Space Charge in High Current Lower Energy Electron Bunches

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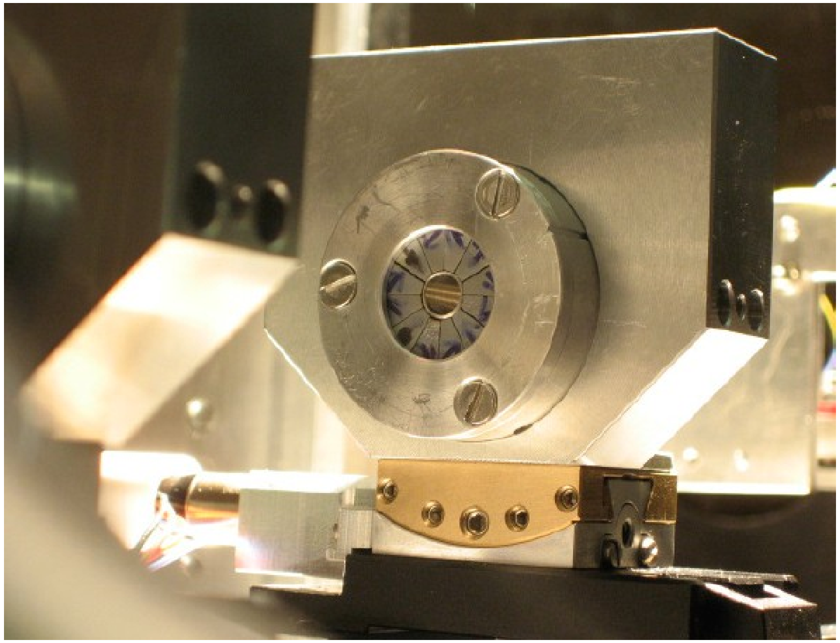
- Experimental Status
- Self acceleration:
  - Visual explanation
  - Renormalization of the electron mass
- GPT: The applied calculation method
- Retardation
- Simulating 2 GeV, 1 nC
  - Point 2 Point and Poisson-Solver still differ
  - Convergence of Point 2 Point
- Simulation Benchmark: Energy conservation assuming mass renormalization

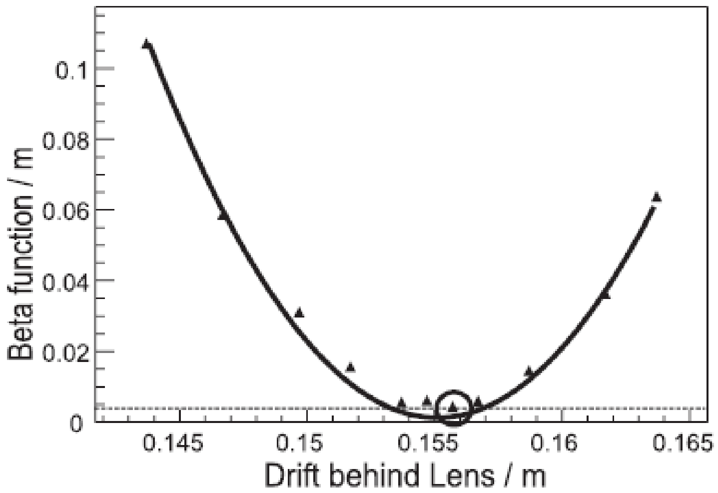
Experimental Status: Beam transport

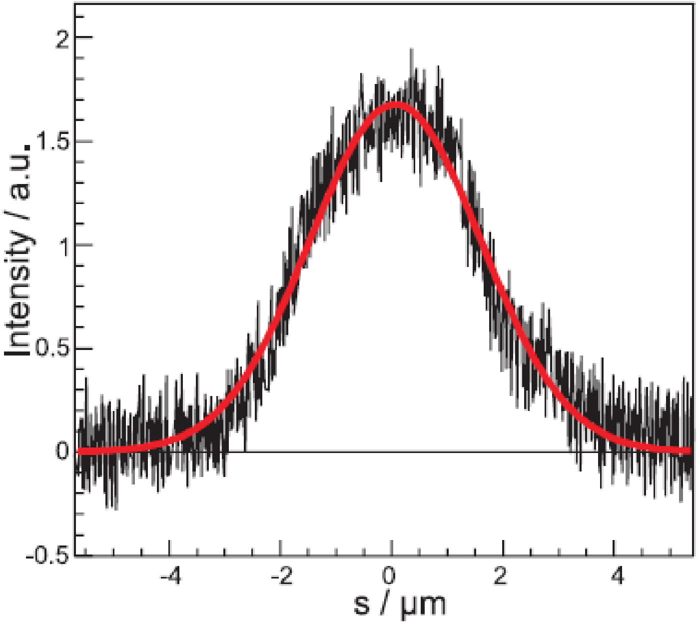
Quadrupoles:

- Aperture 6 mm (5 mm effective)
- Field gradient 503 T/m



Measurements at MaMi Mainz:

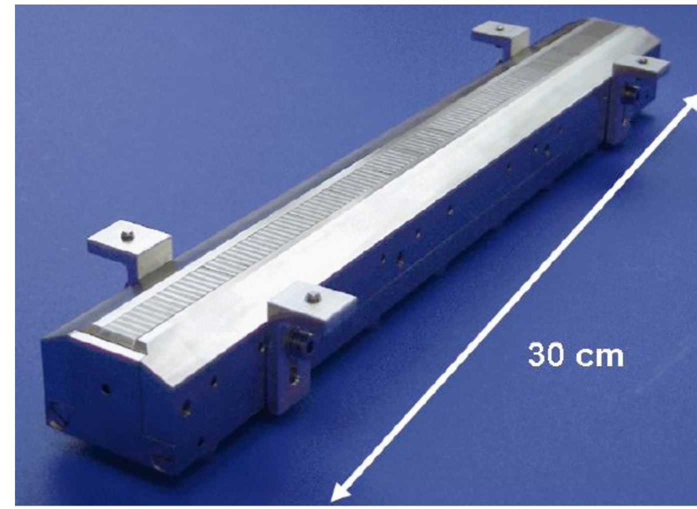




Experimental Status: Undulator

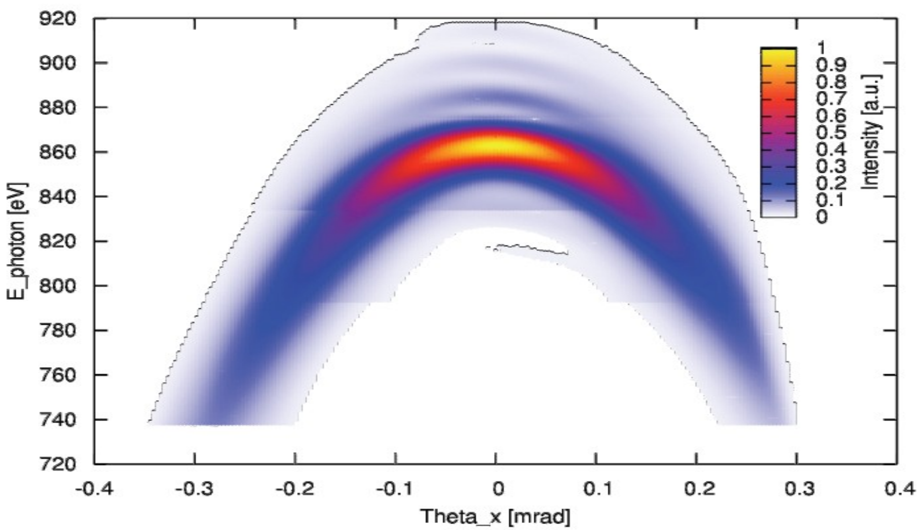
Test Undulator:

- 60 Periods
- 5 mm period length
- 0.9 T field on axis at a gap of 2.5 mm

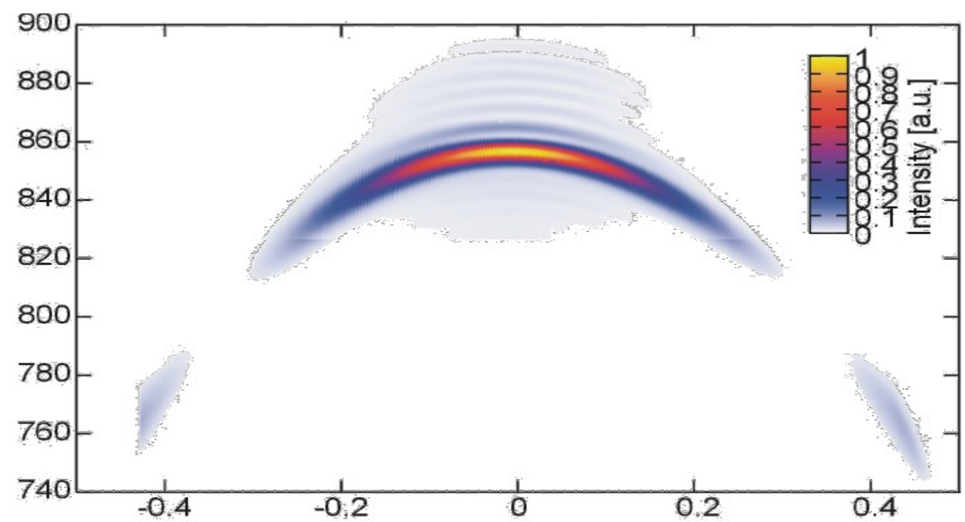


Measurements at MaMi Mainz:

1st Harmonic
electron energy = 705 MeV

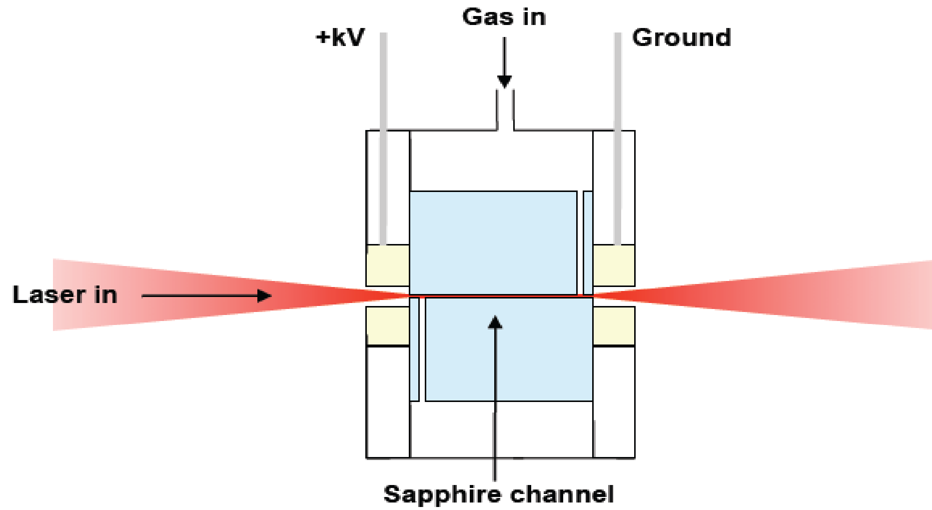


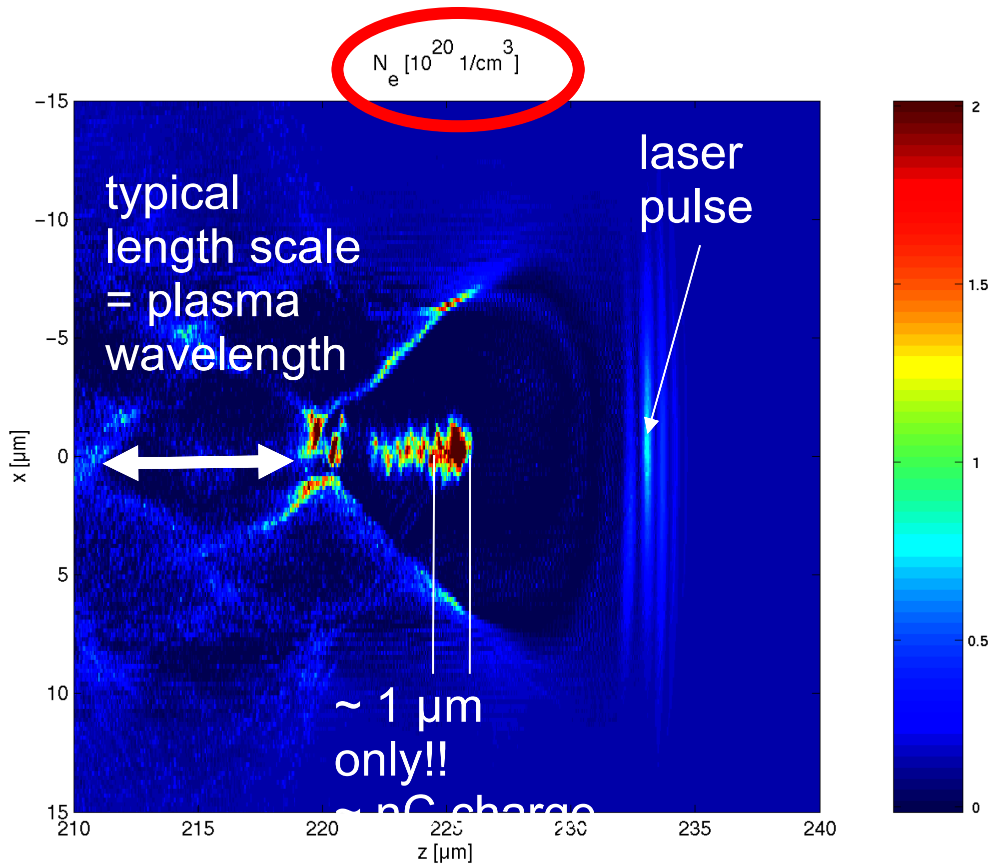
3rd Harmonic
electron energy = 405 MeV

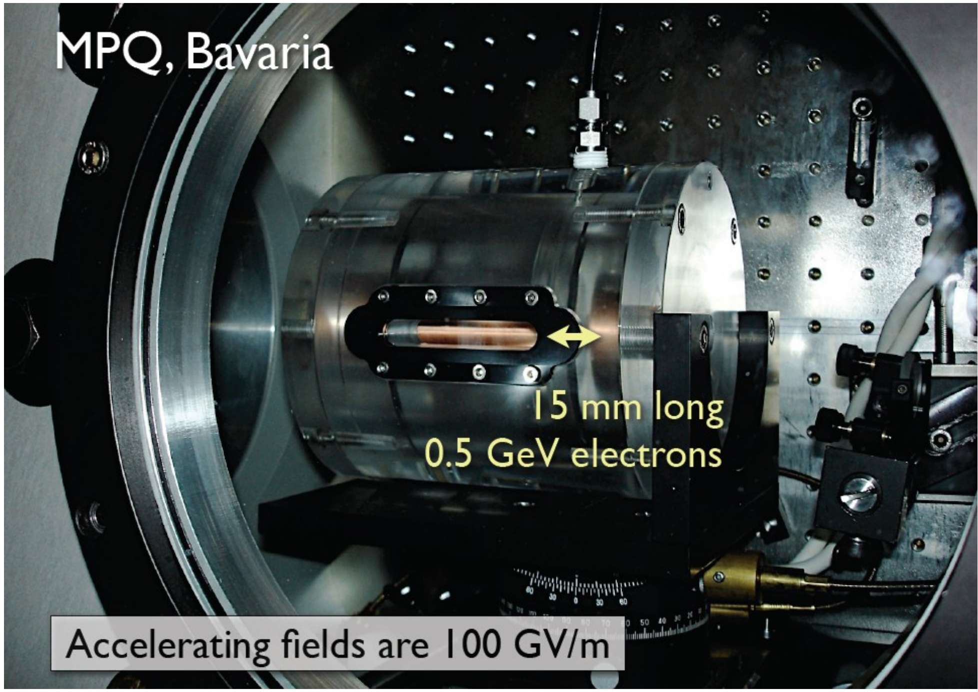


Experimental Status: Electron acceleration

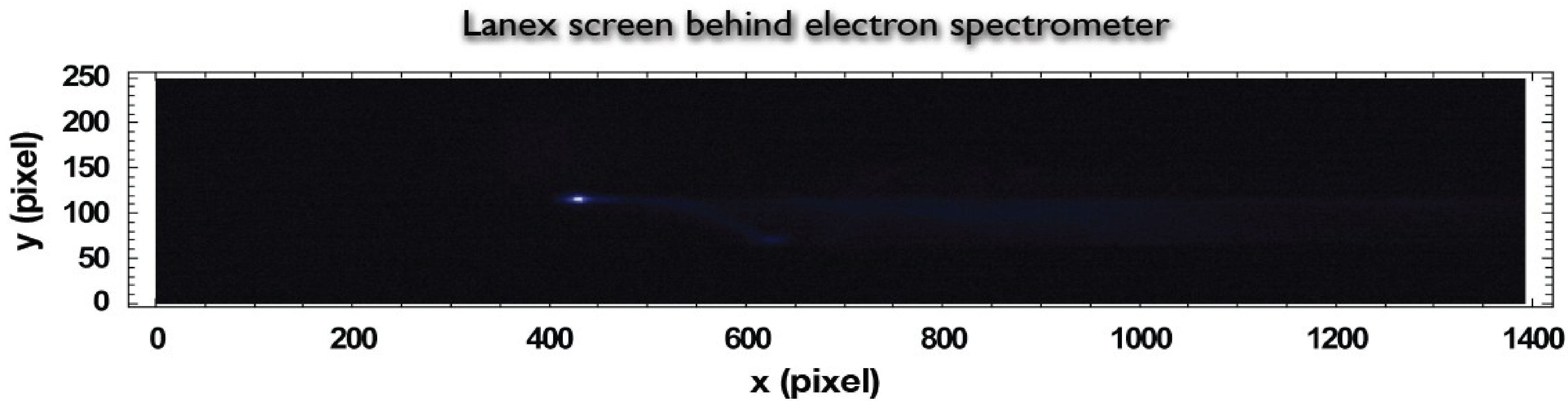
Electron acceleration at MQP:

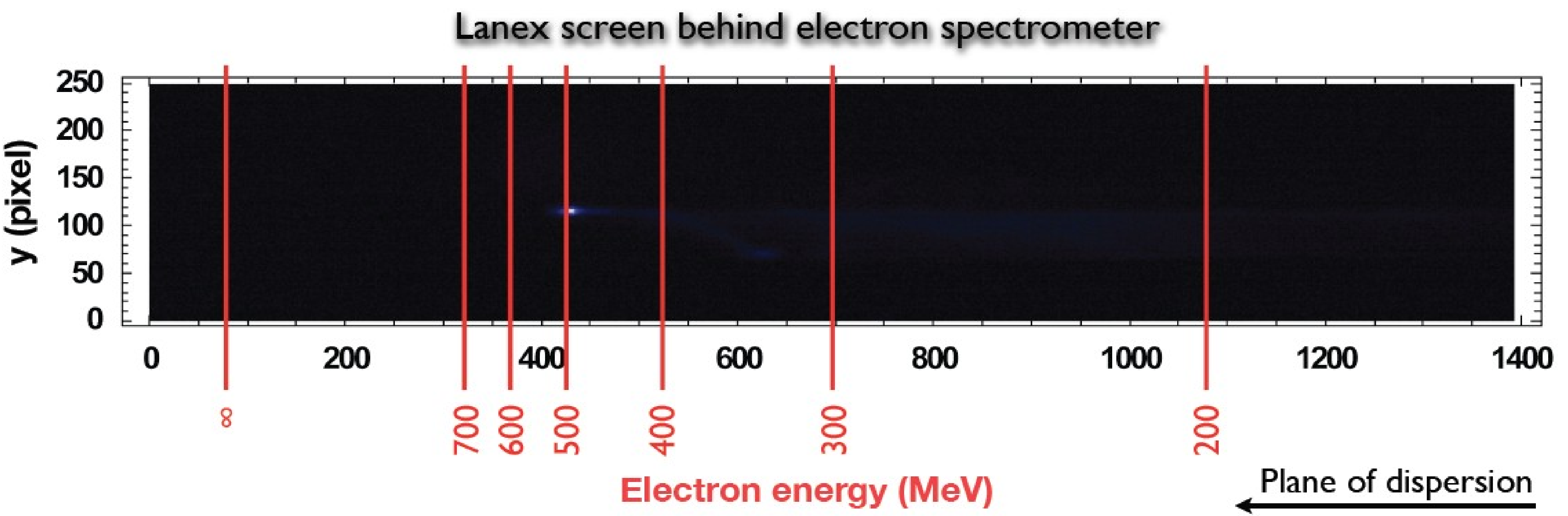






Experimental Status: Electron acceleration





Self Acceleration

The Regime:

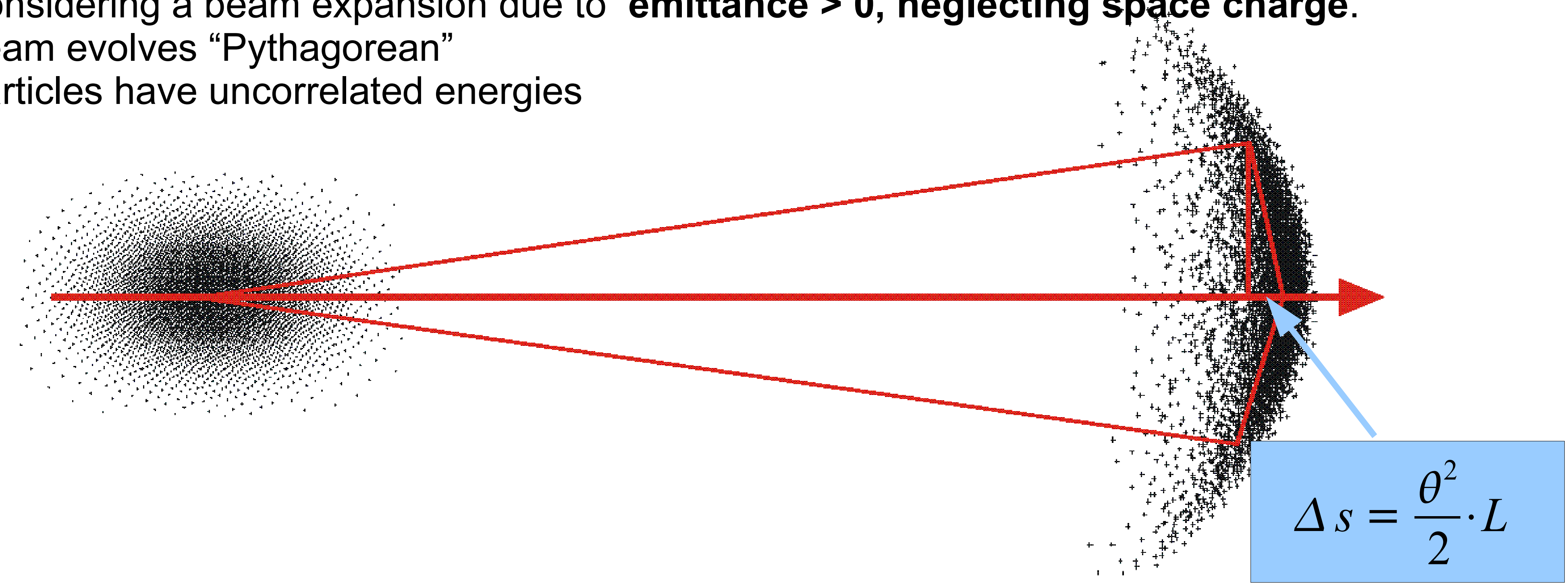
“Extreme” case:

- $\sigma_x = \sigma_y = \sigma_z = 1\ \mu m$
- Charge: 1.25 nC -> I = 150 kA
- Initial distribution: Gaussian

Self Acceleration

Considering a beam propagation with **no emittance, neglecting space charge**.

Considering a beam expansion due to **emittance > 0, neglecting space charge**.
Beam evolves "Pythagorean"
Particles have uncorrelated energies

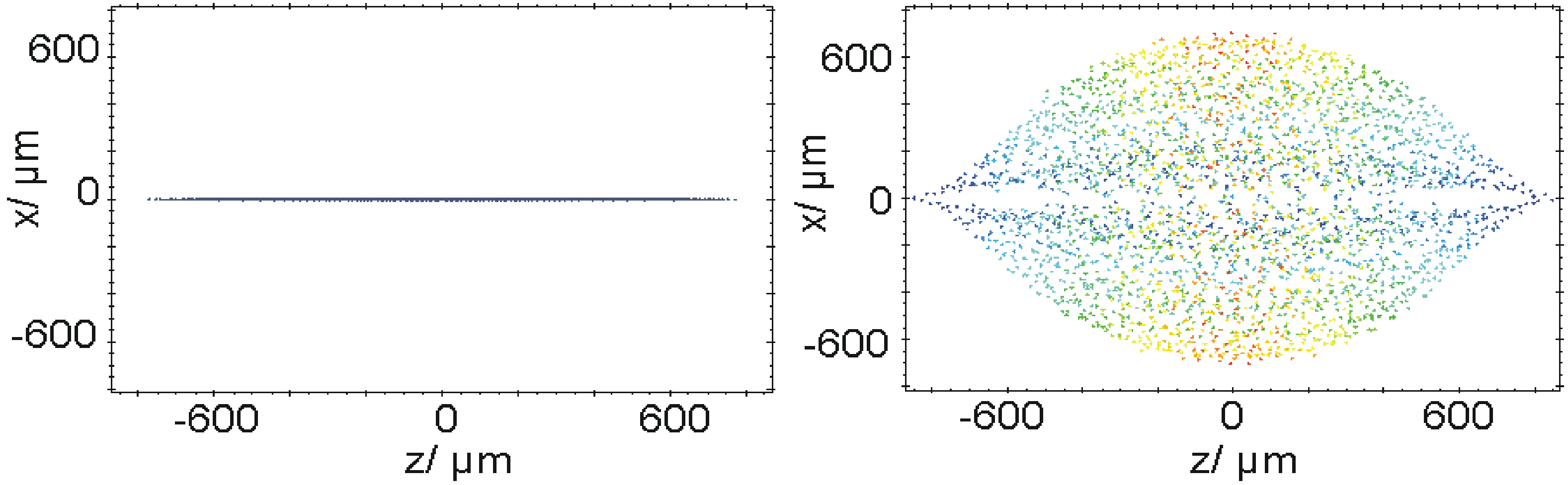


Self Acceleration

Considering a Coulomb explosion: Assuming $\sigma_x = \sigma_y = \sigma_z$ in the lab frame.

Due to relativistic contraction, the longitudinal dimension becomes larger: $\sigma_z{}' = \gamma \cdot \sigma_z$

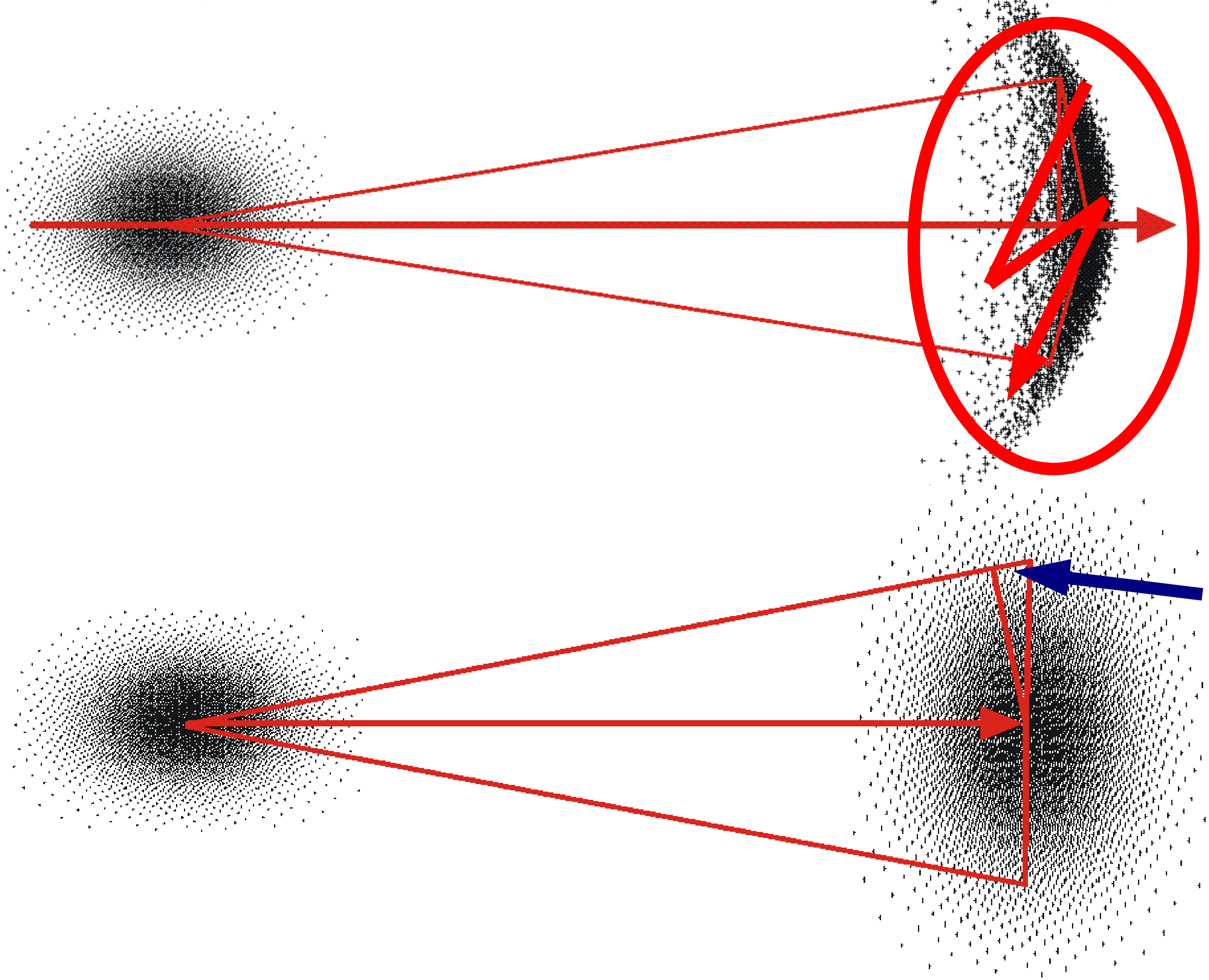
$$\gamma = 260$$



→ "Explosion" is transversal dominated.

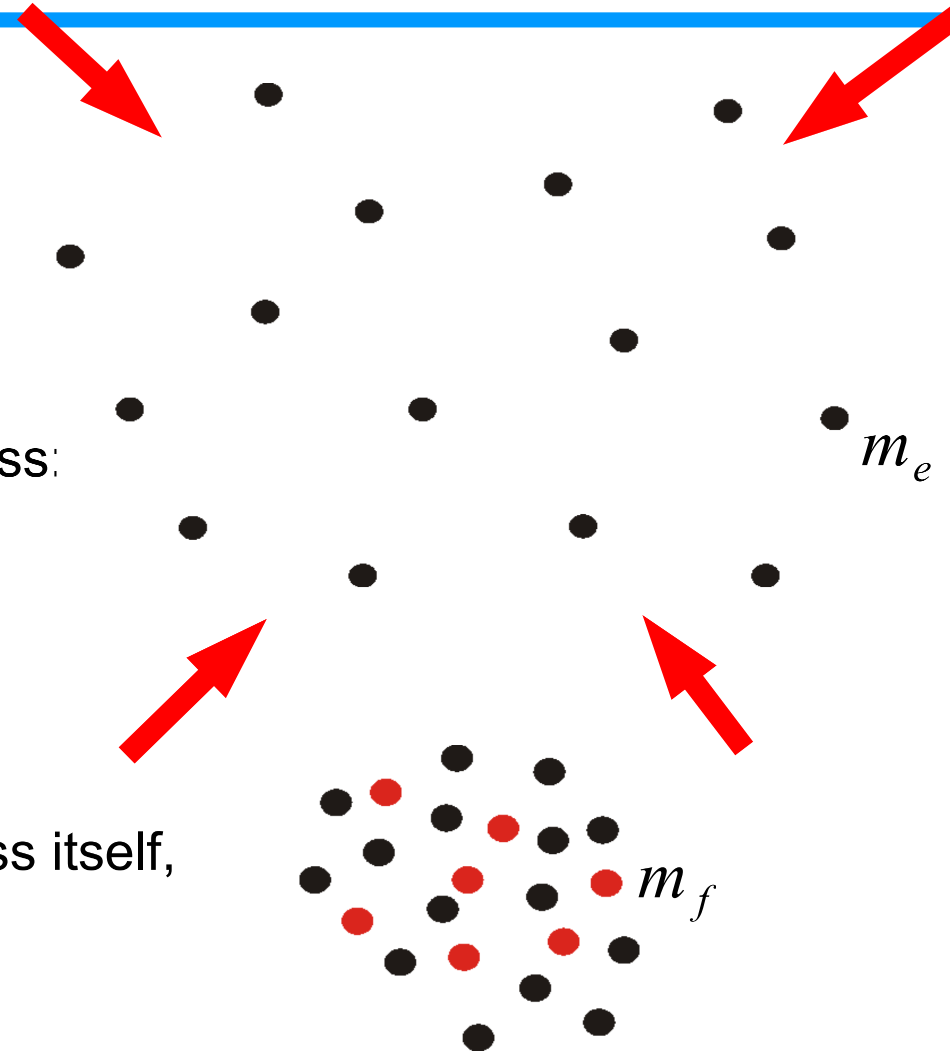
Self Acceleration

Considering a beam expansion with **no emittance, respecting space charge**.



Electrons off axis have to gain speed, hence **gain energy** to “keep up” with the electrons on axis.

Self Acceleration



Quantitative approach:
Compressing electrons corresponds to a gain of potential energy, which can be expressed in terms of a mass:

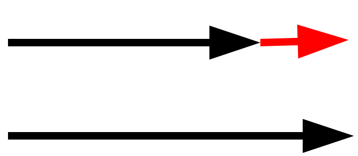
$$E = m c^2$$

The self field can be considered as a mass itself, which "dissolves" as the beam expands:

$$E_f = m_f c^2 = U'$$

Momentum before Expansion:

Momentum after Expansion:



Mass Renormalization

Electron's rest frame: (All energies E are normalized on one electron)

Potential energy leads to a mass of the field: $E_f = m_f c^2$

Total energy in the electrons' mean rest frame:

$$E' = E_0 + E_f = m_0 c^2 + E_f = (m_0 + m_f) c^2$$

$$E' = m' c^2 = \gamma' m_0 c^2$$ after potential energy "released"

Mean Lorentz factor due to Coulomb explosion:

$$\gamma' = 1 + \frac{m_f}{m_0}$$

Potential energy in average is estimated to be of the order of **74 keV / electron**

Laboratory frame: Electron bunch leaves the plasma with γ_0

Total energy:

$$E = \gamma_0 m' c^2 = \gamma_0 m_0 c^2 \cdot \left(1 + \frac{m_f}{m_0}\right)$$

Kinetic energy gains by a factor of γ':

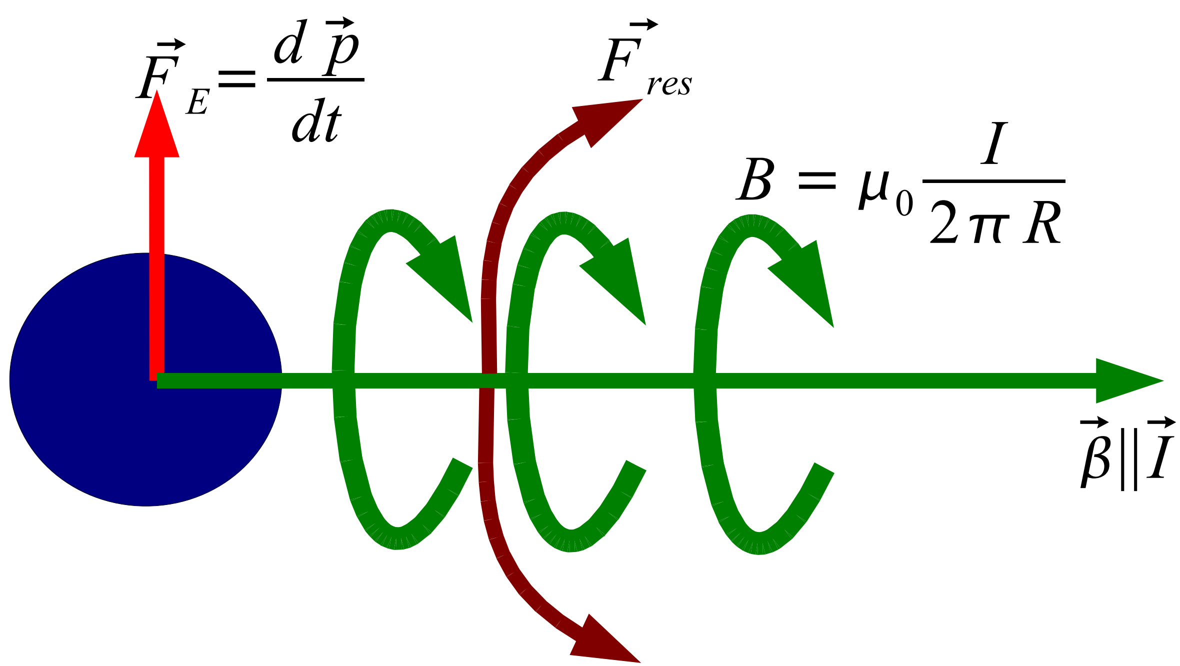
$$E = \gamma_0 \gamma' m_0 c^2 = \gamma m_0 c^2$$ after potential energy "released"

$$\gamma = \gamma_0 \cdot \gamma'$$

Self Acceleration: Lab Frame

Space charge (in the mean rest frame) acts *transversally*.

Where does the *longitudinal momentum* come from in the Laboratory Frame?



GPT

- GPT adapts time steps dynamically -> energy conservation
- Calculation precision is defined by relative momentum changes stepwise
- Space charge is considered point to point using Lorentz transformation. Calculation time t ~ N²

For equation of motion:

$$\mathbf{F}_i = q\left(\mathbf{E}_i + \mathbf{v}_i \times \mathbf{B}_i\right)$$

Interaction in rest frame:

$$\mathbf{E}'_{j\to i} = \frac{Q\mathbf{r}'_{ji}}{4\pi\varepsilon_0 \left|\mathbf{r}'_{ji}\right|^3}$$

From Lorentz transformations:

$$\mathbf{r}_{ji} = \mathbf{r}_i - \mathbf{r}_j$$

$$\mathbf{r}'_{ji} = \mathbf{r}_{ji} + \frac{\gamma_j^2}{\gamma_j + 1}\left(\mathbf{r}_{ji} \cdot \boldsymbol{\beta}_j\right)\boldsymbol{\beta}_j$$

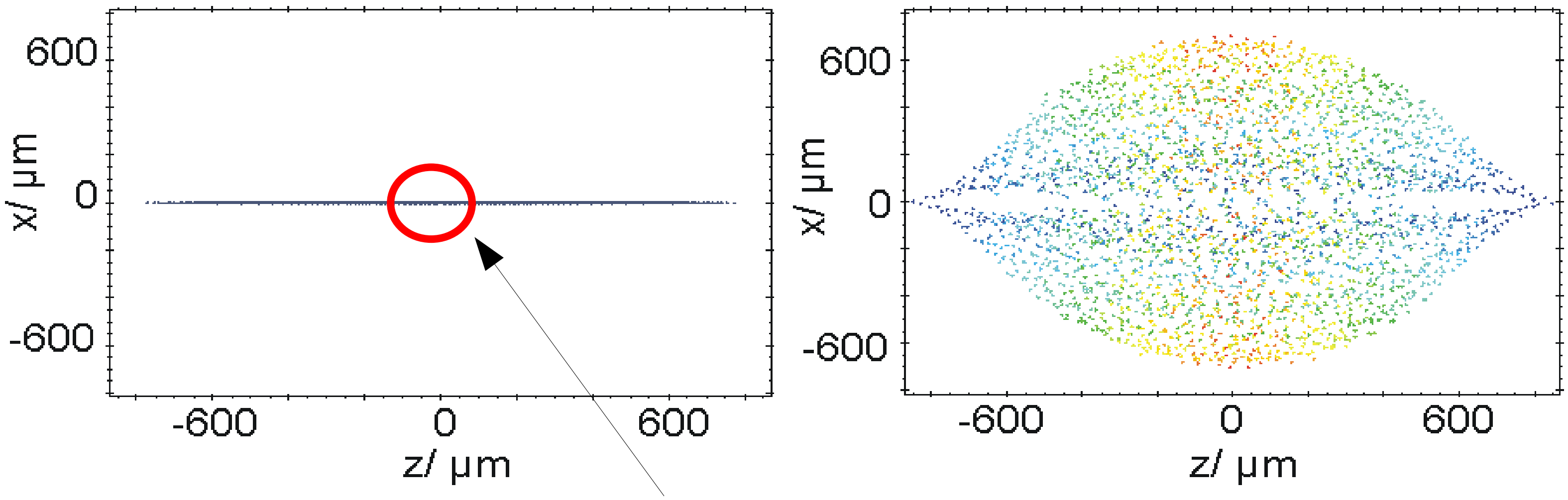
$$\mathbf{E}_i = \sum_{j \neq i} \gamma_j \left[\mathbf{E}'_{j\to i} - \frac{\gamma_j}{\gamma_j + 1}\left(\boldsymbol{\beta}_j \cdot \mathbf{E}'_{j\to i}\right)\boldsymbol{\beta}_j \right]$$

$$\mathbf{B}_i = \sum_{j \neq i} \frac{\gamma_j \boldsymbol{\beta}_j \times \mathbf{E}'_{j\to i}}{c}$$

Discussion of limitations due to Retardation Effects

Assuming the bunch is “born” instantaneously- fields did not “spread” yet:

Going to the electron bunch's mean rest frame:



"Event horizon"

$$\mathbf{E}(\mathbf{r}) = \frac{q}{4\pi\epsilon_0}\frac{\mathbf{r}-\mathbf{r}_0}{|\mathbf{r}-\mathbf{r}_0|^3} \sim \frac{1}{r^2}$$

→ The contribution of close neighbors is strongest

Discussion of limitations due to Retardation Effects

Possibilities of respecting retardation in calculations:

Interaction is described in terms of

$$\mathbf{E}_i = \sum_{j \neq i} \gamma_j \left[\mathbf{E}'_{j \to i} - \frac{\gamma_j}{\gamma_j + 1} \left(\boldsymbol{\beta}_j \cdot \mathbf{E}'_{j \to i} \right) \boldsymbol{\beta}_j \right]$$

$$\mathbf{B}_i = \sum_{j \neq i} \frac{\gamma_j \boldsymbol{\beta}_j \times \mathbf{E}'_{j \to i}}{c}$$

$$\mathbf{r}_{ji} = \mathbf{r}_i - \mathbf{r}_j$$

$$\mathbf{r}'_{ji} = \mathbf{r}_{ji} + \frac{\gamma_j^{\,2}}{\gamma_j + 1} \left(\mathbf{r}_{ji} \cdot \boldsymbol{\beta}_j \right) \boldsymbol{\beta}_j$$

Possible solution:

Interaction only on case of " r' < Event Horizon "

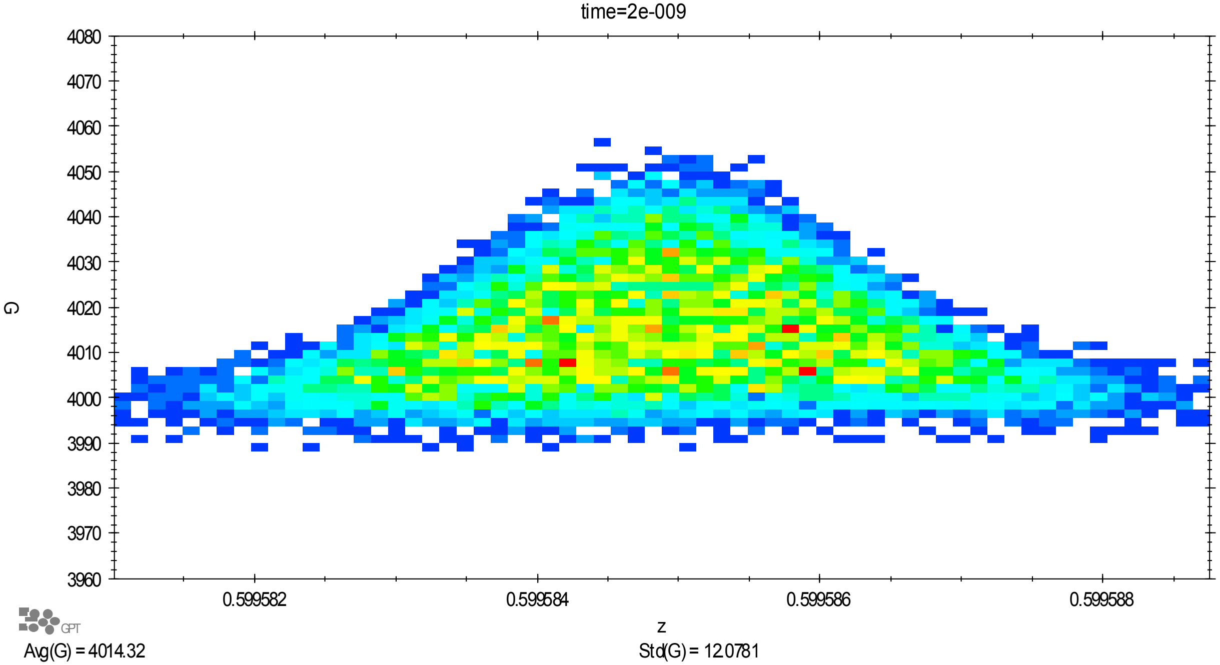
Simulating 2 GeV, 1 nC

Initial distribution:

$$E = 2\ GeV \rightarrow \gamma = 3880$$
$$\sigma_\gamma / \gamma = 0.001$$

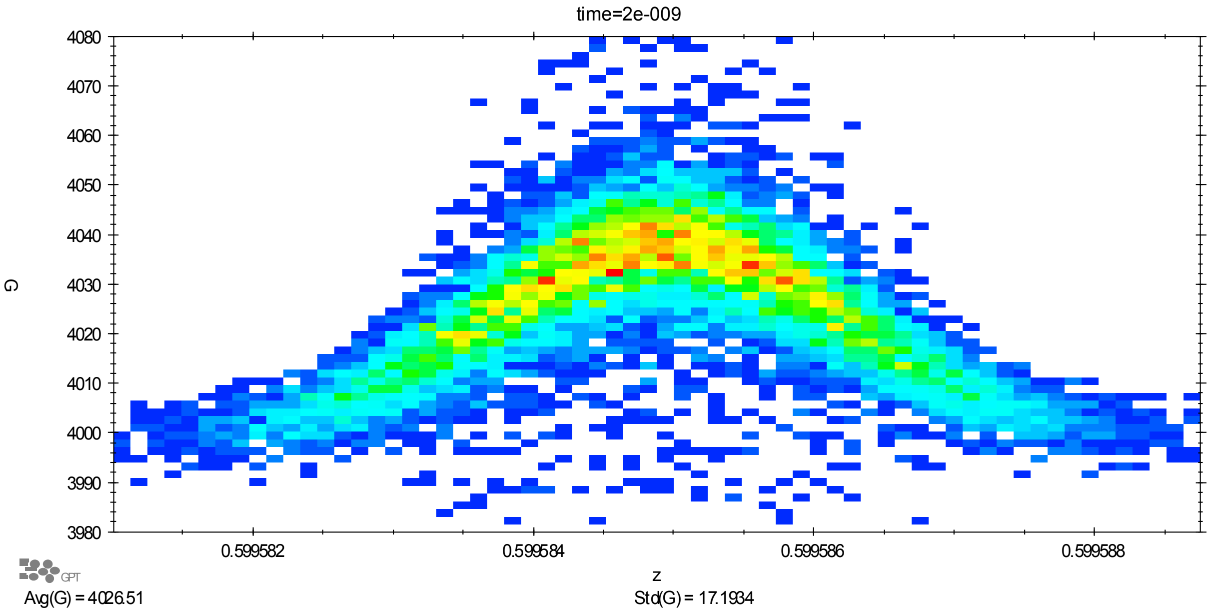
Result of Poisson-Solver:
after propagation of 60cm:

$$\bar{\gamma} = 4014$$
$$\sigma_\gamma / \gamma = 0.003$$



Result of Point2Point
after propagation of 60cm:

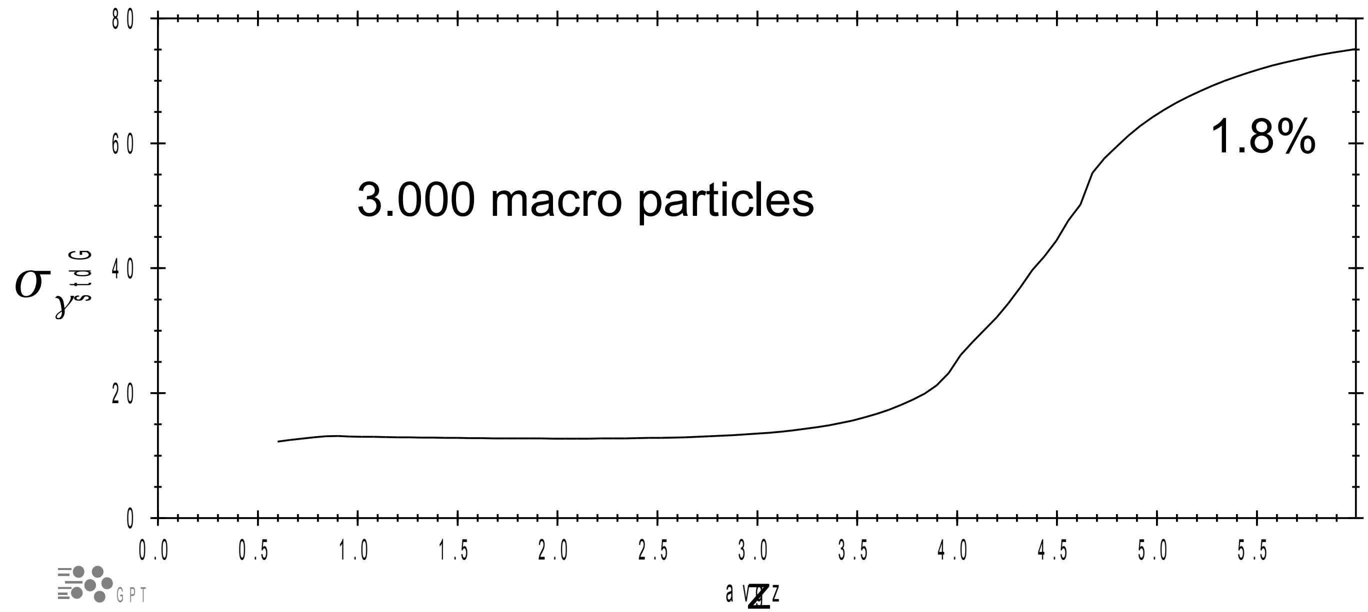
$$\bar{\gamma} = 4027$$
$$\sigma_\gamma / \gamma = 0.0044$$



Simulating 2 GeV, 1 nC

Simulation of designed beam line for E = 2 GeV

Triplet focusing-
Waist at approx. 5m

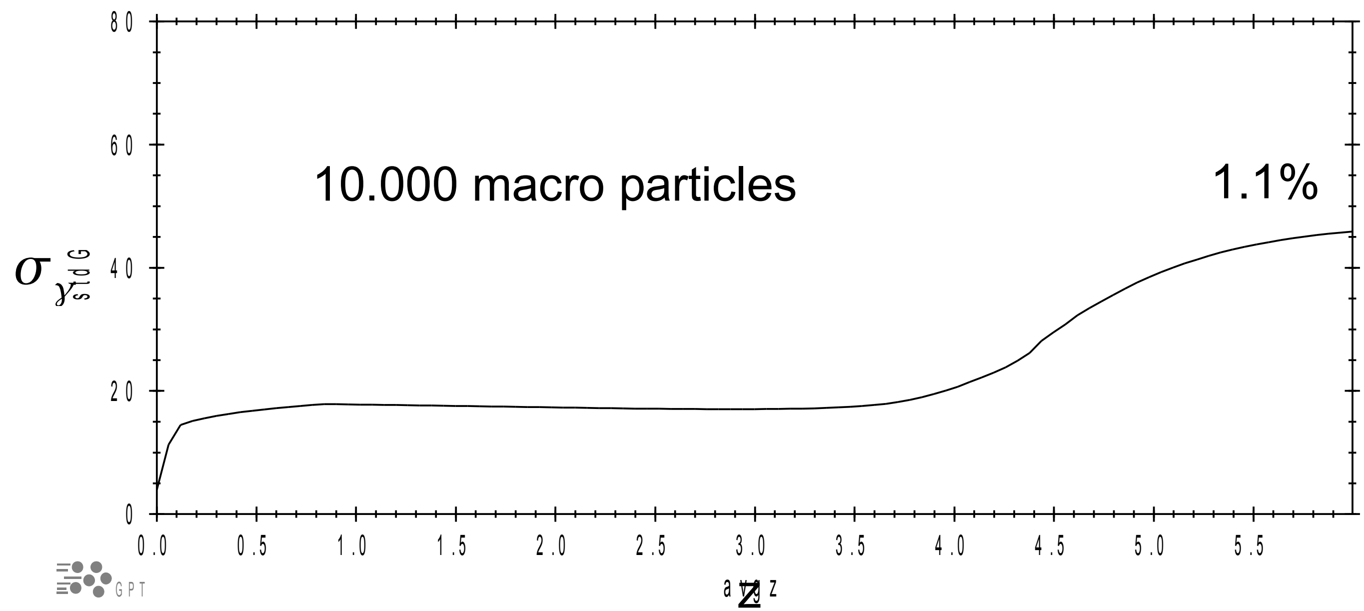


Due to adaptive time steps, scaling is worse than

$t \sim n^2$

Did not reach convergence

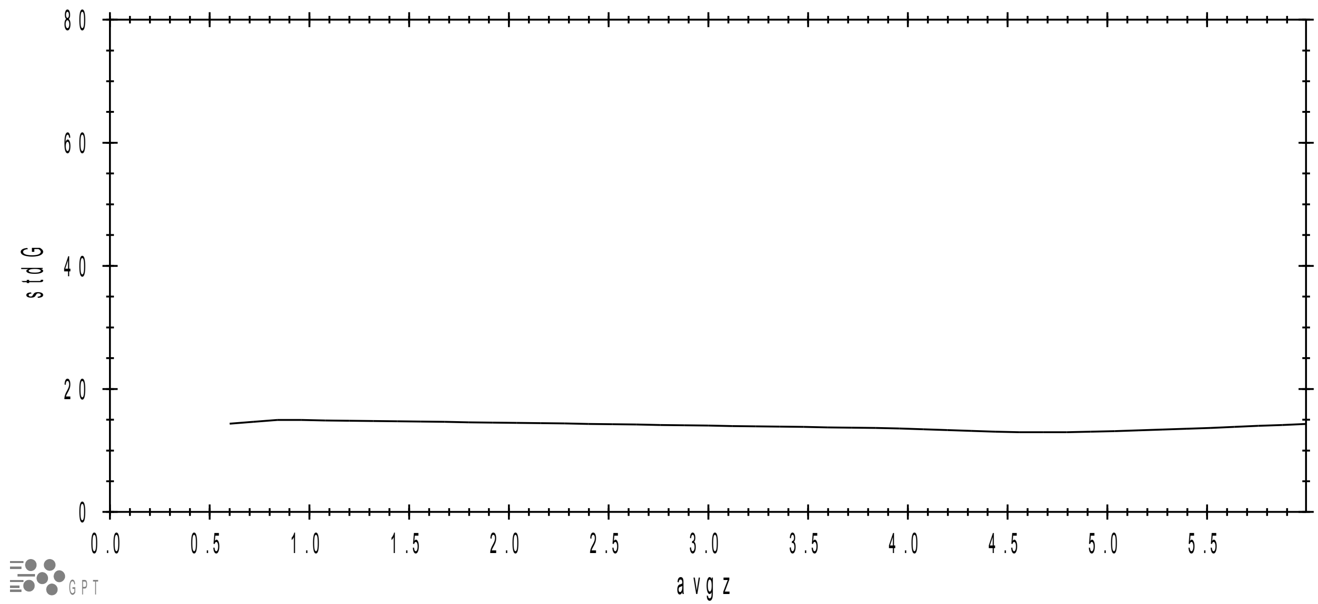
-> End of Point 2 Point ?!



Simulating 2 GeV, 1 nC

GPT-Poisson solver:

Energy spread decreases!



TODO: Try other solvers:

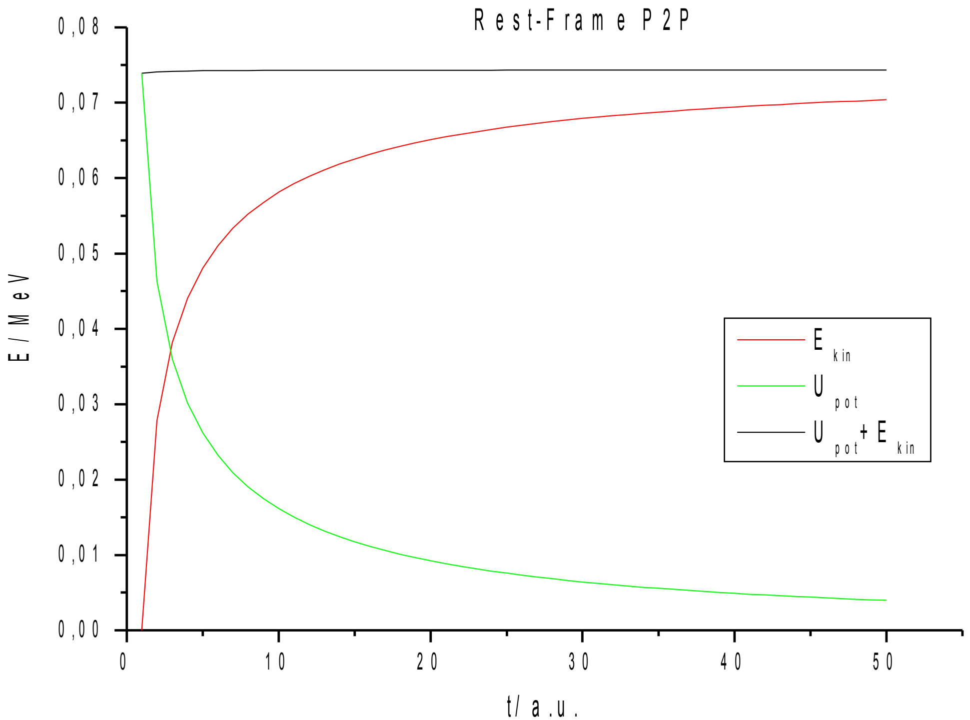
ASTRA

Simulation Benchmark: Energy conservation assuming mass renormalization

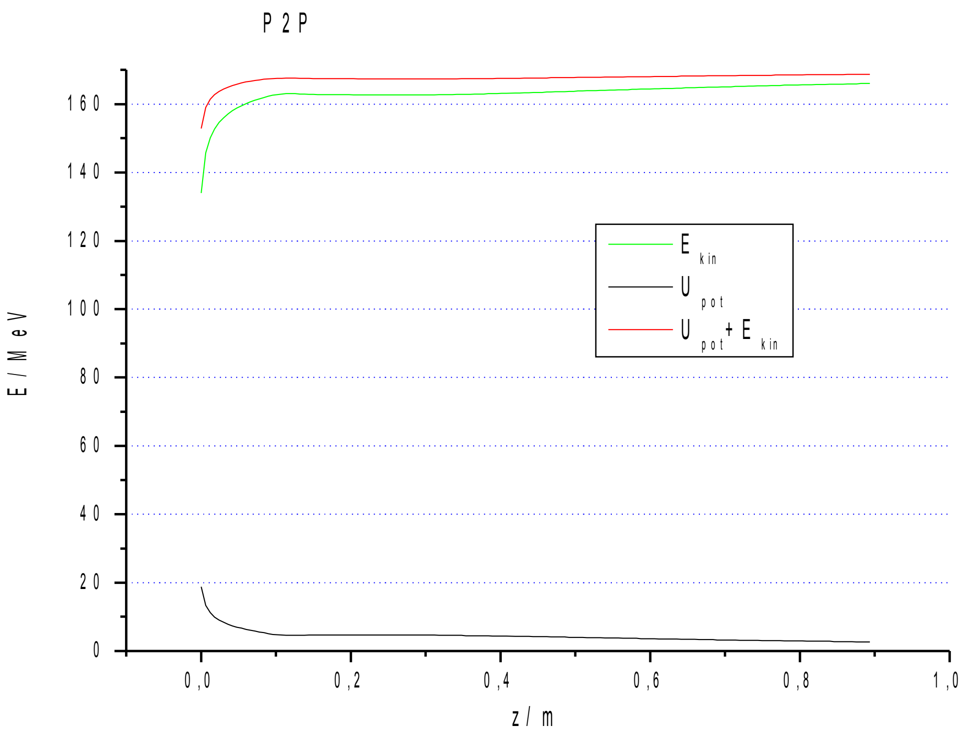
"Benchmark test" for the "extreme case": 130 MeV 1.2 nC: $\sigma_x = \sigma_y = \sigma_z = 1$ µm

Point 2 Point:

Mean rest frame:



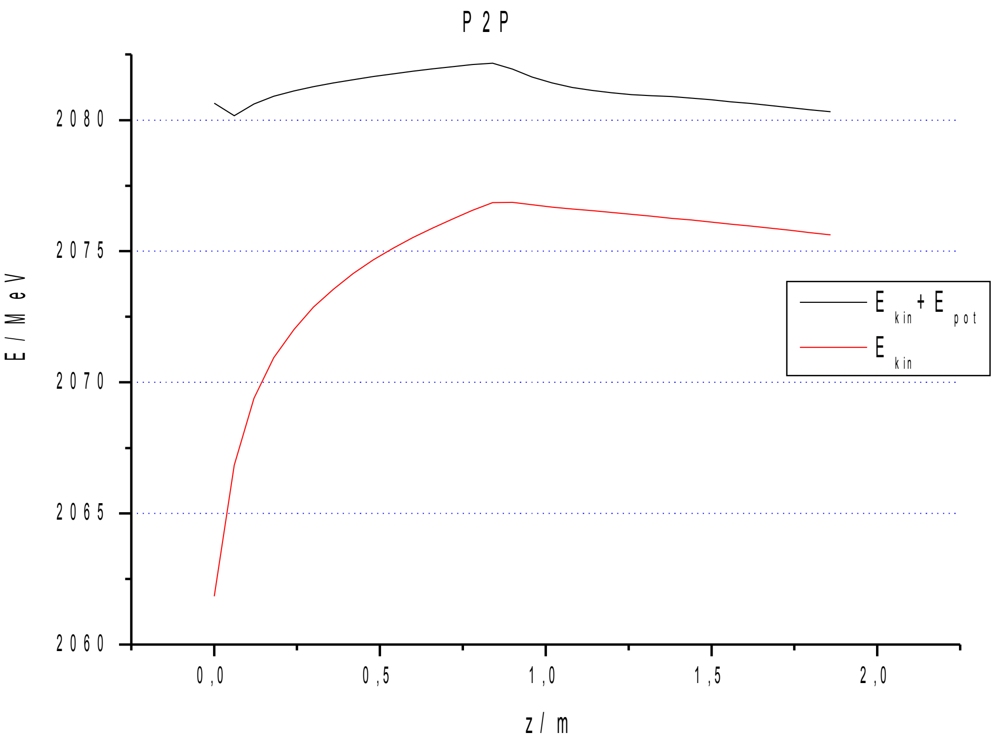
Laboratory frame:



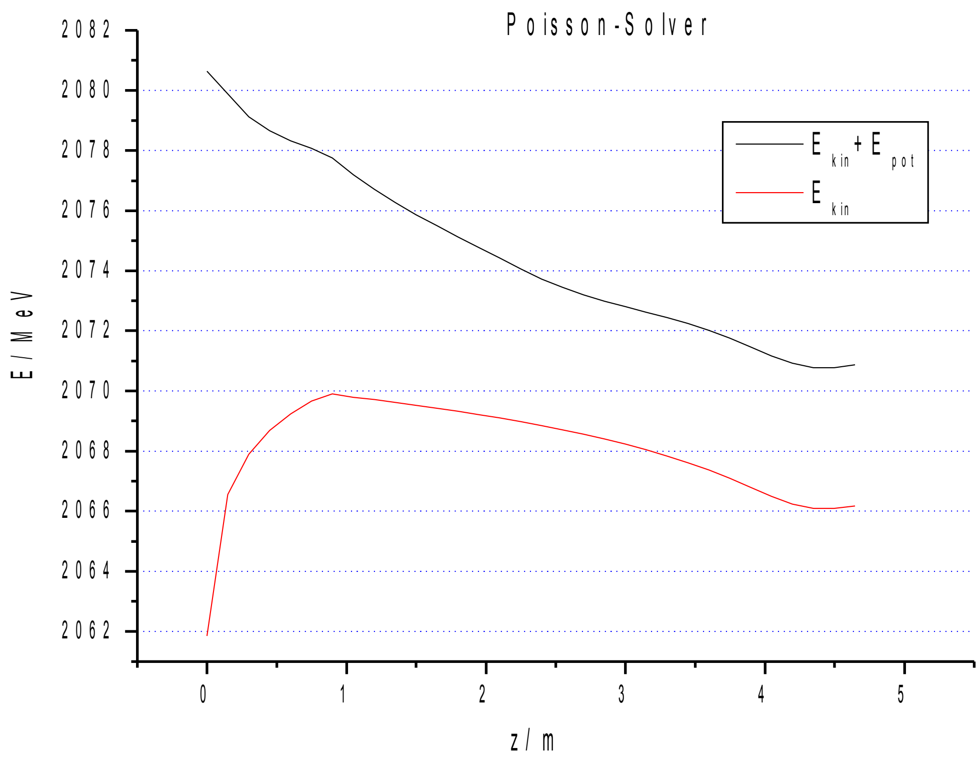
Simulation Benchmark: Energy conservation assuming mass renormalization

"Benchmark test" for 2 GeV 1.0 nC: $\sigma_x = \sigma_y = \sigma_z = 1\ \mu m$

Point 2 Point



Poisson-Solver



Simulation Benchmark: Energy conservation assuming mass renormalization

Calculation of potential energy:
Lorentz Transformation into mean beam rest frame:

$$U' = \frac{1}{8\pi\epsilon_0}\sum_{i\neq j}\frac{q_i q_j}{|r'_i - r'_j|}$$

$$r_x' = r_x$$
$$r_y' = r_y$$
$$r_z' = \gamma_z r_x$$

$$p_z = \beta_z \gamma m_0 c$$
$$\gamma_z' \approx p_z / m_0 c$$

$$\boxed{E_{pot} = \gamma_z U'}$$

Summary

- Experimental Status
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  - Renormalization of the electron mass
- GPT: The applied calculation method
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- Simulating 2 GeV, 1 nC
  - Point 2 Point and Poisson-Solver still differ
  - Convergence of Point 2 Point
- Simulation Benchmark: Energy conservation assuming mass renormalization

Thank you !!!