Linear Coupling Correction at Tevatron using TBT data

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Basic Relations

Hamiltonian perturbation theory (developed by Yuri Alexahin).
Solution of unpertubed (linear and uncoupled) motion

$$z = |A_z|\sqrt{\beta}_z \mathrm{e}^{(i\mu_z+\psi_z)} + c.c. \equiv a_z\sqrt{\beta}_z \mathrm{e}^{i(\mu_z-Q_z\theta)} + c.c. \qquad z = x, y$$

Any solution can be written in terms of the one-revolution matrix (periodic) eigenvectors

$$\vec{z} = V(\theta)\vec{a}(\theta)$$

$$v_{z1} = \sqrt{\beta}_z \mathrm{e}^{(i\mu_z - Q_z\theta)}$$

$$v_{z2} = \frac{i-\alpha_z}{\sqrt{\beta}_z}\mathrm{e}^{(i\mu_z - Q_z\theta)}$$

The coefficients $a_z(\theta)$ propagate as

$$a_z(\theta) = a_z(0)\mathrm{e}^{iQ_z\theta}$$

and the unperturbed hamiltonian is

$$U_0 = i(Q_x a_x a_x^* + Q_y a_y a_y^*)$$

In presence of perturbartions, $\vec{z} = V(\theta)\vec{a}(\theta)$ is a change of variables; new hamiltonian

$$U = U_0 + H_1(\vec{z}) = U_0 + U_1(\vec{a})$$

Linear coupling

$$U_1(\vec{a}) = \frac{i}{2}[C_+(\theta)a_x a_y + C_+^*(\theta)a_x^* a_y^* + C_- a_x a_y^* + C_-^* a_x^* a_y]$$

with

$$C_\pm(\theta) \equiv \frac{R\sqrt{\beta_x\beta_y}}{2B\rho}\Big\{\Big(\frac{\partial B_x}{\partial x}-\frac{\partial B_y}{\partial y}\Big)+B_\theta\Big[\Big(\frac{\alpha_x}{\beta_x}-\frac{\alpha_y}{\beta_y}\Big)-i\Big(\frac{1}{\beta_x}\mp\frac{1}{\beta_y}\Big)\Big]\Big\}\mathrm{e}^{i(\Phi_x\pm\Phi_y)}$$

and

$$\Phi_z \equiv \mu_z - Q_z\theta$$

"Ansatz"

$$a_x(\theta) = a_{x0}(\theta) + w_-^*(\theta)a_{y0}(\theta) + w_+^*(\theta)a_{y0}^*(\theta)$$

$$a_y(\theta) = a_{y0}(\theta) - w_-(\theta)a_{x0}^*(\theta) + w_+^*(\theta)a_{x0}^*(\theta)$$

Inserting into the equation of motion and keeping the first order terms one finds the equation for $w_\pm$

$$C_\pm(\theta) = 2i\mathrm{e}^{-iQ_\pm\theta}\frac{d}{d\theta}\mathrm{e}^{iQ_\pm\theta}w_\pm(\theta)$$

which periodic solution is

$$w_\pm(\theta) = -\int_0^{2\pi} d\theta' \frac{C_\pm(\theta')}{4\sin\pi Q_\pm}\mathrm{e}^{-iQ_\pm[\theta-\theta'-\pi\mathrm{sign}(\theta-\theta')]}$$

with

$$Q_\pm \equiv Q_x \pm Q_y$$

The functions $\tilde{w}_\pm \equiv w_\pm \mathrm{e}^{iQ_\pm\theta}$ are

- constant in coupler free regions
- experience a discontinuity $-iC_\pm\ell/2R$ at coupler locations
- on the resonances $Q_x \pm Q_y = int$ are *constant.*

Minimum tune split

$$\Delta \equiv |\bar{C}_-^{n_-}| \qquad \bar{C}_\pm^{n_\pm} = \frac{1}{2\pi}\int_0^{2\pi} d\theta\ C_\pm \mathrm{e}^{in_\pm\theta} = \frac{n_\pm - Q_\pm}{\pi}\int_0^{2\pi} d\theta\ w_\pm \mathrm{e}^{in_\pm\theta}$$

with

$$n_\pm \equiv \mathrm{Round}(Q_x \pm Q_y)$$

TBT beam position at the j-th vertical BPM following a horizontal kick

$$y_n^j = \left[\sqrt{\beta_y^j}\left(\mathrm{e}^{-i\Phi_y^j} w_+^j - \mathrm{e}^{i\Phi_y^j} w_-^j\right) - \sqrt{\beta_x^j}\mathrm{e}^{i\Phi_x^j}\sin\chi_j\right] A_x \mathrm{e}^{iQ_x(\theta_j+2\pi n)} + c.c.$$

TBT beam position at the j-th horizontal BPM following a vertical kick

$$x_n^j = \left[\sqrt{\beta_x^j}\left(\mathrm{e}^{-i\Phi_x^j} w_+^j + \mathrm{e}^{i\Phi_x^j} w_-^{*j}\right) + \sqrt{\beta_y^j}\mathrm{e}^{i\Phi_y^j}\sin\chi_j\right] A_y \mathrm{e}^{iQ_y(\theta_j+2\pi n)} + c.c.$$

($\chi_j \equiv$ tilt of the j-th BPM).

FFT of z^j gives the *twiss functions*

$$\beta_z^j = |Z_j(Q_z)|^2/A_z^2 \qquad\qquad \mu_z^j = \arg\,(Z_j) - \psi_z$$

with

$$Z^j(Q_z) \equiv \text{ Fourier component of } z^j$$

$$A_z = |A_z|\mathrm{e}^{i\psi_z} \equiv \text{ constant of motion}$$

Amplitude fit

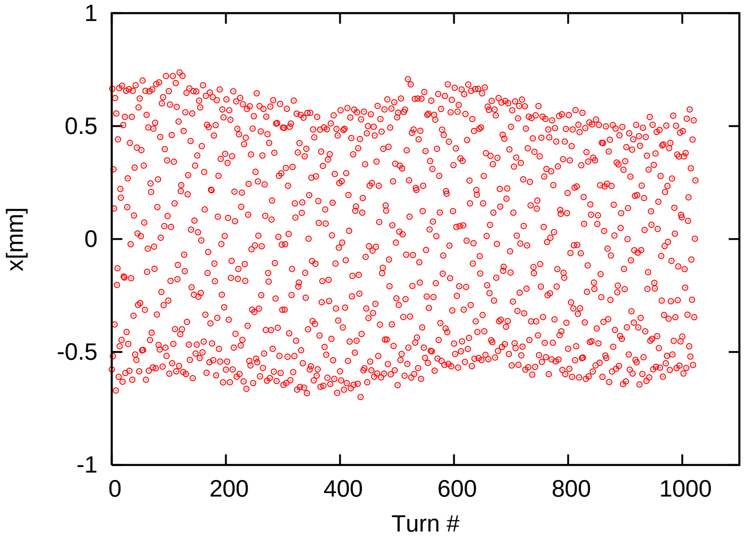
$$|A_z|^2 = \frac{\sum_j 1/\beta_{0z}^j}{\sum_j 1/|Z^j(Q_z))|^2}$$

The FFT of y^j at Q_x, $Y^j(Q_x)$, for a horizontal kick, or $X^j(Q_y)$) for a vertical one, is proportional to the *coupling functions* $w_\pm(\theta_j)$. Assuming χ_j known, we get two equations per BPM in 4 unknowns. Assuming that between two consecutive monitors there are no strong source of coupling, one can solve in favour of $w_\pm(\theta_j) = w_\pm(\theta_{j+1})$.

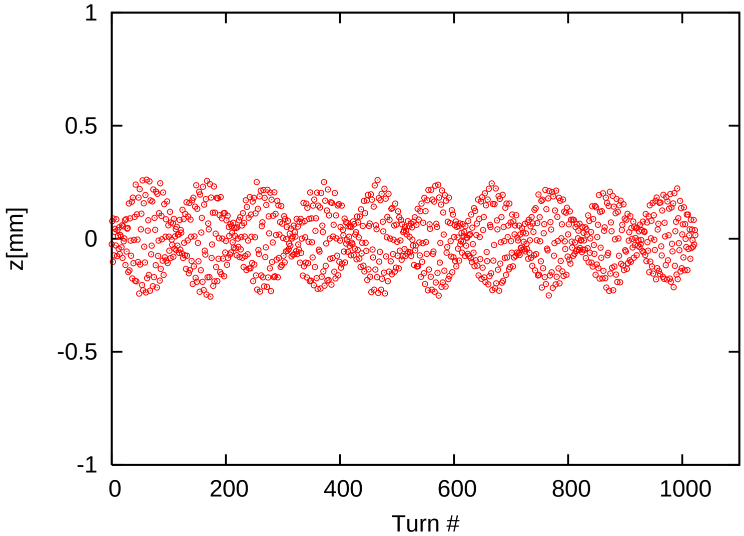
Some Results for TEVATRON

TEVATRON has 118 horizontal and 118 vertical BPM's. They can store 8192 positions data per BPM. The recent upgrade of their electronics allows a precise measurement of the TBT beam position (resolution $\simeq 50\ \mu$m) making possible the use of TBT techniques.

Under "ideal" conditions the oscillations following a kick last some thousand turns

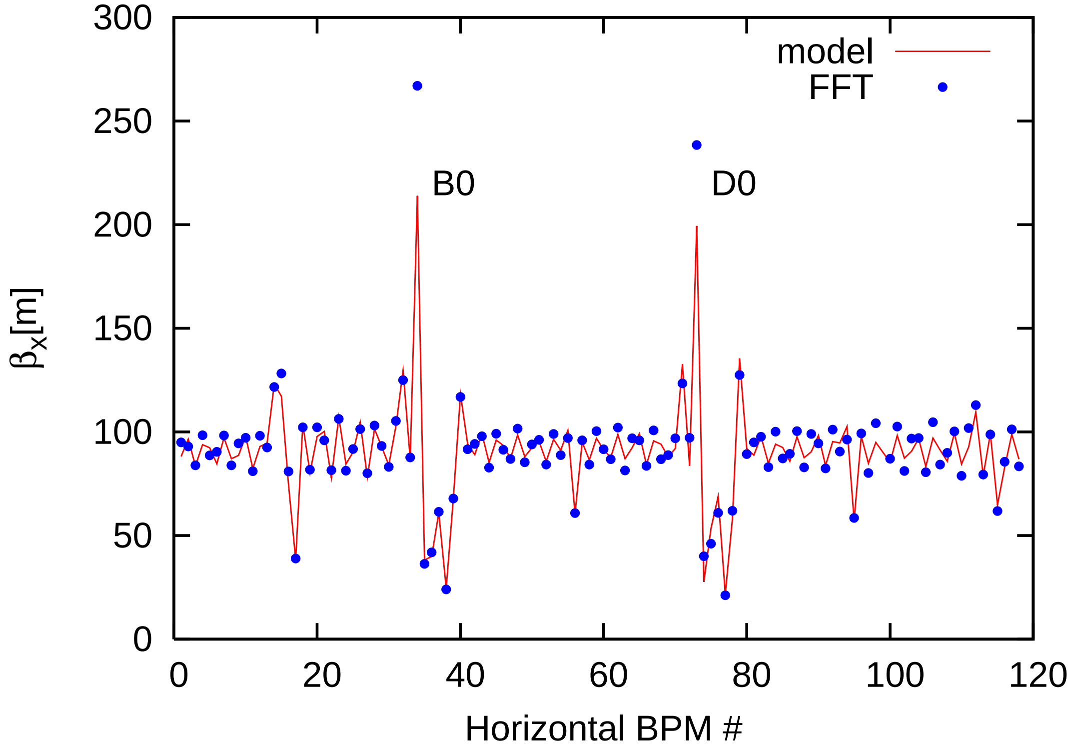


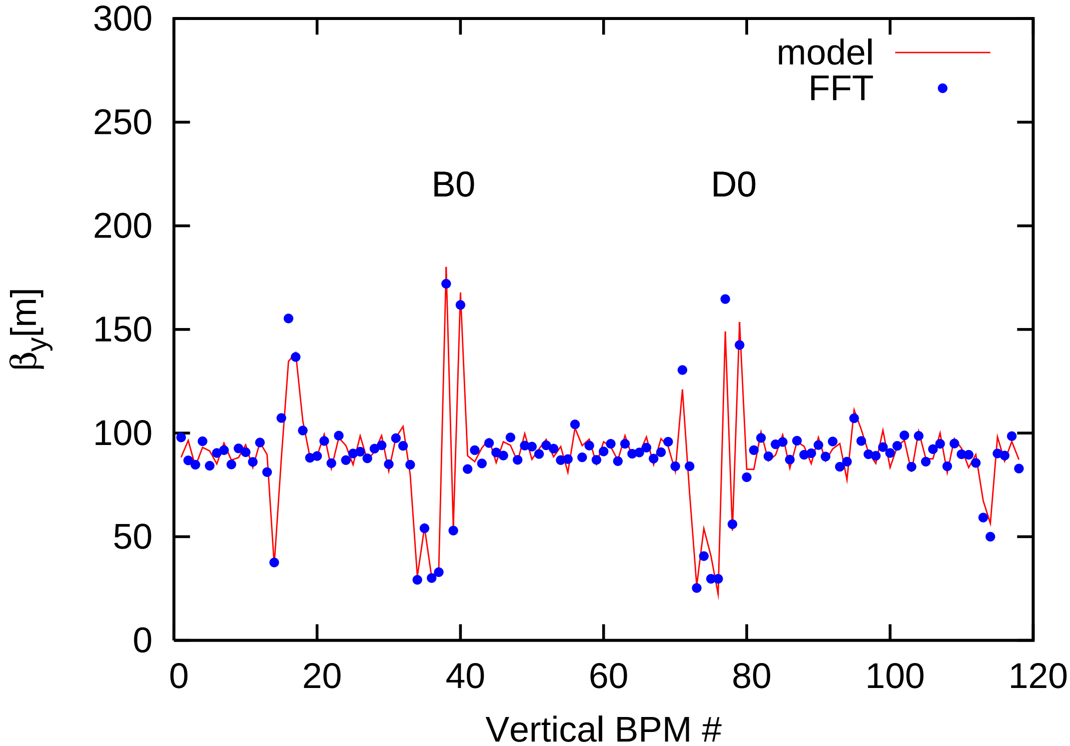
TBT beam position at HBPMF19 following a h-kick.



Resulting TBT beam position at VBPMF18.

Reconstructed (injection) optics





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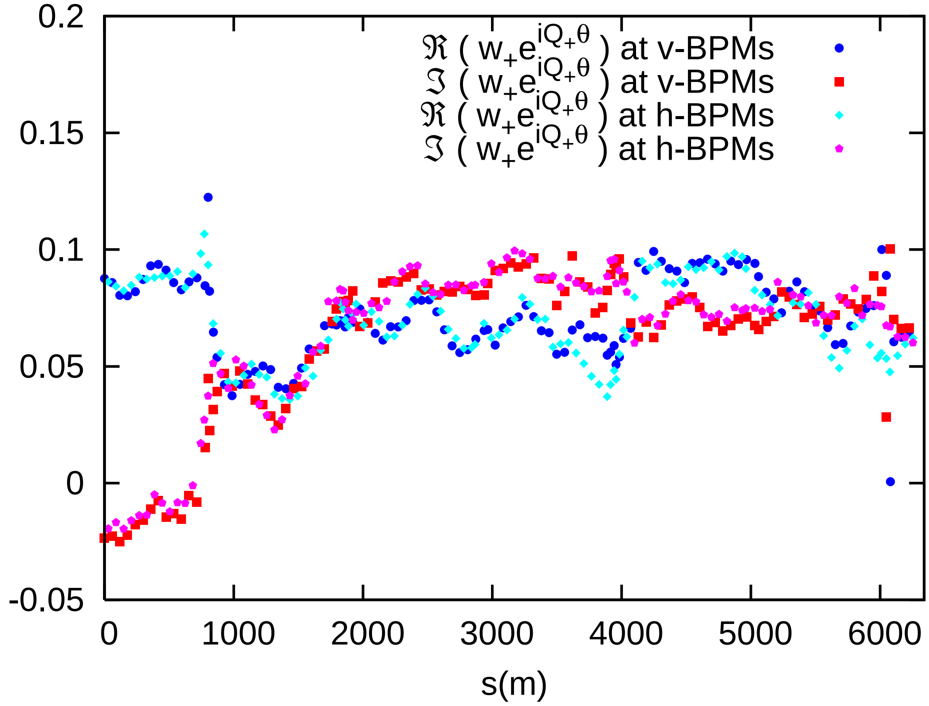
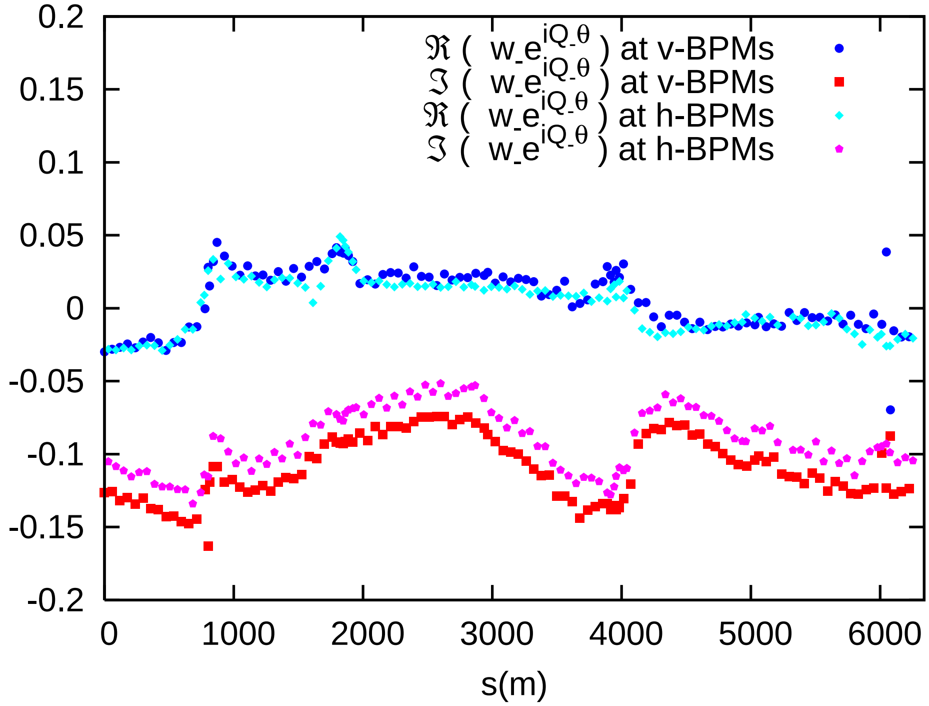
Reconstructed coupling functions





BPM's calibration errors affect the value of β_z^j $(z = x, y)$ computed through the Fourier analysis.

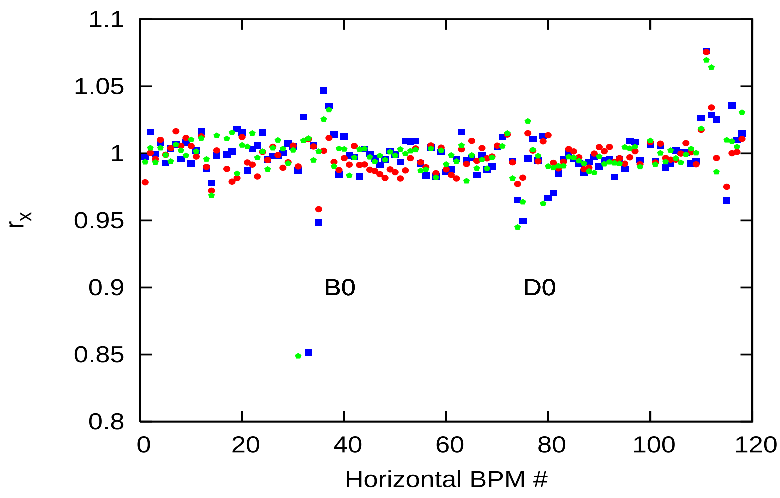
The effect of *random* calibration errors results in a unphysical beta-beating which is likely to average away when computing the oscillation amplitude.

A *systematic* calibration error has no effect on the evaluation of the β functions, but results in a wrong estimate of the oscillation amplitude and therefore of $w_\pm^j$ (unless the error is the same, for both horizontal and vertical BPM's).

By requiring

$$M_{12}^{meas} = M_{12}^{theo}$$

one can compute β_z^j resorting only on the measured phase advance. This requires (at least) three (consecutive) BPM's. Comparison with the value computed through the Fourier analysis may be used to calibrate the BPM's involved.



Example of h-BPM's relative calibration; average ratio=0.9987

However this will *not* correct for a possible systematic calibration error. Through simulations we have estimated that the error on the evaluation of $|\bar{C}_-|$ is

- $\sim 2.5\%$ for 5% systematic calibration error of either horizontal or vertical BPM's (they cancel out when the error has the *same* value)
- 0.5% for 5% random calibration errors
- a systematic tilt by 1^0 of all BPM's results in a error $|\delta\bar{C}_-| \simeq 0.0002$
- the error due to random tilts is negligible.

The application program

An application program for the TBT analysis has been integrated in the TEVATRON control system mainly as a fast tool for correcting the coupling during *shot set up*. The program

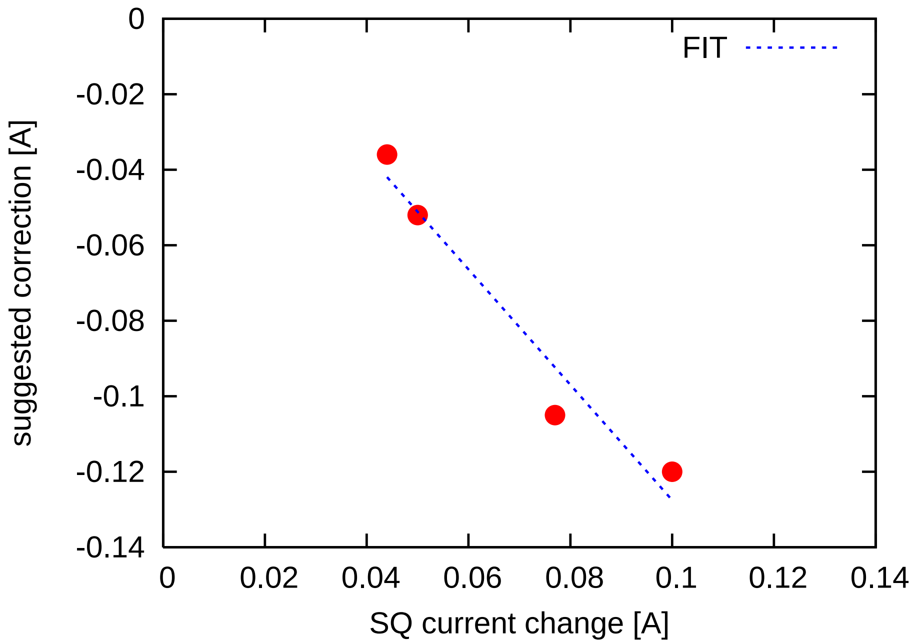
- fires the horizontal or vertical kicker
- computes the linear twiss and coupling functions
- computes and applies the needed corrections to the skew quadrupole circuits SQA0 and SQ.

The time needed to retrieve the data is too large (for instance 7 minutes for 256 turns and 236 BPM's) to make use of all BPM's during routine operation.

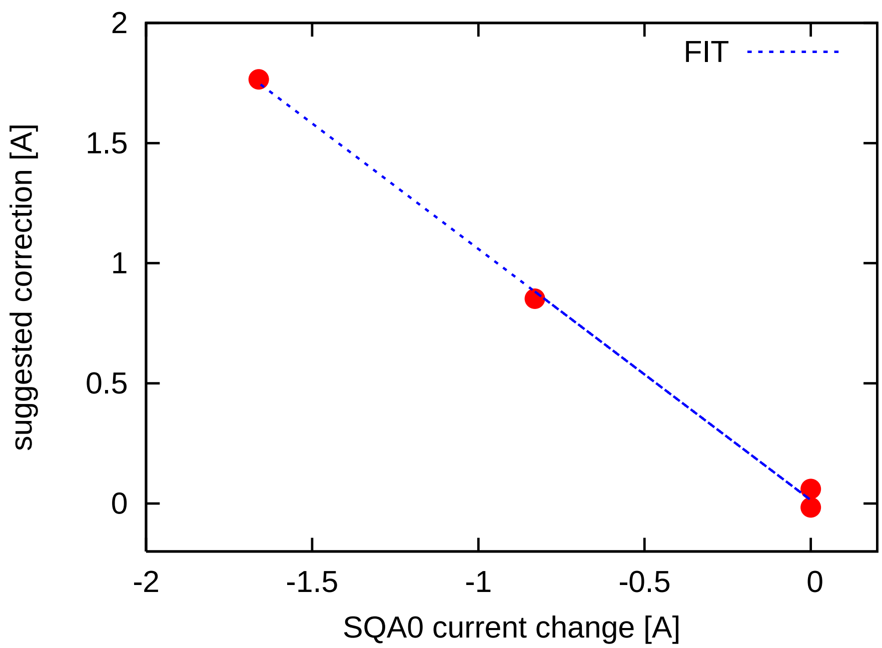
The TEVATRON working point (Q_x=20.584 and Q_z=20.574) being close to $Q_x \pm Q_y = int$, typically we use just 5 horizontal and 5 vertical BPM's.

Off-line analysis using all BPM's has shown only little differences.

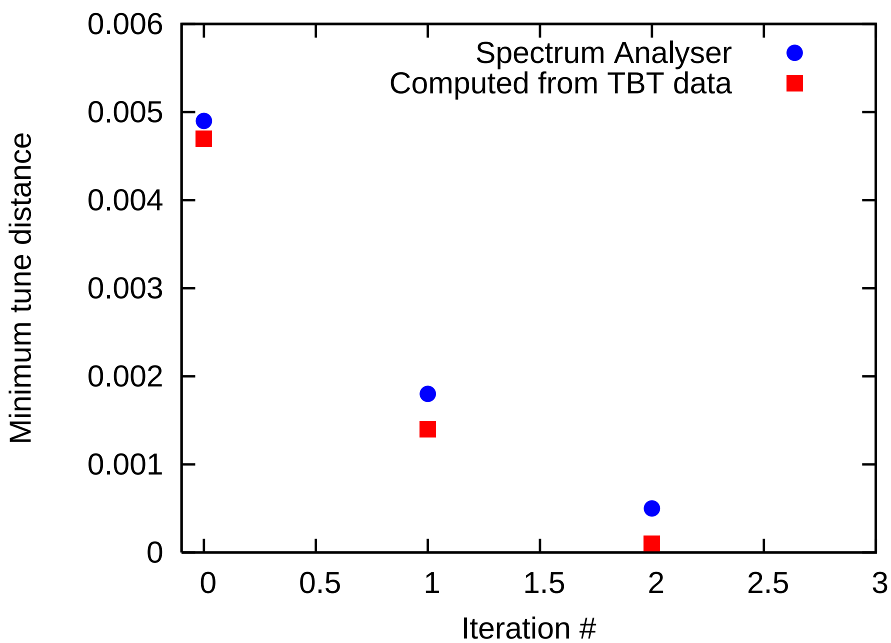
Correcting SQ current vs. SQ excitation



Correcting SQA0 current vs. SQA0 excitation



Minimum tune split measured with S.A. and computed from TBT data

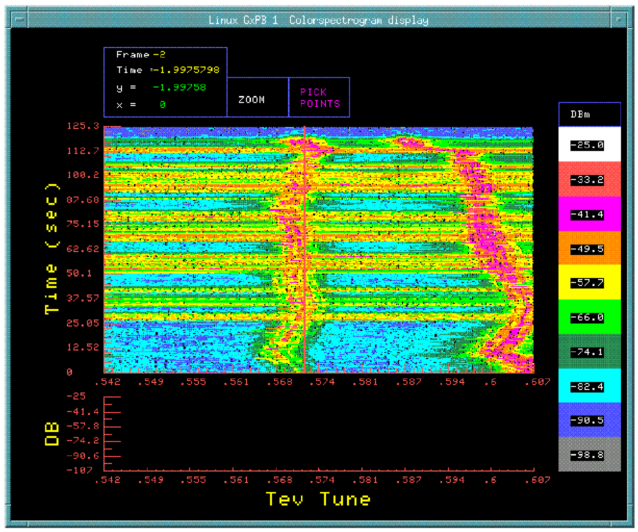


Dependence upon kick amplitude

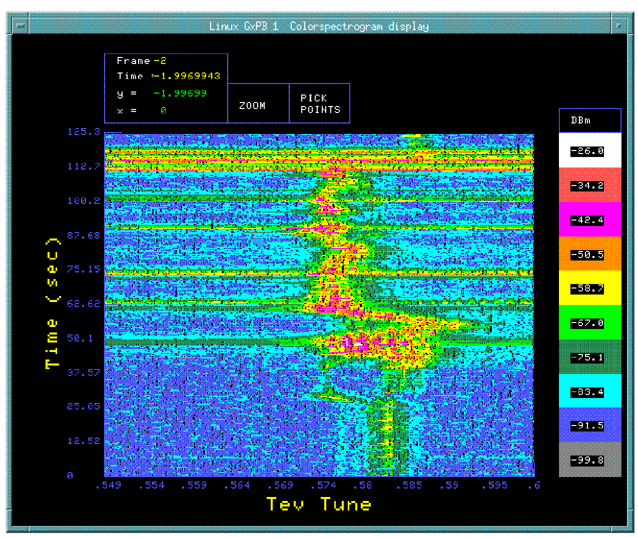
A_x(a.u.)	$\bar{C}_+$	$\bar{C}_-$
38.7	-.0244 -i .0047	.00268 +i .00136
105.2	-.0246 -i .0046	.00271 +i .00152
127.3	-.0250 -i .0043	.00279 +i .00159
136.6	-.0253 -i .0043	.00272 +i .00174

TEVATRON being a fast ramping machine (83 seconds from 150 to 980 GeV), the TBT analysis is the only practical method for measuring optics and coupling also during *acceleration*. The application was used in June after last shut-down for decoupling on the ramp.

First ramp after shut down



Ramp tunes after coupling correction through TBT analysis



Summary

- TBT analysis has been integrated in the TEVATRON control system as a fast tool for correcting the coupling during shot set up
- the method can be applied during acceleration and has been used after last shut down to set up the ramp instead of the time consuming "trial and error" method used before
- the speed of the BPM's data transmission should be improved