusing strength
  - Tune shift is proportional to alpha function, the slope of beta function
  - Tune shift is proportional to the square of fringe extension

$$\Delta\nu = \Delta\nu_{in} + \Delta\nu_{out} = \frac{1}{48\pi} K_0 (\alpha_{in} - \alpha_{out}) f_1^2$$

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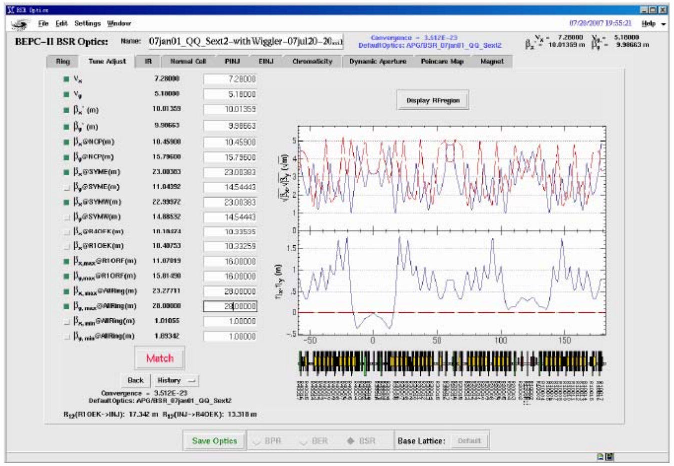
Tune shift computation using SAD

- SAD can treat fringe fields of quads and bends
- Choose SR mode: **BSR_07jan01**
- Totally 7 types of quads in SR ring



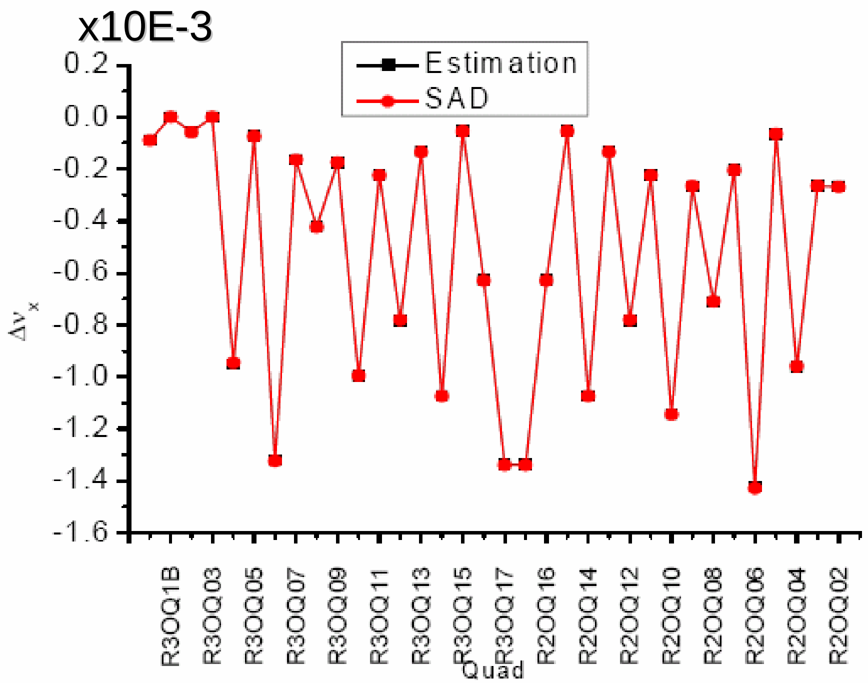
Adjacent to Sexts (105Q) — **Special quads in IR** (Q1A, Q1B, Q2/Q3)

Quad. type	105Q	110Q	160Q	Q1A	Q1B	Q2/Q3	QSR
Effective length (m)	0.31/0.34	0.408	0.646	0.254	0.464	0.548	0.24
Aperture (mm)	52.5	55	80	58	67	52	52.5
Fringe length (f_1) (m)	0.154	0.167	0.238	0.115	0.172	0.133	0.109
Number	44	10	6	2	2	4	1

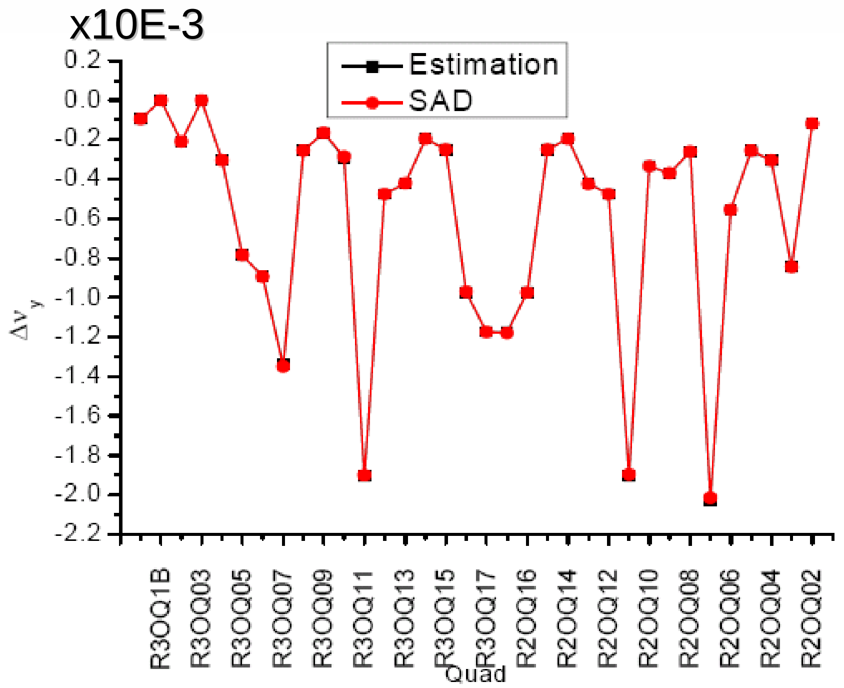
Radius (aperture)

Tune shift computation using SAD (cont)

- Tune shift for each quad (half SR ring)



Horizontal tune



Vertical tune

Estimation: $\Delta\nu = \Delta\nu_{in} + \Delta\nu_{out} = \frac{1}{48\pi} K_0 (\alpha_{in} - \alpha_{out}) f_1^2$

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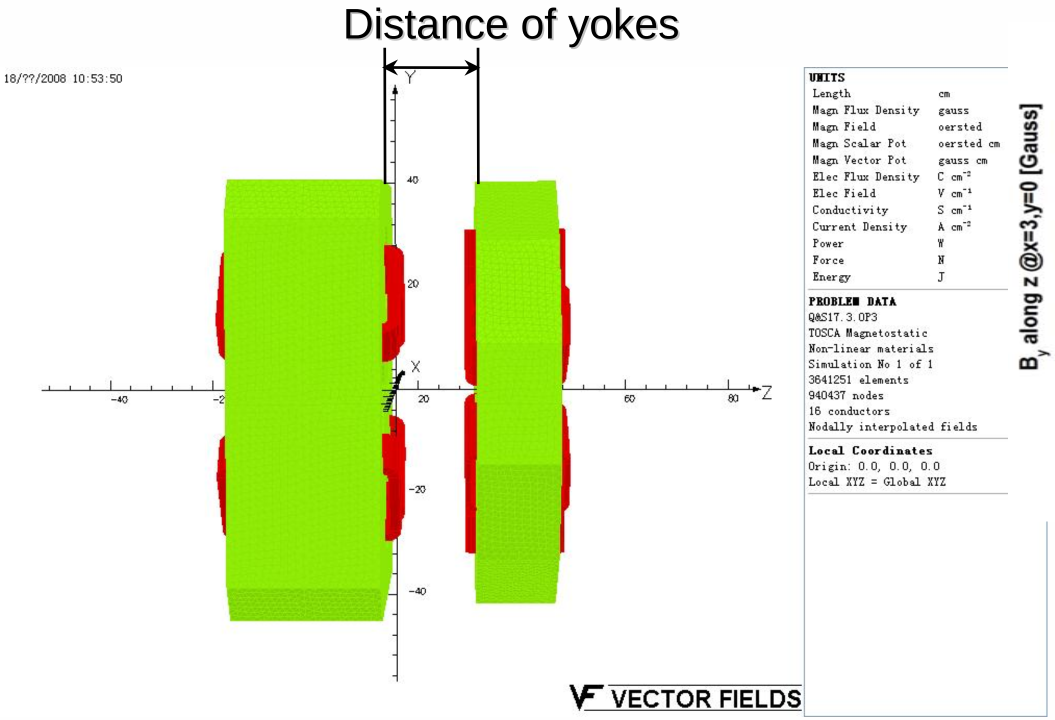
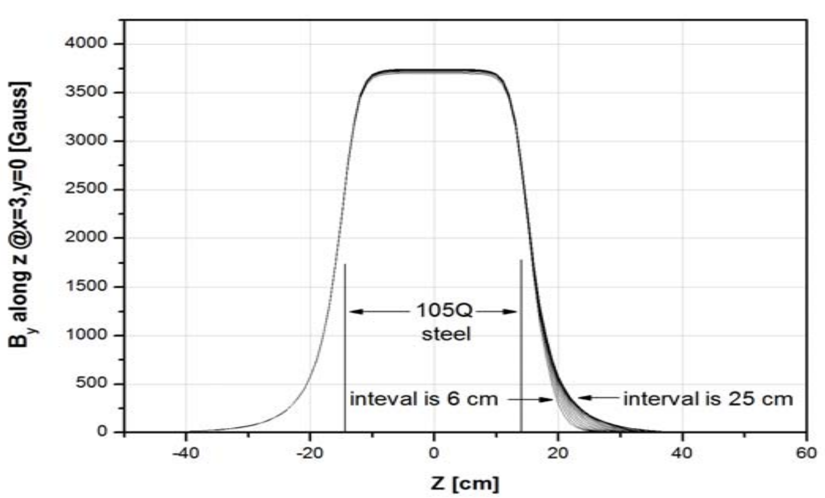
Simulation using OPERA-3D/TOSCA (Y. Chen)

- Quad (105Q) and Sext (130S) assembly
- 36 sexts in a ring divided into 4 groups
- Distance of yokes: from 6cm to 25cm



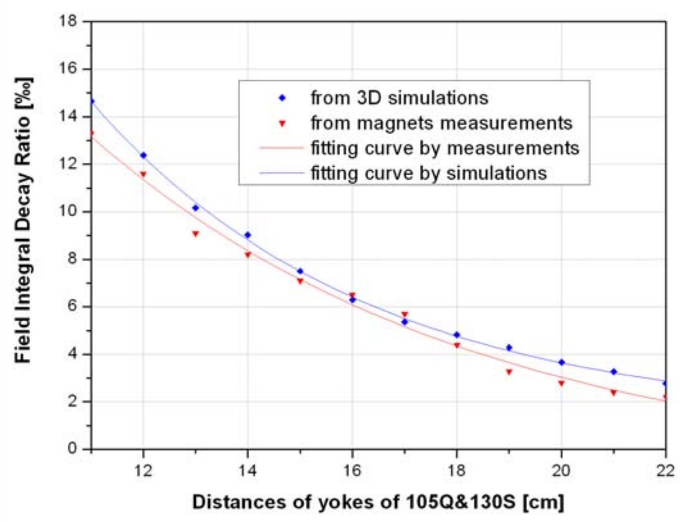


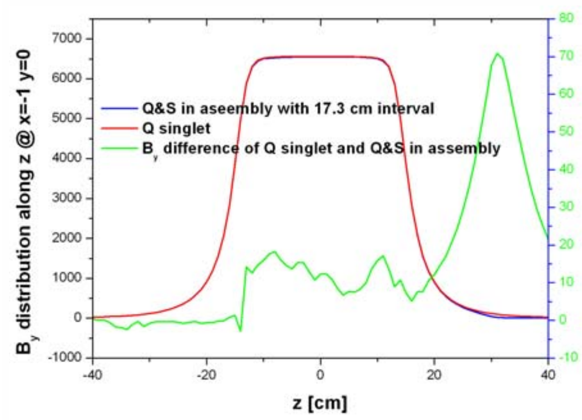
Simulation using OPERA-3D/TOSCA (Y. Chen) (cont)

- Quad field integral decay

  ~0.6% @17.3cm of yoke distance (BEPCII case)
- Simulation agreed well with point-to-point measurement
- Distance of yokes should be larger than **25cm**,

  if decay<0.1% required. Quad aperture radius: **5.25cm**





Measurement

Decay vs. yoke distance
Simulation and measurement

field gradient difference (simulation)
@17.3cm of yoke distance

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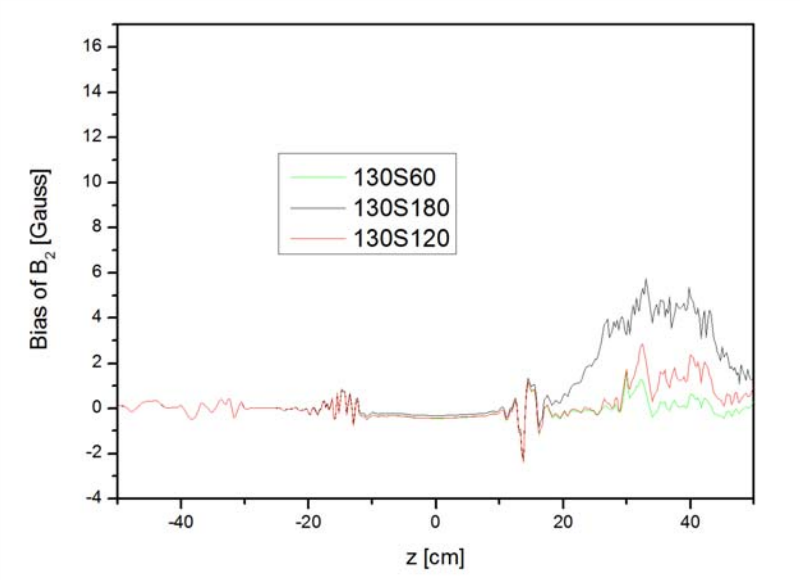
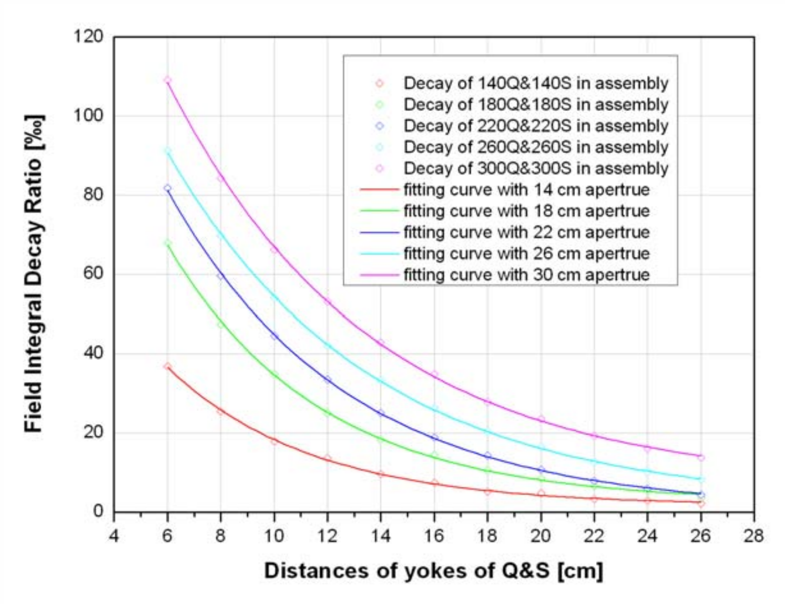
Simulation using OPERA-3D/TOSCA (Y. Chen) (cont)

Large aperture=>long fringe extension=>large field integral decay



Bias of B_2 vs. Sext current



Field integral decay vs. magnet aperture

Tune shift computation using SAD -- summary

- BSR_07jan01
  - Turn on the **bend** fringe fields: (0.0, -0.0226)
  - Turn on the **quad** fringe fields: (-0.0360, -0.0402)
  - Turn on both the bend and quad fringe fields: (-0.0360, -0.0632)
  - Turn on magnetic interference between quads and sexts:
    (-0.028, -0.037)
  - Turn on fringe fields and magnetic interference: (-0.064, -0.102)
  - Measured tune shift with beam: (-0.065, -0.09)
- **Basically, estimated tune shift agreed well with beam based measurement (measured tune shift and fudge factors)**

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Lie Algebra technique

- Hamiltonian system
- Solve the problem analytically
- Perturbation treatment if necessary
- Preserve the semplecticity of the solution

$$\vec{r}'' = f(\vec{r}, \vec{r}') \rightarrow X_i' = [H, X_i]$$

$$X^{(f)} = e^{-:\int_0^t H(X,t')dt':} X^{(i)}$$

Generating function: $F(t) = \int_0^t H(X,t')dt'$

Step 1: s-dependent Hamiltonian in the field of a normal quad

- **Frenet-Serret coordinate system**
- **On-momentum particle**
- **Expand H(s) in polynomials**

$$H(q,p,t) = e\phi + c\sqrt{(\vec{P} - c\vec{A})^2 + m_0^2 c^2}$$

ϕ **: scalar potential**

$\vec{A}$ **: vector potential**

$$H(s) = \frac{1}{2}(p_x^2 + p_y^2) + \frac{1}{2}K(s)(x^2 - y^2) - \frac{1}{4}K'(s)(xp_x + yp_y)(x^2 - y^2)$$
$$-\frac{1}{12}K''(s)(x^4 - y^4) + \frac{1}{32}K'^2(s)(x^4 - y^4)(x^2 - y^2)$$
$$+\frac{1}{48}K'''(s)(xp_x + yp_y)(x^4 - y^4) + \frac{1}{256}K^{(4)}(s)(x^4 - y^4)(x^2 + y^2) + O(X^8)$$

Ref.[1] J. Irwin and C.X. Wang

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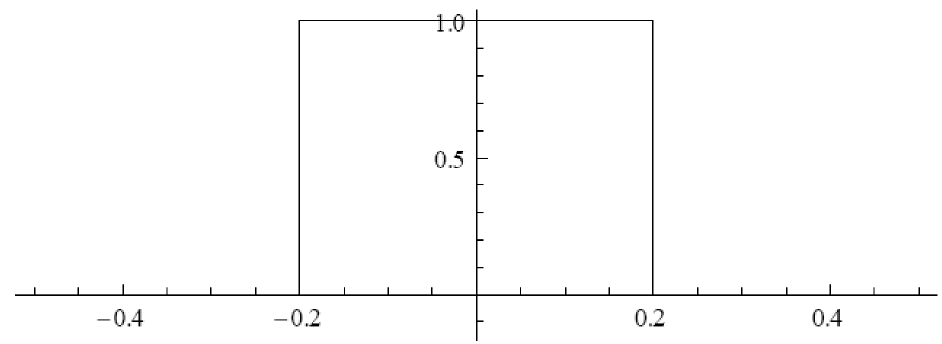
Step 2: Perturbation treatment

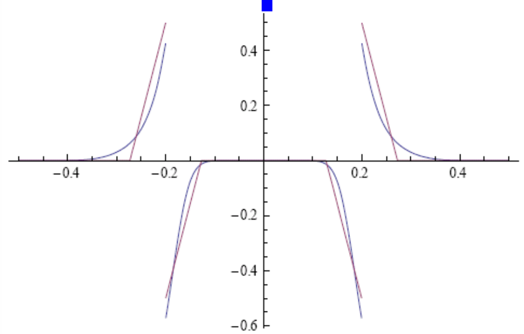
- Solutions for s-dependent Hamiltonian system are hard to be found, even for linear system
- Offer clear physical picture of perturbations
- Evaluate the significancy of fringe field effect

$$H(s) \cong \frac{1}{2}(p_x^2 + p_y^2) + \frac{1}{2}K(s)(x^2 - y^2) = H_0(s) + \widetilde{H}(s)$$

$$H_0(s) = \begin{cases} \frac{1}{2}(p_x^2 + p_y^2) + \frac{1}{2}K_0(x^2 - y^2) & s_1 \le s \le s_0 \\ \frac{1}{2}(p_x^2 + p_y^2) & s_0 < s \le s_2 \end{cases}$$

$$\widetilde{H}(s) = \frac{1}{2}\widetilde{K}(s)(x^2 - y^2) = \begin{cases} \frac{1}{2}[K(s) - K_0](x^2 - y^2) & s_1 \le s \le s_0 \\ \frac{1}{2}K(s)(x^2 - y^2) & s_0 < s \le s_2 \end{cases}$$

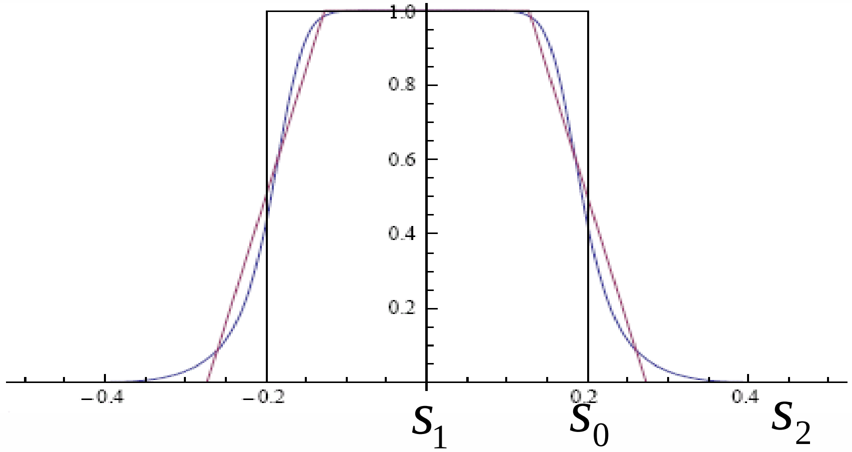




Ref.[1] J. Irwin and C.X. Wang

Step 3: Linear map (from quad center to far right side)

- Map of ideal quad
- Map of fringe
- Map of drift



$$M(s_1 \to s_2) = R_-(s_1 \to s_0) R_+(s_0 \to s_2)$$

$$R_-(s_1 \to s_0) = M_Q(s_1 \to s_0) e^{:f_2^-:}$$

$$R_+(s_0 \to s_2) = e^{:f_2^+:} M_{drift}(s_0 \to s_2)$$

$$R_f = e^{:f_2^-:} e^{:f_2^+:} = e^{:f_2:}$$

$$M_Q(s_1 \to s) \leftrightarrow \begin{bmatrix} \cos\sqrt{K_0}s & \frac{\sin\sqrt{K_0}s}{\sqrt{K_0}} & 0 & 0 \\ -\sqrt{K_0}\sin\sqrt{K_0}s & \cos\sqrt{K_0}s & 0 & 0 \\ 0 & 0 & \cosh\sqrt{K_0}s & \frac{\sinh\sqrt{K_0}s}{\sqrt{K_0}} \\ 0 & 0 & \sqrt{K_0}\sinh\sqrt{K_0}s & \cosh\sqrt{K_0}s \end{bmatrix}$$

$$M_{drift}(s_0 \to s) \leftrightarrow \begin{bmatrix} 1 & s - s_0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & s - s_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ref.[1] J. Irwin and C.X. Wang

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Step 4: Generating functions

- s-dependent dynamical variables: Taylor expansion
- Assumption: **fringe region is short**
- 2rd BCH formula is enough

$$X = [x, p_x, y, p_y]^T$$

$$f_2^- = -\int_{s_1}^{s_0} \overline{H}(s)ds + \frac{1}{2}\int_{s_1}^{s_0} ds \int_{s}^{s_0} ds'[\overline{H}(s), \overline{H}(s')]$$

$$f_2^- = -\int_{s_0}^{s_2} \overline{H}(s)ds + \frac{1}{2}\int_{s_0}^{s_2} ds \int_{s}^{s_2} ds'[\overline{H}(s), \overline{H}(s')]$$

$$\overline{H}(s) = \begin{cases} \widetilde{H}(s, M_Q(s_0 \to s)X) & s_1 \le s \le s_0 \\ \widetilde{H}(s, M_{drift}(s_0 \to s)X) & s_0 \le s \le s_2 \end{cases}$$

$$\widetilde{H}(s) = \frac{1}{2}\widetilde{K}(s)(x^2 - y^2) = \begin{cases} \frac{1}{2}[K(s) - K_0](x^2 - y^2) & s_1 \le s \le s_0 \\ \frac{1}{2}K(s)(x^2 - y^2) & s_0 < s \le s_2 \end{cases}$$

$$M_Q(s_1 \to s) \leftrightarrow \begin{bmatrix} \cos\sqrt{K_0}s & \frac{\sin\sqrt{K_0}s}{\sqrt{K_0}} & 0 & 0 \\ -\sqrt{K_0}\sin\sqrt{K_0}s & \cos\sqrt{K_0}s & 0 & 0 \\ 0 & 0 & \cosh\sqrt{K_0}s & \frac{\sinh\sqrt{K_0}s}{\sqrt{K_0}} \\ 0 & 0 & \sqrt{K_0}\sinh\sqrt{K_0}s & \cosh\sqrt{K_0}s \end{bmatrix}$$

$$\downarrow$$

$$M_Q(s_0 \to s) \leftrightarrow \begin{bmatrix} 1 - \frac{1}{2}K_0(s)\Delta s^2 & \Delta s - \frac{1}{6}K_0(s)\Delta s^3 & 0 & 0 \\ -K_0\Delta s & 1 - \frac{1}{2}K_0(s)\Delta s^2 & 0 & 0 \\ 0 & 0 & 1 + \frac{1}{2}K_0(s)\Delta s^2 & \Delta s + \frac{1}{6}K_0(s)\Delta s^3 \\ 0 & 0 & K_0\Delta s & 1 + \frac{1}{2}K_0(s)\Delta s^2 \end{bmatrix}$$

Step 4: Generating functions (cont)

- **Represented by fringe field integrals (FFI)**

$$f_2^- \cong -\frac{1}{2}I_0^-(x^2-y^2) - I_1^-(xp_x - yp_y) - \frac{1}{2}I_2^-(p_x^2-p_y^2) + \frac{1}{2}K_0 I_2^-(x^2+y^2) + \frac{2}{3}K_0 I_3^-(xp_x - yp_y) + \frac{1}{2}\Lambda_2^-(x^2+y^2)$$

$$f_2^+ \cong -\frac{1}{2}I_0^+(x^2-y^2) - I_1^+(xp_x - yp_y) - \frac{1}{2}I_2^+(p_x^2-p_y^2) + \frac{1}{2}\Lambda_2^+(x^2+y^2)$$

$$f_2 \cong f_2^- + f_2^+ + \frac{1}{2}[f_2^-, f_2^+]$$

$$\approx -(I_1^- + I_1^+)(xp_x - yp_y) - \frac{I_2^- + I_2^+}{2}(p_x^2 - p_y^2) + \frac{K_0 I_2^-}{2}(x^2+y^2) + \frac{2K_0 I_3^-}{3}(xp_x + yp_y) + \frac{\Lambda_2^- + \Lambda_2^+}{2}(x^2+y^2) - \frac{1}{2}I_0^+(I_1^- + I_1^+)(x^2+y^2) - \frac{1}{2}I_0^+(I_2^- + I_2^+)(xp_x + yp_y)$$

$$I_0^- + I_0^+ \equiv 0$$

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Fringe field integrals

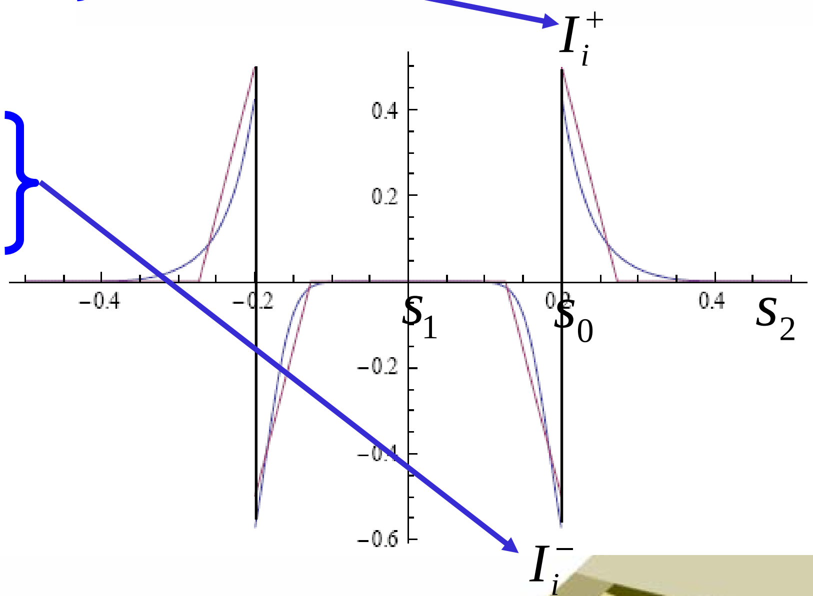
- Anti-symmetric assumption not necessary

$$I_0^- = \int_{s_1}^{s_0} \widetilde{K}(s)ds \qquad I_1^- = \int_{s_1}^{s_0} \widetilde{K}(s)(s-s_0)ds$$

$$I_2^- = \int_{s_1}^{s_0} \widetilde{K}(s)(s-s_0)^2 ds \qquad I_3^- = \int_{s_1}^{s_0} \widetilde{K}(s)(s-s_0)^3 ds$$

$$I_0^+ = \int_{s_0}^{s_2} \widetilde{K}(s)ds \qquad I_1^+ = \int_{s_0}^{s_2} \widetilde{K}(s)(s-s_0)ds$$

$$I_2^+ = \int_{s_0}^{s_2} \widetilde{K}(s)(s-s_0)^2 ds \qquad I_3^+ = \int_{s_0}^{s_2} \widetilde{K}(s)(s-s_0)^3 ds$$



$$\Lambda_2^- = \int_{s_1}^{s_0} ds \int_{s}^{s_0} ds' K(s)K(s')(s'-s)$$

$$\Lambda_2^+ = \int_{s_0}^{s_2} ds \int_{s}^{s_2} ds' K(s)K(s')(s'-s)$$

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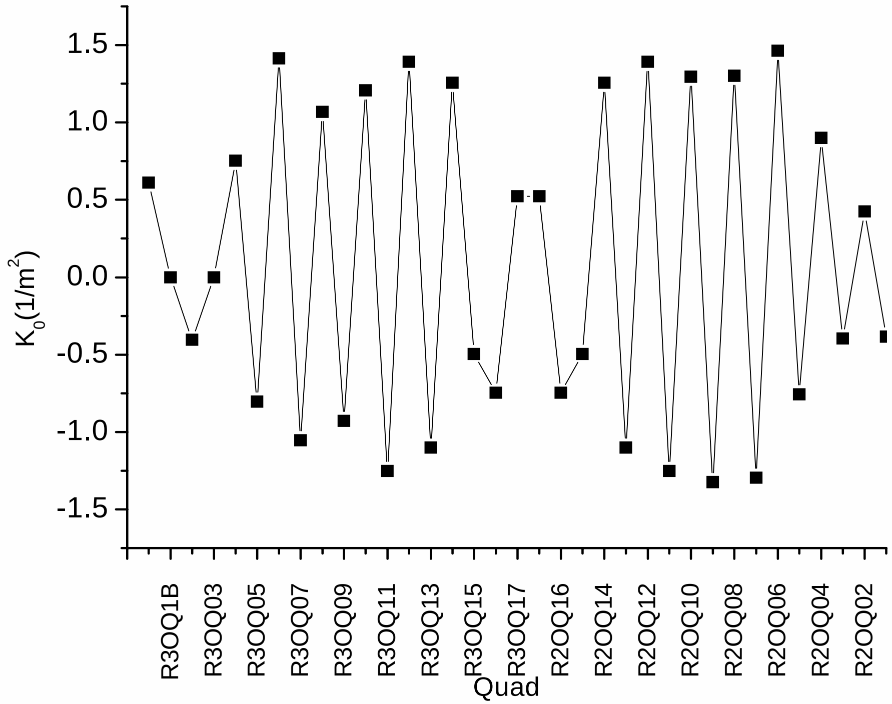
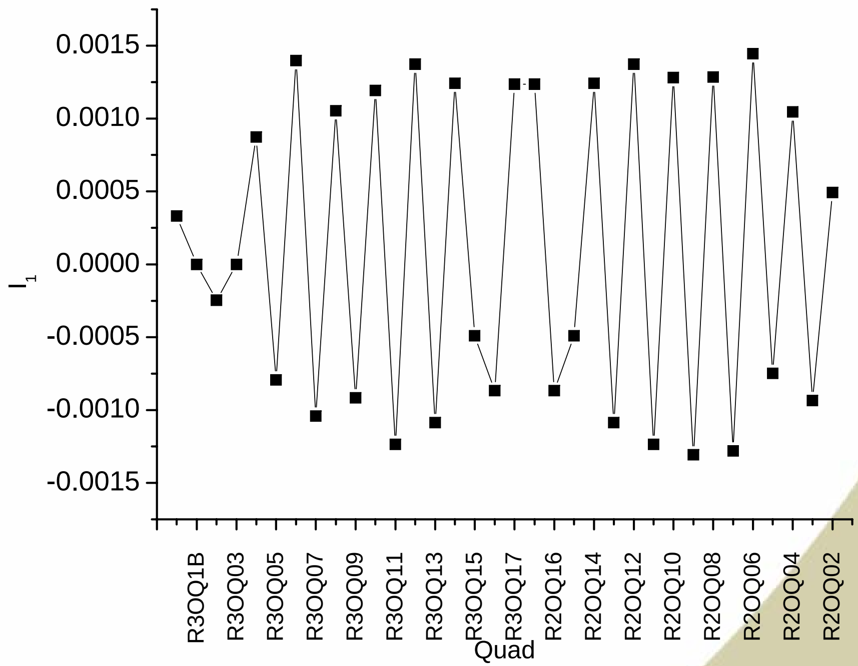
Fringe field integrals (cont)

Case of BEPCII SR ring





$$I_1 = I_1^- + I_1^+ = \frac{1}{24} K_0 f_1^2$$

Proportional to square of fringe extension

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Step 5: Correction matrix of fringe field

- Linear fringe effects
  - Scale change
  - Drift
  - Quadrupole

$$f_2 \cong f_2^- + f_2^+ + \frac{1}{2}[f_2^-, f_2^+]$$

$$\approx -(I_1^- + I_1^+)(xp_x - yp_y) - \frac{I_2^- + I_2^+}{2}(p_x^2 - p_y^2)$$

$$+ \frac{K_0 I_2^-}{2}(x^2 + y^2) + \frac{2K_0 I_3^-}{3}(xp_x + yp_y) + \frac{\Lambda_2^- + \Lambda_2^+}{2}(x^2 + y^2)$$

$$-\frac{1}{2} I_0^+ (I_1^- + I_1^+)(x^2 + y^2) - \frac{1}{2} I_0^+ (I_2^- + I_2^+)(xp_x + yp_y)$$

$$MR_x = \begin{bmatrix} 1 & 0 \\ J_3 & 1 \end{bmatrix} \begin{bmatrix} 1 & J_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e^{J_1} & 0 \\ 0 & e^{-J_1} \end{bmatrix}$$

$$J_1 = (I_1^- + I_1^+) - \frac{2K_0 I_3^-}{3} + \frac{1}{2} I_0^+ (I_2^- + I_2^+)$$

$$J_2 = I_2^- + I_2^+ \quad J_3 = K_0 I_2^- + (\Lambda_2^- + \Lambda_2^+) - I_0^+ (I_1^- + I_1^+)$$

SAD linear fringe:

$$MR_x(\mathrm{SAD}) = \begin{bmatrix} 1 & J_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e^{I_1} & 0 \\ 0 & e^{-I_1} \end{bmatrix}$$

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First derivation from linear fringe map

Corrected focal length

$$f^{-1} = -T_{21} \cong \sqrt{K_0}\sin(\sqrt{K_0}L_0)e^{-2J_1} - 2J_3\cos(\sqrt{K_0}L_0)$$

Recalculation of tune shift

$$MR_x = \begin{bmatrix} 1 & 0 \\ J_3 & 1 \end{bmatrix}\begin{bmatrix} 1 & J_2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} e^{J_1} & 0 \\ 0 & e^{-J_1} \end{bmatrix}$$

$$\begin{bmatrix} \cos(2\pi Q_0)+\alpha\sin(2\pi Q_0) & \beta\sin(2\pi Q_0) \\ -\frac{1+\alpha^2}{\beta}\sin(2\pi Q_0) & \cos(2\pi Q_0)-\alpha\sin(2\pi Q_0) \end{bmatrix}_{out} MR_x$$

$$= \begin{bmatrix} \cos(2\pi Q)+\alpha\sin(2\pi Q) & \beta\sin(2\pi Q) \\ -\frac{1+\alpha^2}{\beta}\sin(2\pi Q) & \cos(2\pi Q)-\alpha\sin(2\pi Q) \end{bmatrix}_{out}$$

$$\Delta Q_{out} = Q - Q_0 \cong -\frac{\alpha_{out}}{2\pi}J_1 + \frac{1}{4\pi}\beta_{out}J_3 - \frac{1+\alpha_{out}^2}{4\pi\beta_{out}}J_2 \cong -\frac{\alpha_{out}K_0 f_1^2}{48\pi}$$

$$J_1 \approx I_1 = \frac{1}{24}K_0 f_1^2$$

Outline

- Introduction
- Estimation of tune shift
  - Quad fringe fields
  - Magnetic interference
- Linear fringe map
- Equivalent hard edge model
- Numerical test
- Summary

Second derivation from linear fringe map – equiv. H. E. model

- **Two parameters for symmetric longitudinal field distribution**
  - **Equivalent strength**
  - **Equivalent length**

$$M(-s_2 \to s_2) = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} 1 & \lambda \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\sqrt{K_{eq}}L_{eq} & \dfrac{\sin\sqrt{K_{eq}}L_{eq}}{\sqrt{K_{eq}}} \\ -\sqrt{K_{eq}}\sin\sqrt{K_{eq}}L_{eq} & \cos\sqrt{K_{eq}}L_{eq} \end{bmatrix} \begin{bmatrix} 1 & \lambda \\ 0 & 1 \end{bmatrix}$$

$$=$$

$$M(-s_2 \to s_2) = \begin{bmatrix} 1 & \lambda \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \Delta L \\ 0 & 1 \end{bmatrix} MR_x \begin{bmatrix} \cos\sqrt{K_0}L_0 & \dfrac{\sin\sqrt{K_0}L_0}{\sqrt{K_0}} \\ -\sqrt{K_0}\sin\sqrt{K_0}L_0 & \cos\sqrt{K_0}L_0 \end{bmatrix} ML_x \begin{bmatrix} 1 & \Delta L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \lambda \\ 0 & 1 \end{bmatrix}$$

$$T_{11} = T_{22} \qquad T_{11} \cdot T_{22} - T_{12} \cdot T_{21} = 1$$

Equivalent strength and length in the focusing plane

$$K_{eq}^{F} \approx K_0[1-(\frac{6A}{L_0^2}-\frac{54A^2}{L_0^4}+\frac{12B}{L_0^3}+\ldots)+(\frac{2A}{5}+\ldots)K_0+O(K_0^2)]$$

$$L_{eq}^{F} \approx L_0[1+(\frac{6A}{L_0^2}-\frac{18A^2}{L_0^4}+\frac{12B}{L_0^3}+\ldots)-(\frac{2A}{5}+\ldots)K_0+O(K_0^2)]$$

$$K_{eq}^{F}L_{eq}^{F} \approx K_0L_0[1+(-\frac{3A^2}{L_0^2}+\frac{2B}{L_0}-\frac{2C}{L_0}+\ldots)K_0+O(K_0^2)]$$

$$2J_1 = A\cdot K_0 + D\cdot K_0^2 \quad J_2 = B\cdot K_0 \quad J_3 = C\cdot K_0^2$$

A, *B*, *C*, and *D*: parameters on fringe profile

$$I_0^- = \int_{s_1}^{s_0} \widetilde{K}(s)ds \qquad I_1^- = \int_{s_1}^{s_0} \widetilde{K}(s)(s-s_0)ds$$

$$I_2^- = \int_{s_1}^{s_0} \widetilde{K}(s)(s-s_0)^2 ds \qquad I_3^- = \int_{s_1}^{s_0} \widetilde{K}(s)(s-s_0)^3 ds \qquad \propto K_0$$

$$\Lambda_2^- = \int_{s_1}^{s_0} ds \int_{s}^{s_0} ds' K(s)K(s')(s'-s)$$

$$\Lambda_2^+ = \int_{s_0}^{s_2} ds \int_{s}^{s_2} ds' K(s)K(s')(s'-s) \qquad \propto K_0^2$$

$$J_1 = (I_1^- + I_1^+) - \frac{2K_0 I_3^-}{3} + \frac{1}{2} I_0^+ (I_2^- + I_2^+)$$

$$J_2 = I_2^- + I_2^+ \qquad J_3 = K_0 I_2^- + (\Lambda_2^- + \Lambda_2^+) - I_0^+ (I_1^- + I_1^+)$$

$$2J_1 = A \cdot K_0 + D \cdot K_0^2 \qquad J_2 = B \cdot K_0 \qquad J_3 = C \cdot K_0^2$$

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Equivalent strength and length in the defocusing plane

- Easily derived from results of focusing plane using analogy

$$\begin{bmatrix} \cosh\sqrt{K}L & \dfrac{\sinh\sqrt{K}L}{\sqrt{K}} \\ \sqrt{K}\sinh\sqrt{K}L & \cosh\sqrt{K}L \end{bmatrix} = \begin{bmatrix} \cos\sqrt{-K}L & \dfrac{\sin\sqrt{-K}L}{\sqrt{-K}} \\ -\sqrt{-K}\sin\sqrt{-K}L & \cos\sqrt{-K}L \end{bmatrix}$$

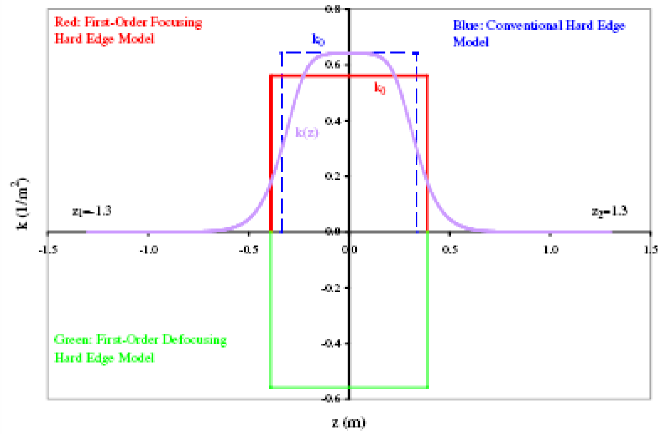
$$K_{eq}^{D} \approx K_0[1-(\frac{6A}{L_0^2}-\frac{54A^2}{L_0^4}+\frac{12B}{L_0^3}+\ldots)-(\frac{2A}{5}+\ldots)K_0+O(K_0^2)]$$

$$L_{eq}^{D} \approx L_0[1+(\frac{6A}{L_0^2}-\frac{18A^2}{L_0^4}+\frac{12B}{L_0^3}+\ldots)+(\frac{2A}{5}+\ldots)K_0+O(K_0^2)]$$

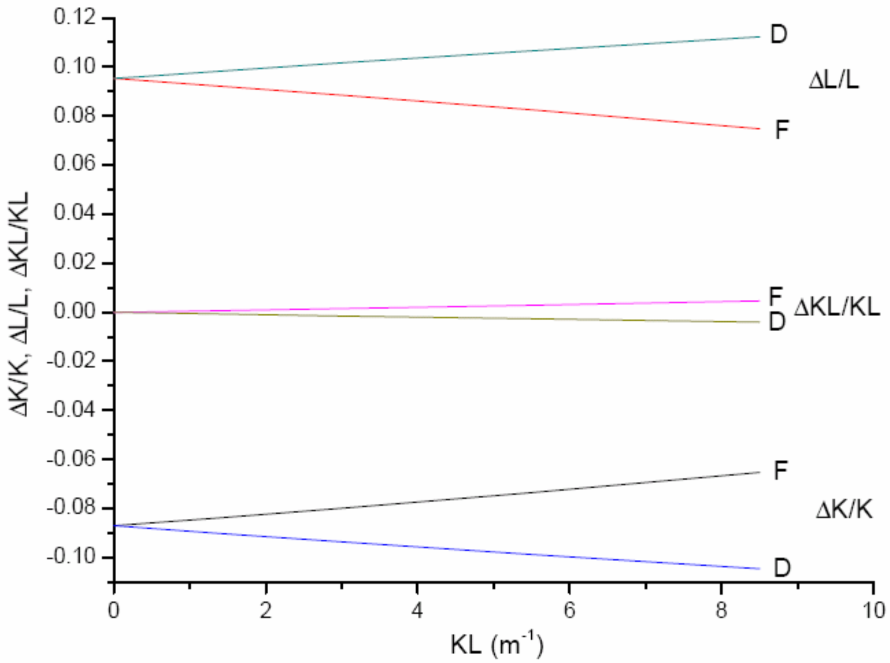
$$K_{eq}^{D}L_{eq}^{D} \approx K_0L_0[1-(-\frac{3A^2}{L_0^2}+\frac{2B}{L_0}-\frac{2C}{L_0}+\ldots)K_0+O(K_0^2)]$$

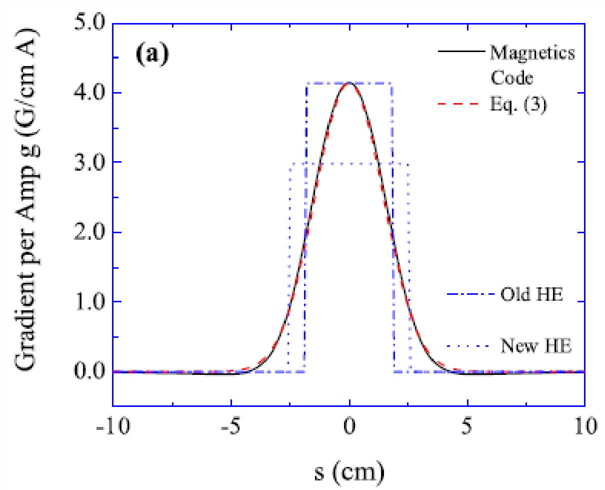
Properties of equiv. H.E. model

- Equiv. length is longer
- Equiv. strength is lower
- Deviation of Equiv. product is small



Ref.[2] Courtesy J.G. Wang





Ref. [3] Courtesy S. Bernal, et al.

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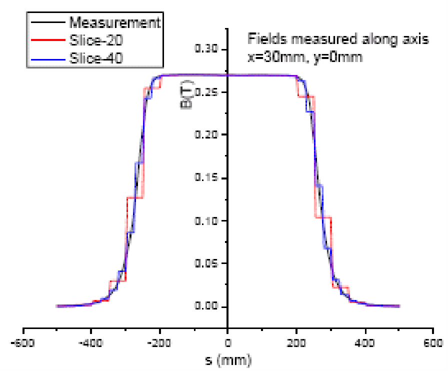
Numerical test of the equiv. H.E. model

- **Purpose of numerical test**
  - **Accuracy of linear fringe map**
  - **Validity of equiv. H.E. model**
- **Comparison**
  - **Numerical calculation using "slicing" method**
  - **Non-truncated equiv. H.E. model**
  - **Truncated equiv. H.E. model**



slicing

$$K_{eq}^F \approx K_0[1-(\frac{6A}{L_0^2}-\frac{54A^2}{L_0^4}+\frac{12B}{L_0^3}+...)+(\frac{2A}{5}+...)K_0+O(K_0^2)]$$

$$L_{eq}^F \approx L_0[1+(\frac{6A}{L_0^2}-\frac{18A^2}{L_0^4}+\frac{12B}{L_0^3}+...)-(\frac{2A}{5}+...)K_0+O(K_0^2)]$$

$$K_{eq}^F L_{eq}^F \approx K_0 L_0[1+(-\frac{3A^2}{L_0^2}+\frac{2B}{L_0}-\frac{2C}{L_0}+...)K_0+O(K_0^2)]$$

→

$$K_{eq}^F \approx K_0[1+(-\frac{6A}{L_0^2}+\frac{54A^2}{L_0^4}-\frac{12B}{L_0^3})+\frac{2A}{5}K_0]$$

$$L_{eq}^F \approx L_0[1+(\frac{6A}{L_0^2}-\frac{18A^2}{L_0^4}+\frac{12B}{L_0^3})-\frac{2A}{5}K_0]$$

$$K_{eq}^F L_{eq}^F \approx K_0 L_0[1+(-\frac{3A^2}{L_0^2}+\frac{2B}{L_0}-\frac{2C}{L_0})K_0]$$

Numerical test of the equiv. H.E. model (cont)

- Cases with variables as
  - Effective strength
  - Effective length
  - Fringe extension
  - Full fringe
- Focusing functions
  - Enge function (Default Enge coefficients used in COSY INFINITY
  - Gaussian function

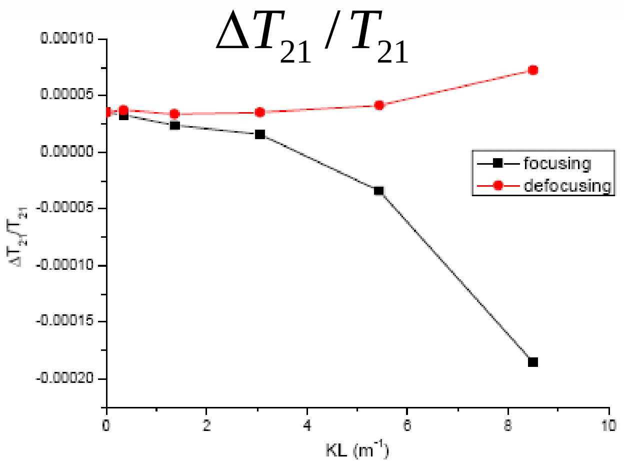
$$G_{ga}(s) = G_0 \exp(-\pi s^2 / d^2)$$

$$E(s) = \frac{1}{1 + \exp[a_1 + a_2(\frac{s}{D}) + a_3(\frac{s}{D})^2 + a_4(\frac{s}{D})^3 + a_5(\frac{s}{D})^4 + a_6(\frac{s}{D})^5]}$$

a_1	a_2	a_3	a_4	a_5	a_6
0.296471	4.533219	-2.270982	1.068627	-0.036391	0.022261

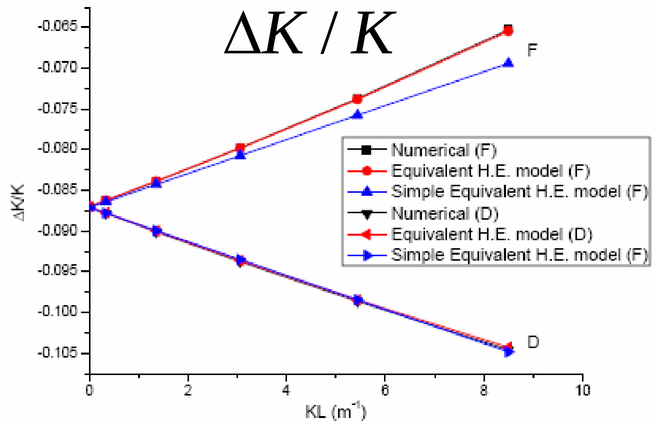
Numerical test of the equiv. H.E. model (cont)

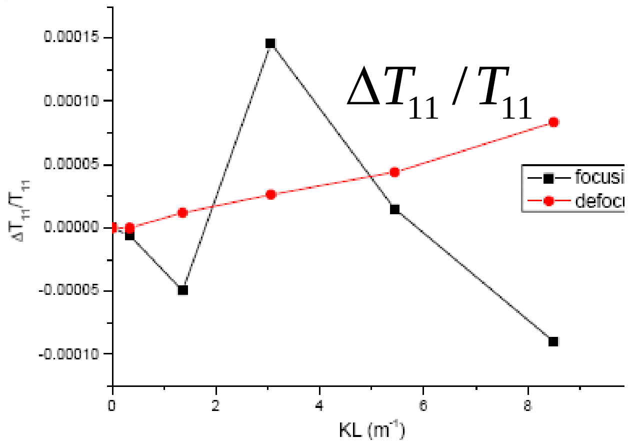
- Effective strength as variable

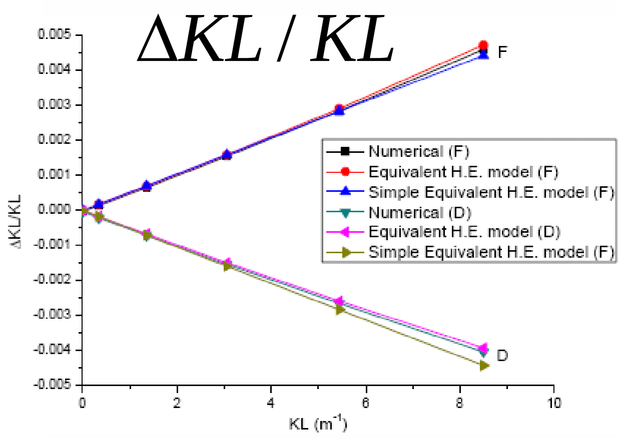


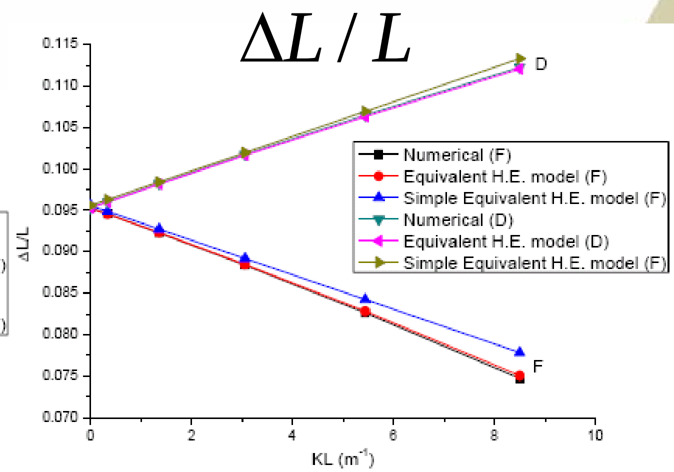
$$L_0 = 0.34m$$

$$D = 0.105m$$





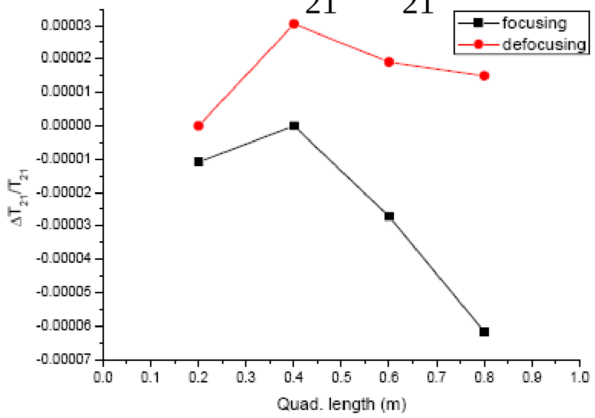


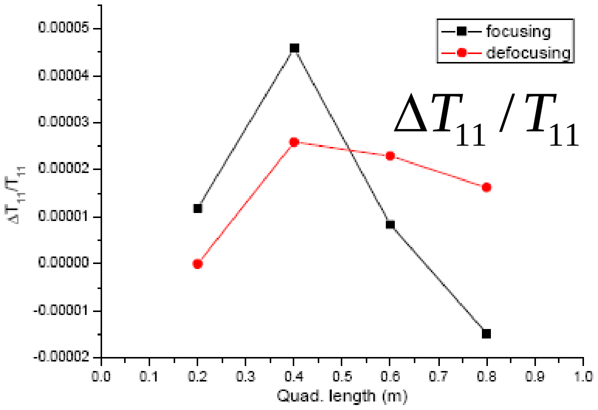


Numerical test of the equiv. H.E. model (cont)

- **Effective length as variable**
  - **short magnet=>full fringe**

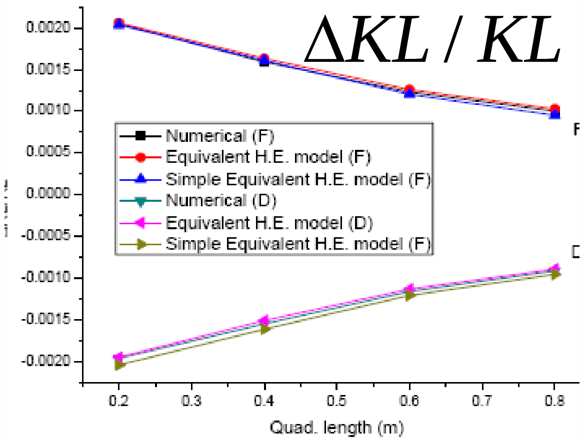
$\Delta T_{21} / T_{21}$

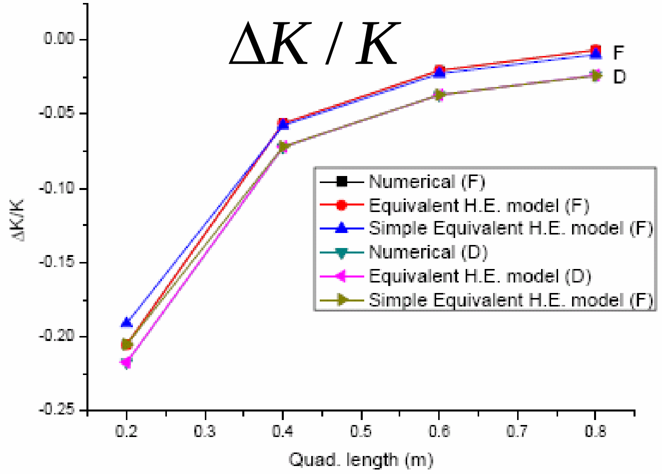


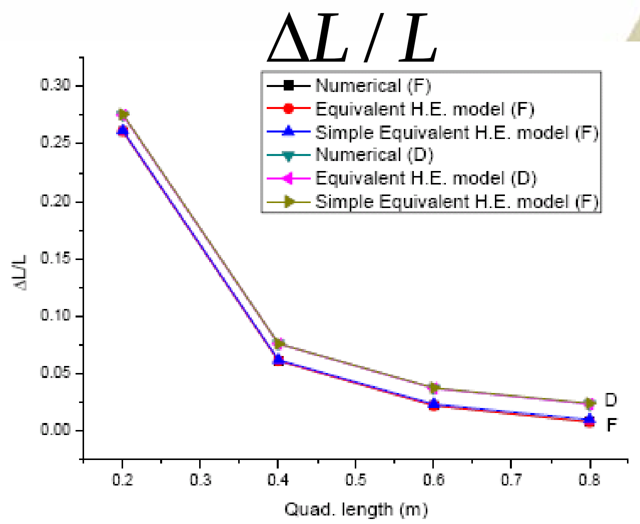


$$K_0 = 10 m^{-2}$$

$$D = 0.105 m$$

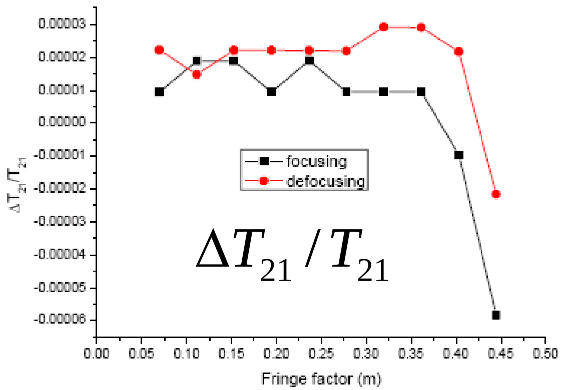






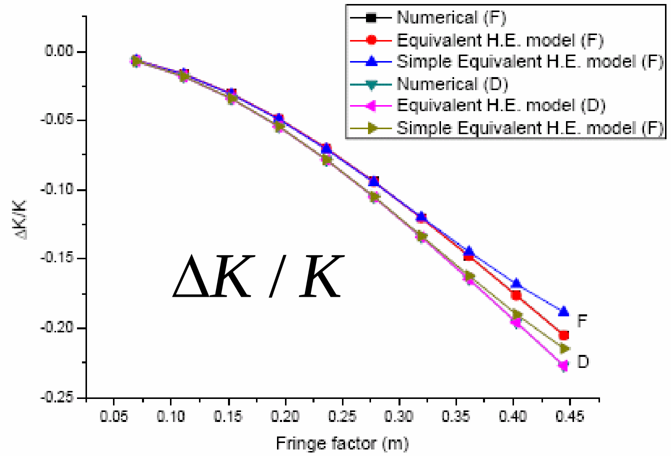
Numerical test of the equiv. H.E. model (cont)

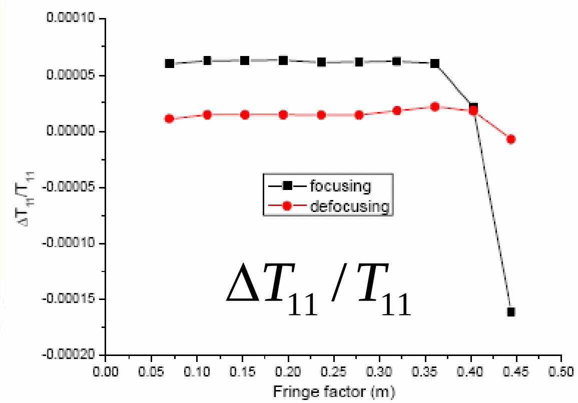
- **Fringe extension as variable**
  - **short fringe: excellent**
  - **long fringe: not so good**

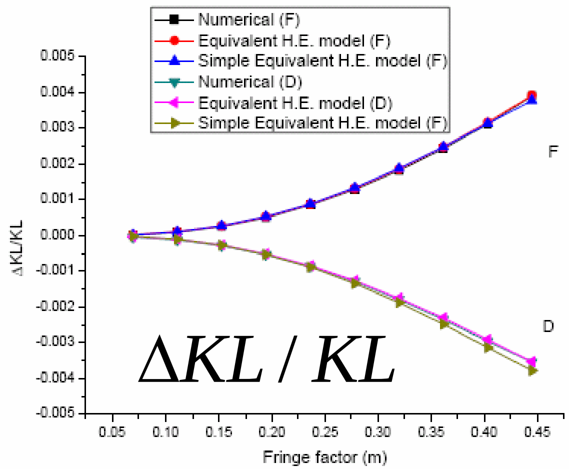


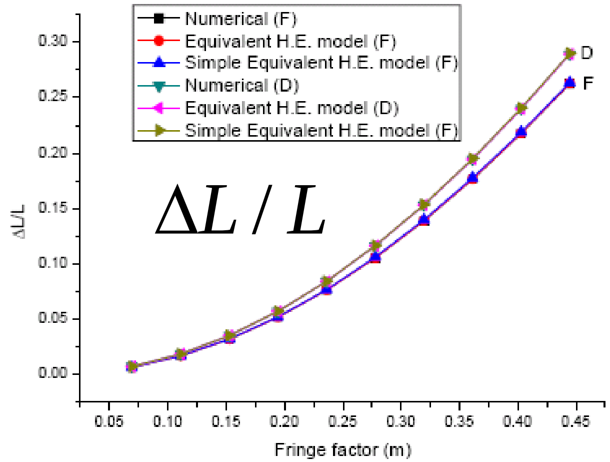
$$K_0 = 2m^{-2}$$

$$L_0 = 0.6m$$



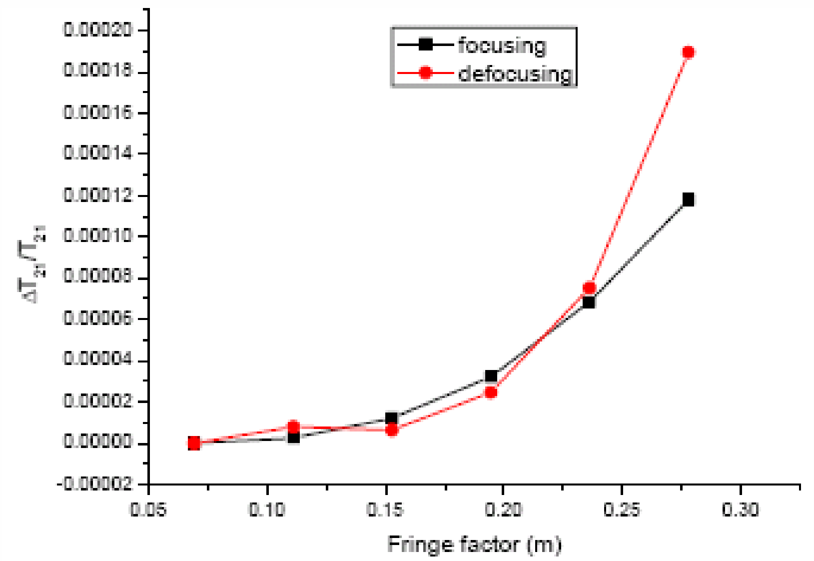




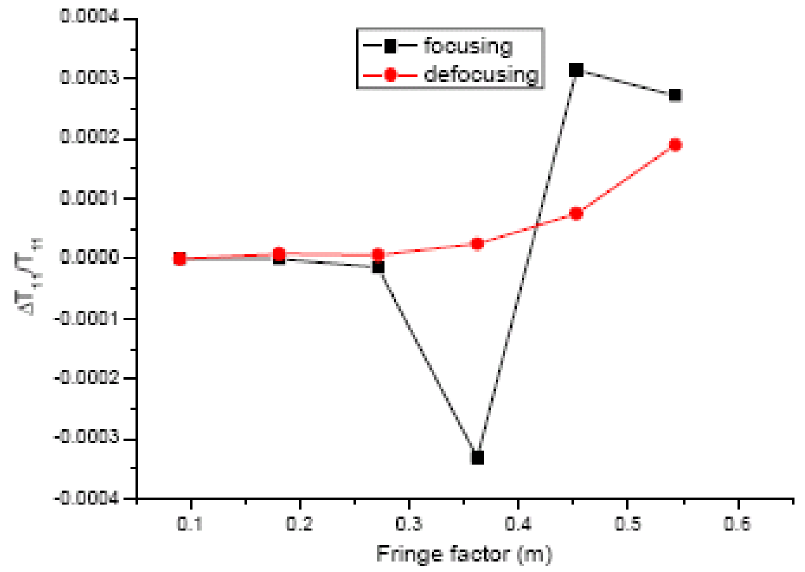


Numerical test of the equiv. H.E. model (cont)

- Case of full fringe
  - fringe map estimation is no quite good for “long” full fringe magnet



$\Delta T_{21} / T_{21}$



$\Delta T_{11} / T_{11}$

$$K_0 = 2m^{-2}$$

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Proposal of a simple H.E. model

- First order correction
- Applicable for both focusing and defocusing planes
- Easy implemented in codes not include fringe fields, such as MAD and AT
- More effective for cases of **small quad field integral** and **short fringe extension**

$$K_{eq} = K_0 \left(1 - \frac{f_1^2}{2L_0^2}\right) \qquad L_{eq} = L_0 \left(1 + \frac{f_1^2}{2L_0^2}\right)$$

$$f_1 = \sqrt{24 \left| \int_0^{\infty} \frac{\widetilde{G}(s)}{G_0} (s - s_0) ds \right|}$$

Proposal of a simple H.E. model (cont)

- Test of the simple model using SAD
  - BSR_07jan01, nominal: (7.28, 5.18)
  - R3OQ02:

$$L_0 = 0.548m \quad K_0 = 0.405 \quad f_1 = 0.133m \quad L_{eq} = 0.564m$$

Turn on fringe: $(7.27994, 5.17979)$

Simple model: $(7.27994, 5.17979)$

  - R2OQ06:

$$L_0 = 0.34m \quad K_0 = 1.46 \quad f_1 = 0.154m \quad L_{eq} = 0.375m$$

Turn on fringe: $(7.27858, 5.17944)$

Simple model: $(7.27856, 5.17945)$

Outline

- Introduction
- Estimation of tune shift
  - Quad fringe fields
  - Magnetic interference
- Linear fringe map
- Equivalent hard edge model
- Numerical test
- Summary

Summary

- Tune shift in BEPCII rings was well explained by effects of fringe fields and magnet interference.
- A simple method was found to calculate the tune shift due to quad fringe fields.
- Perturbation treatment based on Lie technique is a good approach for quad fringe field effects. It is easy to be extended to calculate nonlinear fringe maps, even magnetic interference included.
- The work will also help to estimate the significancy of fringe fields and magnetic interference in small rings such as CSNS, Proton Therapy Accelerator, etc.
- The simple H.E. model may be applied in MAD and LOCO for linear optics design and compensation.

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Thank you for your attention!!

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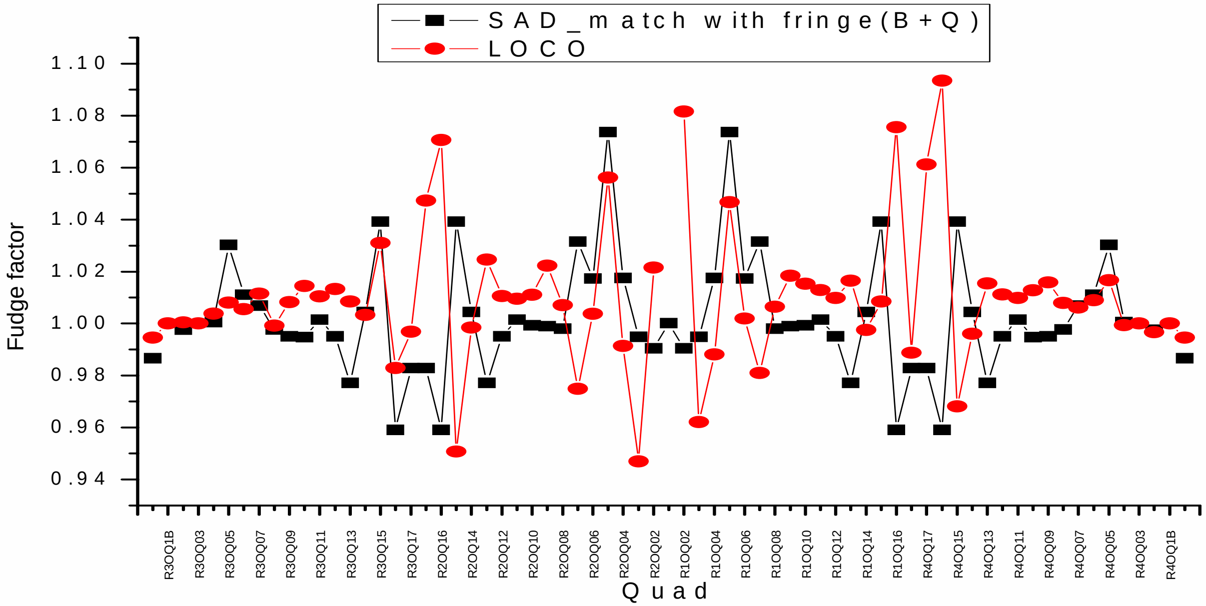
Optics correction

Comparison of SAD and LOCO correction

SAD: with fringe fields of quads and bends, without magnetic interference
LOCO: based on beam measurement, include all imperfections



The figure shows similar correction scheme in SAD and LOCO

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