Smith-Pursell radiation from different kinds of gratings. Comparison of theoretical models and experimental results.

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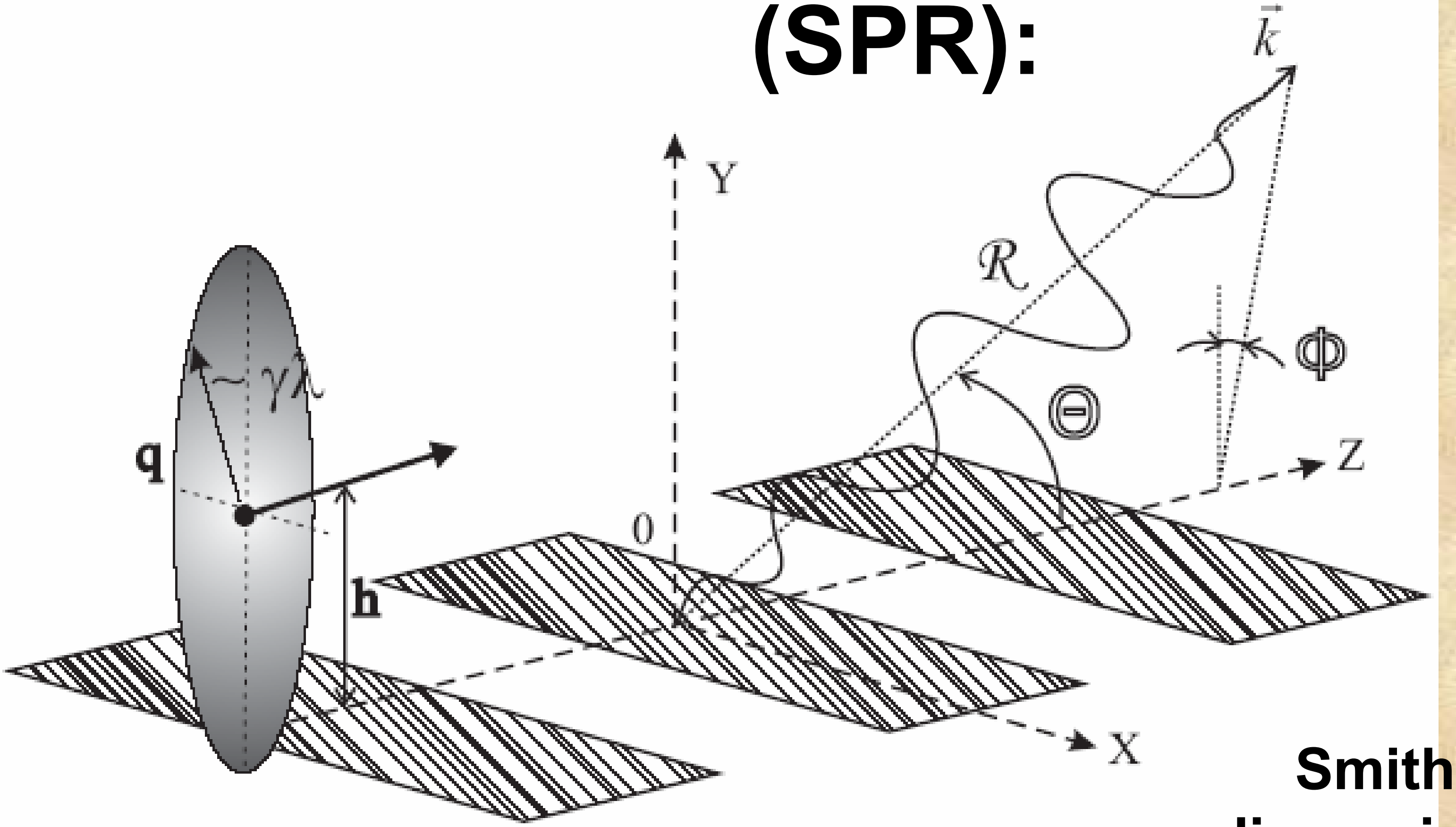
Investigation problems

- **CSPR from different profile targets – comparison and simulations based on the Van den Berg model, surface current model and resonant diffraction radiation one.**
- **Pre-wave zone effect for SPR**
- **Experimental results on coherent SPR**
- **Estimation of the electron bunch length**

Incoherent Smith-Purcell radiation (SPR):



Smith-Purcell dispersion relation:

$$\lambda = \frac{d}{n}\left(1 - Cos[\Theta]\right)$$

First observed by

S.J. Smith and E.M. Purcell,

Phys. Rev. Lett. **92**, 1069 (1953))

Possibilities of SPR application:

1. *Non-invasive low-emittance relativistic beam diagnostics (D.C. Nguyen, Nucl. Instr. Meth. Phys. Res. A **393**,(1997) 514).*
2. *Compact free electron lasers based on SPR in millimeter and sub-millimeter range (V. Kumar and K.-J. Kim, Phys. Rev. E **73**, 026501 (2006)).*
3. *Beam position monitor based on SPR (G. Doucas, M.F.Kimmit, J.H.Brownell et.al, Nucl. Instr. Meth. Phys. Res. A **474**, (2001) 10)*

There are some fuzzy points of SPR features need to clarify:

1. *There are many different models exist to describe features of SPR from gratings of different profile.*

<u>What model is correct for $E_e \leq 10$ MeV and chosen grating profile?</u>

<u>What kind of grating profile may provide the best "coupling" between beam and grating?</u>

2. *All the SPR models describe SPR features in so-called "wave" (or "far") zone where the radiation source can be considered as point-like.*

<u>When does this approximation correct?</u>

<u>What are the main differences of SPR properties in "wave" and "pre-wave" zones (where this approximation is not valid)?</u>

Let us compare SPR models. We will consider the most often used of them:

1. **Van den Berg's model** (P.M. van den Berg, J. Opt. Soc. Am. 63, No.12, 1588-1597 (1973)) – *the relativistic particle field is substituted for a packet of evanescent waves which are diffracting on the grating according to optics laws.*

2. **Surface current model** (J.H. Brownell, J. Walsh, G. Doucas, Phys. Rev. E 57, No.1, p. 1075-1080 (1998)) – *SPR is generated by time-dependent surface current induced by relativistic particle field on the grating.*

3. **Resonant diffraction radiation (RDR) model** (A.P. Potylitsyn, Nucl. Instr. Meth. Phys. Res. B 145, 60-66 (1998)) – *an exact solution of Maxwell's equations for the radiation of relativistic particle moving close to conducting semi-plane was used.*

Difference between SPR intensity calculated using models №1 and №2,№3 ones may achieve 2 orders of magnitude for some gratings and ***$E_e \leq$ 10 MeV***

Experiment investigations of SPR

The experiments those authors came to conclusion about accordance of their data with theoretical predictions of Surface Current model:

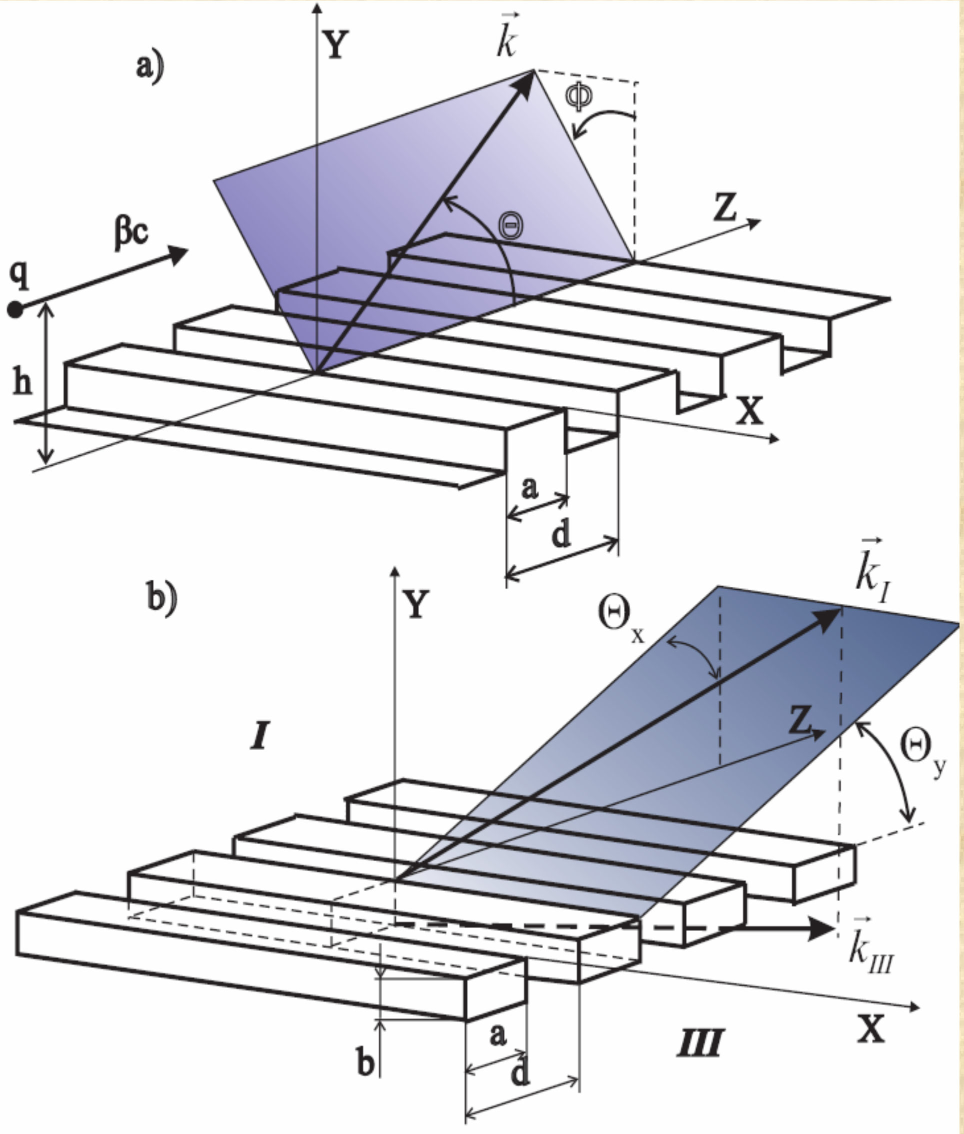
1. *K.J. Woods et al., Phys. Rev. Lett.* ***74****, 3808 (1995).* E_e*=2.8 MeV*
2. *J.H. Brownell et al. J. Phys. D: Appl. Phys.* ***30****, 2478-2481 (1997).* E_e*=3.6 MeV*
3. *A. Doria et al. Nucl. Instr. Meth. Phys. Res. A* ***483****, 263 (2002).* E_e*=2.3 MeV*
4. *S.E. Korbly et al., Phys. Rev. SP-AB* ***9****, 022802 (2006).* E_e*=0.515 MeV*

The experiments those authors came to conclusion about accordance of their data with theoretical predictions of van den Berg's model:

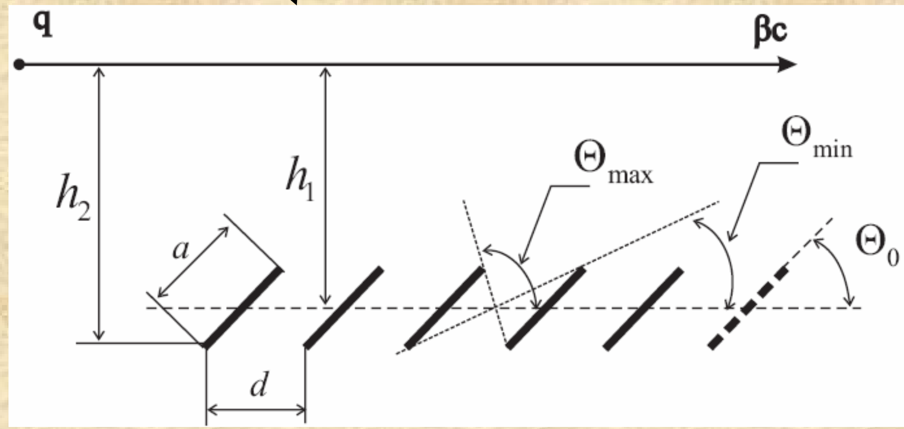
1. *A. Gover et al., J. Opt. Soc. Am. B* ***1****, 723 (1984).* E_e*=0.1 MeV*
2. *G. Kube et al., Phys. Rev. E* ***65****, 056501 (2002).* E_e*=855 MeV*
3. *H. Backe et al., in NATO Workshop "Advanced Radiation Sources and Applications", Springer, 2006, pp. 267 - 282.* E_e*=1.44 MeV*

Grating profiles we will consider:

"Volume" gratings



Flat strip grating



Angular distribution of SPR per an electron per one grating period:

Van den Berg's model:

$$\frac{dW_n}{d\Omega} = \frac{\alpha\hbar c}{2d} n^2 \frac{\sin^2\Theta\cos^2\Phi}{\left(1/\beta - \cos\Theta\right)^3} |\mathfrak{R}_n|^2 \times \exp[-\frac{h}{h_{eff}}\sqrt{1+(\beta\gamma\sin\Theta\sin\Phi)^2}].$$

Surface current model:

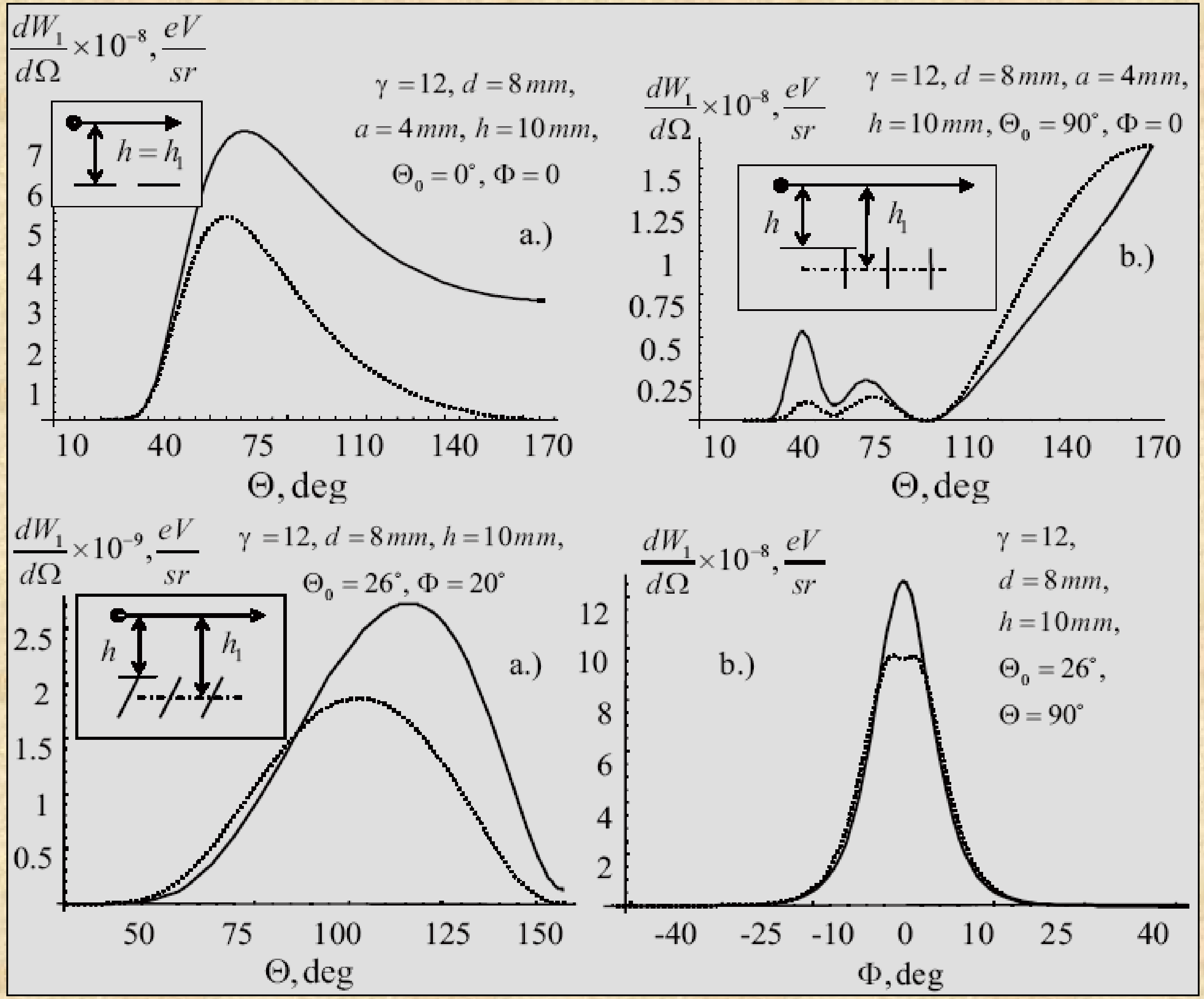
$$\frac{dW_n}{d\Omega} = \alpha\hbar c \frac{2\pi}{d} \frac{n^2}{\left(1/\beta - \cos\Theta\right)^3} \left|\vec{R}_n\right|^2 \times \exp[-\frac{h_2}{h_{eff}}\sqrt{1+(\beta\gamma\sin\Theta\sin\Phi)^2}],$$

$$\left|\vec{R}_n\right|^2 = \left|\left[\vec{n}\times[\vec{n}\times\vec{G}]\right]\right|^2$$

RDR model:

$$\frac{d^2W_{RDR}}{d\omega d\Omega} = \frac{d^2W_{DR}}{d\omega d\Omega} F_{n,cell} F_N$$

Comparison of surface current (SC) and RDR model



Comparison of van den Berg's model and surface current model (RDR)

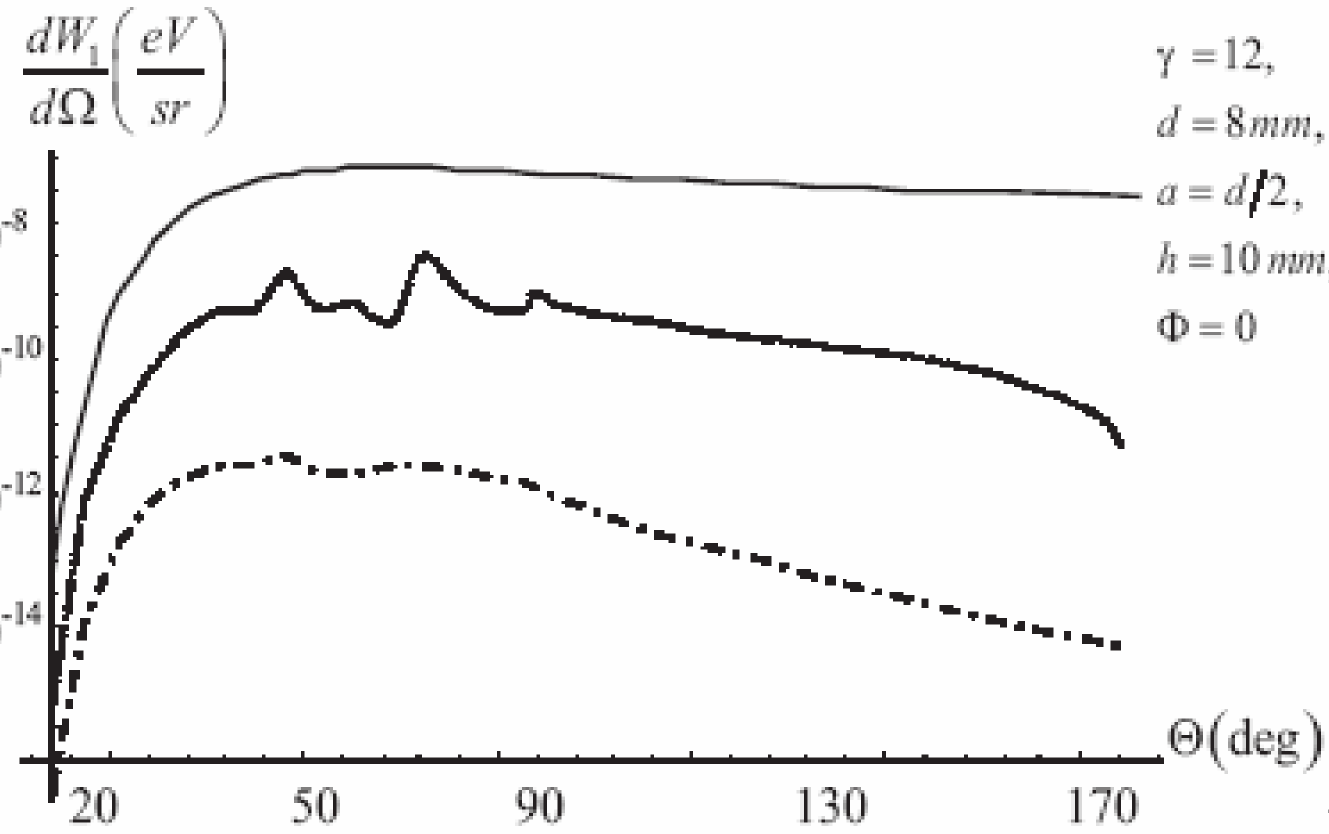


FIG. 6: The angular distribution of the SPR intensity for a "thin" grating with vacuum gaps: a solid line for the RDR model; a dot line (b/d = 1/8) and a dash-dotted line (b/d = 0.001) for van den Berg's model.

Comparison of van den Berg's model and surface current model (RDR)

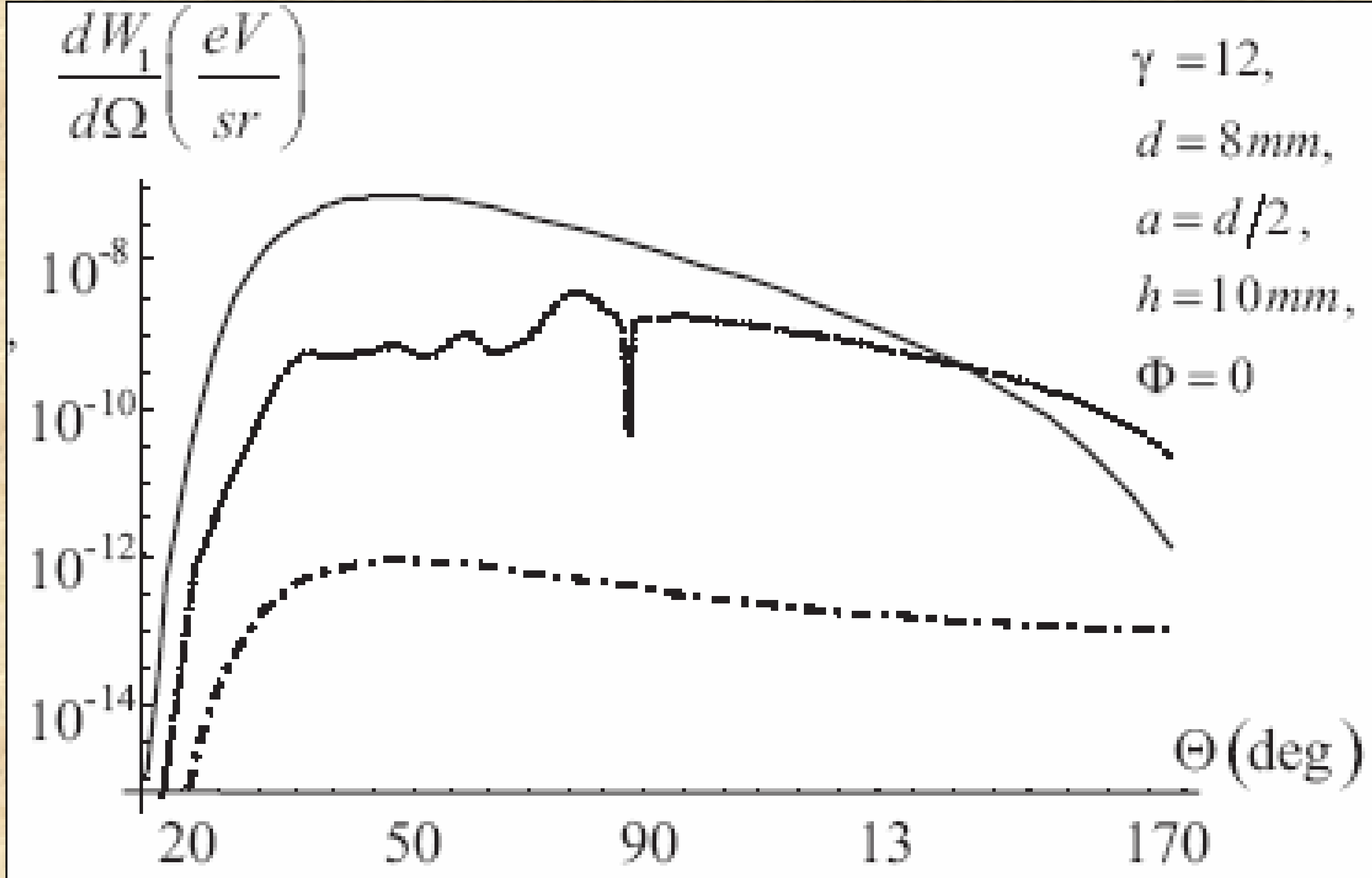


FIG. 7: Angular distribution of the SPR intensity for the lamellar grating: according to the current model - a solid line (b/d = 1/8), according to van den Berg's model - a dot line (b/d = 1/8) and a dash-dotted line (b/d = 0.001).

Comparison of van den Berg's model and surface current model (RDR)

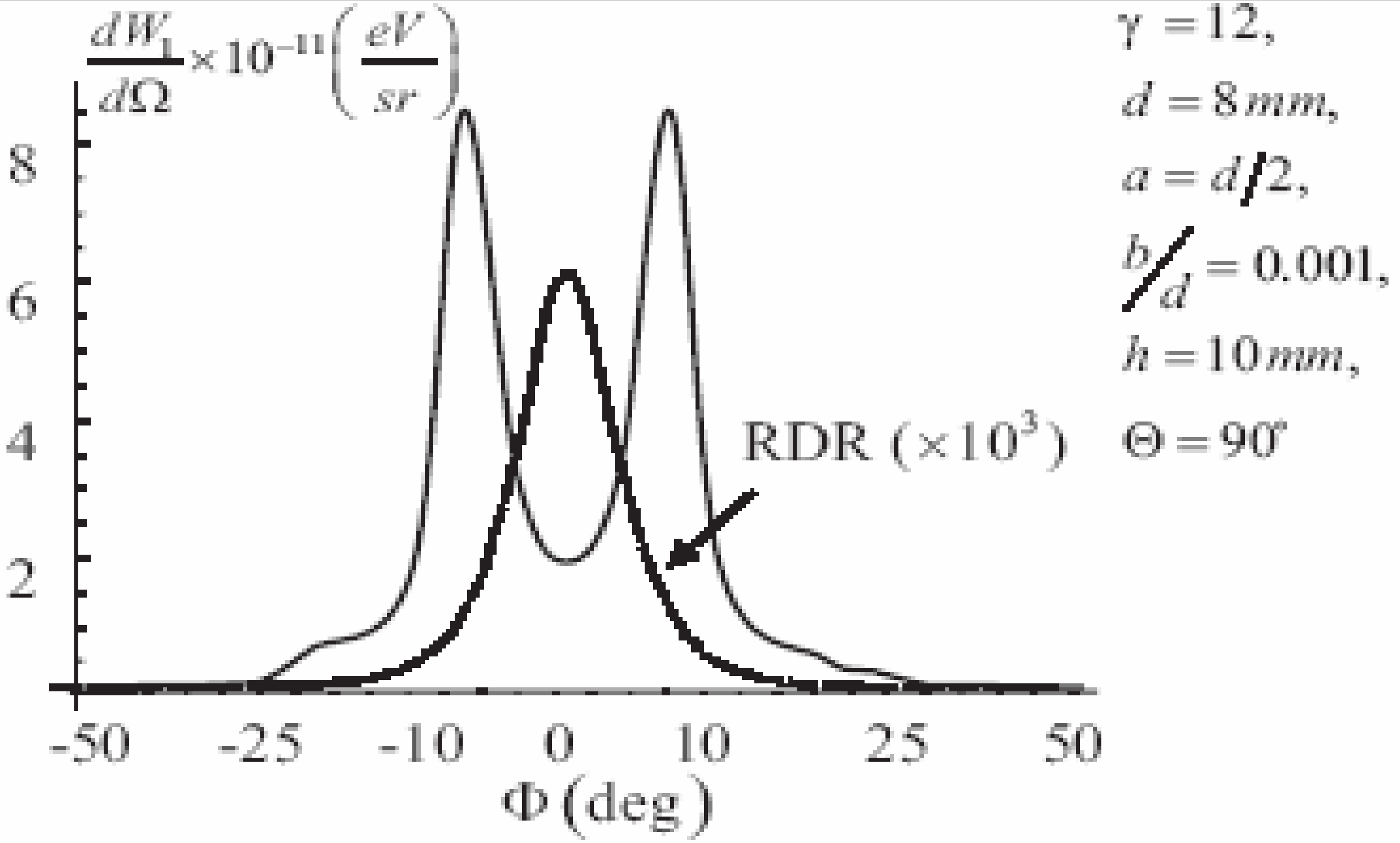
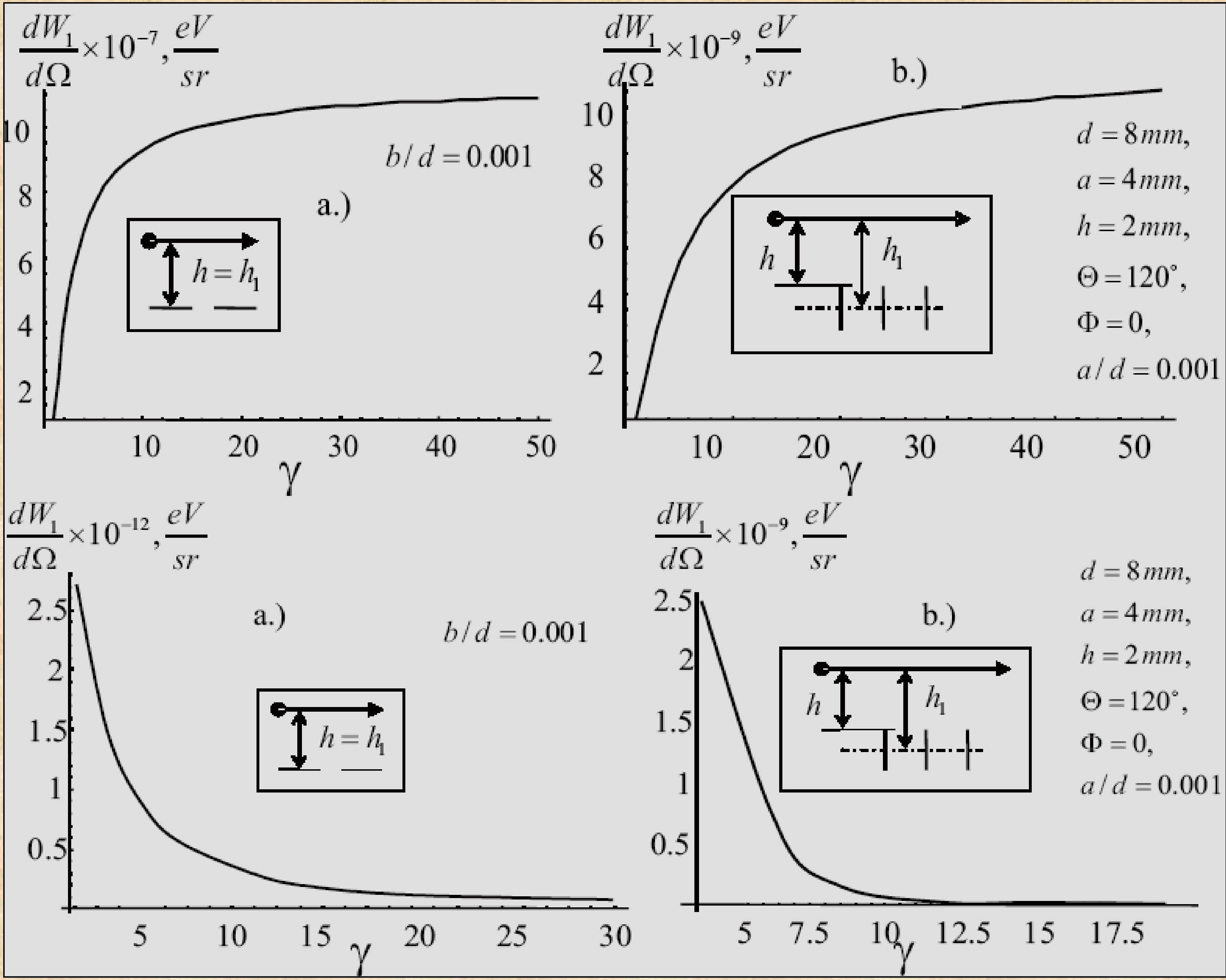


FIG. 11: Azimuth dependence of the SPR intensity for a "flat" grating according to van den Berg's model (a solid line) and according to the RDR model (a dot line, should be multiplied by 10^3).



$\frac{dW_1}{d\Omega}\times10^{-7}, \frac{eV}{sr}$
$b/d = 0.001$
a.)
$h = h_1$
γ
$\frac{dW_1}{d\Omega}\times10^{-9}, \frac{eV}{sr}$
b.)
h
h_1
$d = 8mm,$
$a = 4mm,$
$h = 2mm,$
$\Theta = 120°,$
$\Phi = 0,$
$a/d = 0.001$
γ
$\frac{dW_1}{d\Omega}\times10^{-12}, \frac{eV}{sr}$
a.)
$b/d = 0.001$
$h = h_1$
γ
$\frac{dW_1}{d\Omega}\times10^{-9}, \frac{eV}{sr}$
b.)
h
h_1
$d = 8mm,$
$a = 4mm,$
$h = 2mm,$
$\Theta = 120°,$
$\Phi = 0,$
$a/d = 0.001$
γ

Conclusions from SPR models comparison:

1. There exists a difference in the SPR intensity between the "volume" and "flat" gratings: the SPR yield from the "volume" grating predicted by van den Berg's model for a moderately relativistic case is by two orders of magnitude higher practically for all the polar angles;

2. There exists a difference in the azimuth dependencies for the "flat" gratings: the current and RDR models predict a single-maximum angular distribution with maximum in the plane perpendicular to the grating ($\Phi = 0$), while van den Berg's model predicts the distribution with a minimum in the plane $\Phi = 0$;

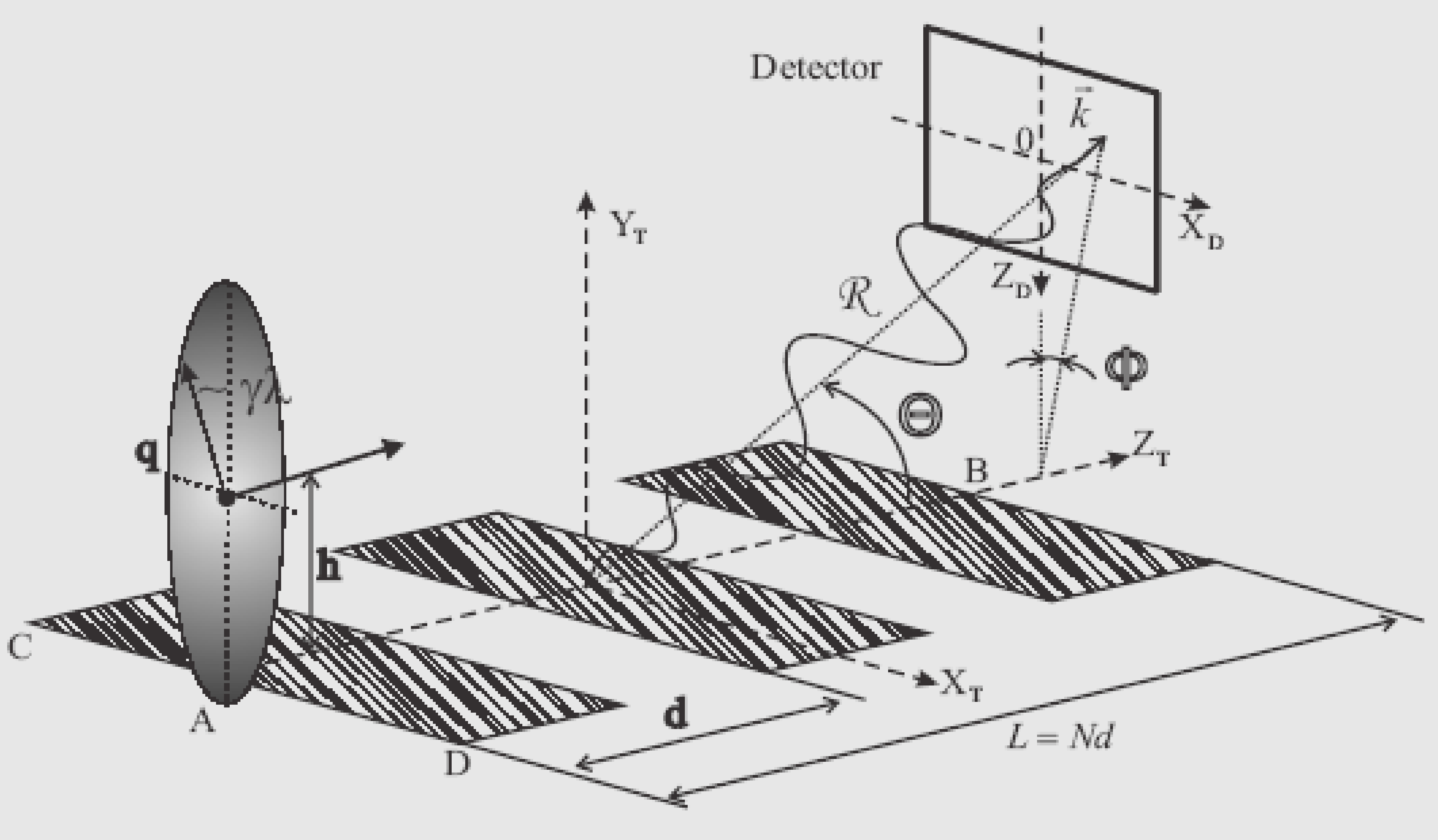
3. There exists a difference in the SPR yield dependence on the Lorentz-factor of a particle: van den Berg's model predicts the yield decrease with the energy growth, while the current and RDR models predict the yield increase.

2. Smith-Purcell radiation in “pre-wave” zone

The unusually large distance corresponds to “far” zone approximation for X-FELs based on undulator radiation. That is because of theirs very long insertion devices. For example, the SLAC insertion device to Linac Coherent Light Source (LCLS) has 140 m length and minimal distance corresponds to “far” zone is about 380 m (*R. Tatchyn, Proc. 27th Int. Free Electron Laser Conf., 21-26 August 2005, Stanford, USA, P.282*).

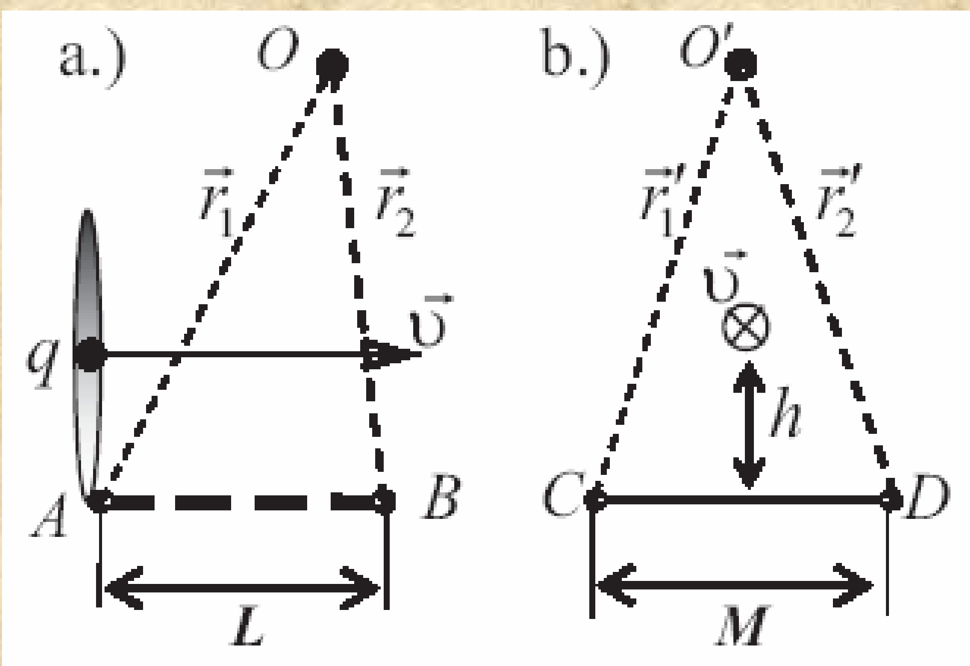
The term “pre-wave zone” was considered for the first time by *V.A. Verzilov, Phys. Letters A 273, 135-140 (2000).* It was shown that for the case of backward transition and diffraction radiation (BTR and BDR respectively) the “far”-zone criterion $R_0 \geq \gamma^2\lambda$ R_0- distance between target and detector centers

SPR geometry



“Far”-zone criterion for SPR

The longitudinal size of radiating surface is equal to grating length ***L* (a)**. The transversal size is equal to grating width (*M*).



Let us consider two waves radiated from opposite sides of grating (a) when a charged particle goes near.

With approximation of far distance from detector (that is much more than grating length) one can write phase difference in the following form:

$$\Delta\varphi = \varphi_B - \varphi_A \approx k\left(r_2 - r_1 + \frac{L}{\beta}\right),$$

“Far”-zone criterion for SPR

Using following expression $\vec{r}_2 = \vec{r}_1 - \vec{L},$

one can obtain:

$$\Delta\varphi \approx k\left(\frac{L^2}{2r_1} - \frac{(\vec{L},\vec{r_1})}{r_1} + \frac{L}{\beta}\right),$$

The two last terms do not depend on the detector distance with respect to the grating unlike the first term that is the first-order correction of “pre”-wave zone.

So, the “far”-zone criterion will be:

$$k\frac{L^2}{2r_1} \ll \pi$$

or:

$$r_1 \gg \frac{L^2}{\lambda_n} \approx N^2 d \frac{n}{\beta^{-1} - \cos\Theta}$$

“Far”-zone criterion for SPR

The same criterion may be obtained for transversal SPR distributions (in case when the grating width less than particle coulomb field radius):

$$r'_1 \gg \frac{M^2}{\lambda_n} \approx M^2 \frac{n}{d(\beta^{-1} - \cos\Theta)}$$

Thus the «far»-zone condition for SPR in relativistic case

does not depend on particle energy!

Let us consider the influence of detector disposition in “pre-wave” zone on the SPR angular distributions.

For that we will use the model proposed by V.A. Verzilov in already cited work. In this model the relativistic particle field is expressed by packet of plane waves.

The field components on the detector plane will be written in the form:

$$\begin{pmatrix} E_x^D \\ E_z^D \end{pmatrix} = const \int_{-M/2}^{M/2} dX_T \int_{-Nd/2}^{Nd/2} dZ_T \begin{pmatrix} X_T \\ h \end{pmatrix} \chi(Z_T) \times$$
$$\times \frac{K_1\left[\frac{2\pi}{\beta\gamma\lambda}\sqrt{X_T^2 + h^2}\right]}{\sqrt{X_T^2 + h^2}} \exp\left[i\Delta\varphi(X_T, Z_T, X_D, Z_D)\right].$$

Radiation intensity will find as usual: $I = const(|E_x^D|^2 + |E_z^D|^2)$

Let us obtain the expression for phase shift:

$$\Delta\varphi = \frac{2\pi}{\lambda}\left(\Delta\mathcal{R} + \frac{Z_T}{\beta}\right)$$

$$\Delta\mathcal{R} \;\; = \;\; \mathcal{R}(Z_T, X_T, X_D, Z_D) \; - \; \mathcal{R}(0, 0, X_D, Z_D)$$

$$\Delta\varphi = \frac{2\pi}{\lambda}\Bigg(\Big[{\mathcal{R}_0}^2 + (X_T - X_D)^2 + Z_T^2 + Z_D^2 + \\ -2Z_T(\mathcal{R}_0\cos\Theta + Z_D\sin\Theta)\Big]^{1/2} - \\ -\Big[{\mathcal{R}_0}^2 + Z_D^2 + X_D^2\Big]^{1/2} + \frac{Z_T}{\beta}\Bigg),$$

Derivation of expression for the phase shift

Assume that:

$$\mathcal{R}_0 \gg X_T, Y_T, X_D, Y_D$$

And using transformation of variables:

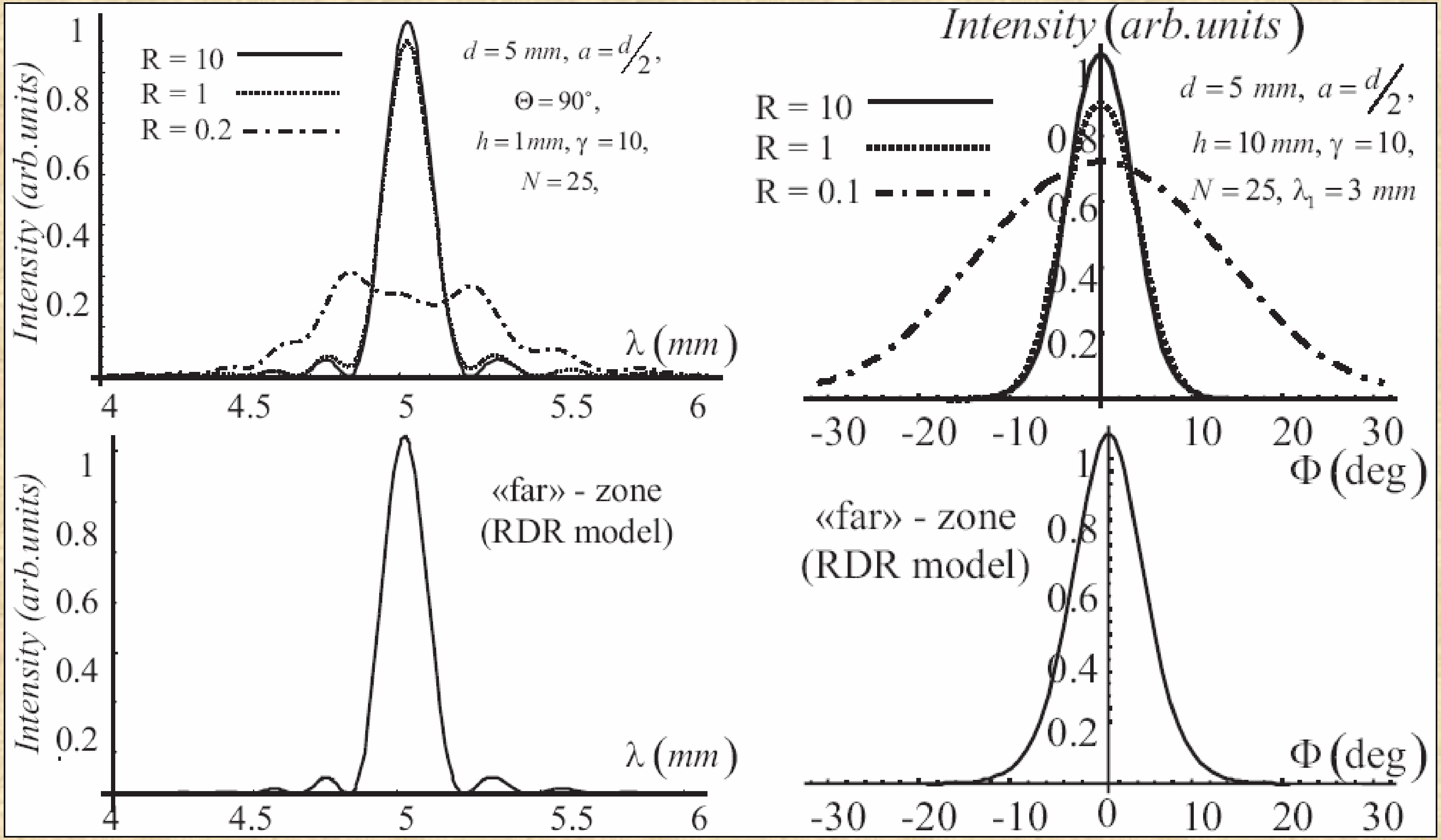
$$\begin{pmatrix} x_T \\ z_T \\ l \end{pmatrix} = \frac{2\pi}{\beta\gamma\lambda}\begin{pmatrix} X_T \\ Z_T \\ L \end{pmatrix}, \quad \begin{pmatrix} x_D \\ z_D \end{pmatrix} = \frac{\beta\gamma}{\mathcal{R}_0}\begin{pmatrix} X_D \\ Z_D \end{pmatrix},$$

$$R = \frac{\mathcal{R}_0}{L^2/\lambda} = 4\pi^2 \frac{1}{l^2}\frac{\mathcal{R}_0}{\gamma^2\lambda}.$$

We obtain more simple expression for the phase shift:

$$\Delta\varphi \approx \pi \frac{{z_T}^2 + {x_T}^2}{l^2 R} - \beta\gamma z_T \cos\Theta - \\ - z_T z_D \sin\Theta - x_T x_D + \gamma z_T .$$

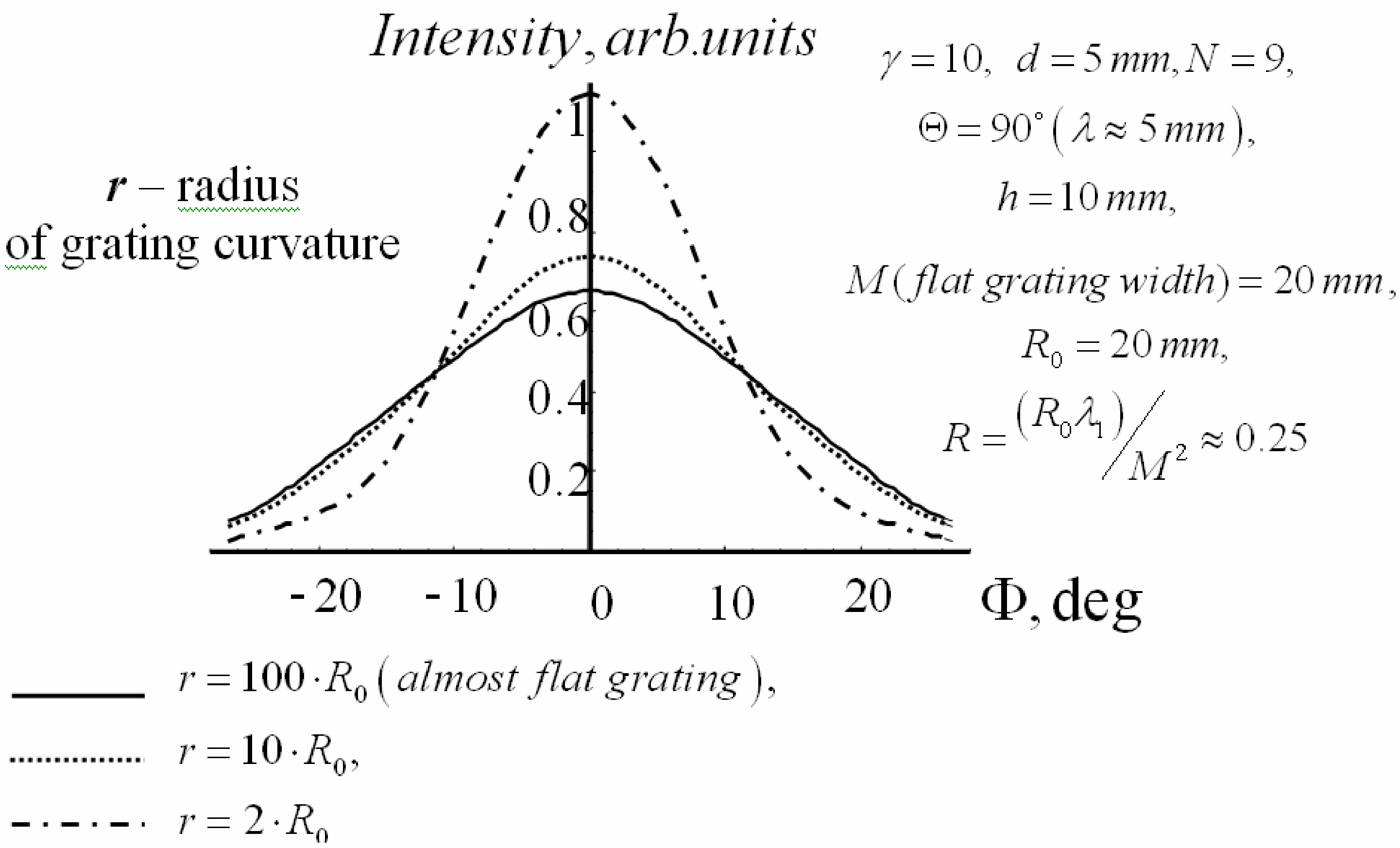
The results obtained



D.V. Karlovets, A.P. Potylitsyn PRST-AB (2006), to be published

$$\tan \Phi \approx \frac{X_D}{\mathcal{R}_0} = \frac{x_D}{\beta\gamma}$$

SPR focusing



Focusing effect for SPR from concave cylindrical grating

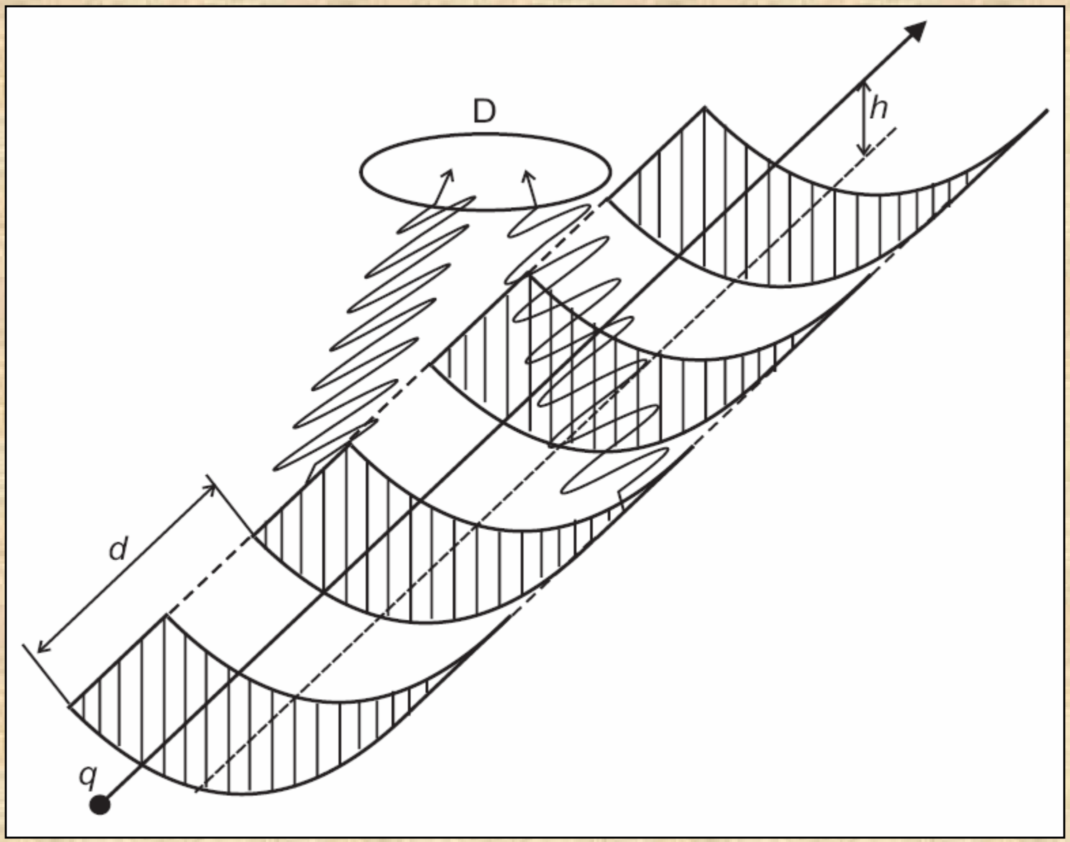
The results obtained

The approach developed allows to calculate SPR characteristics from "concave" grating

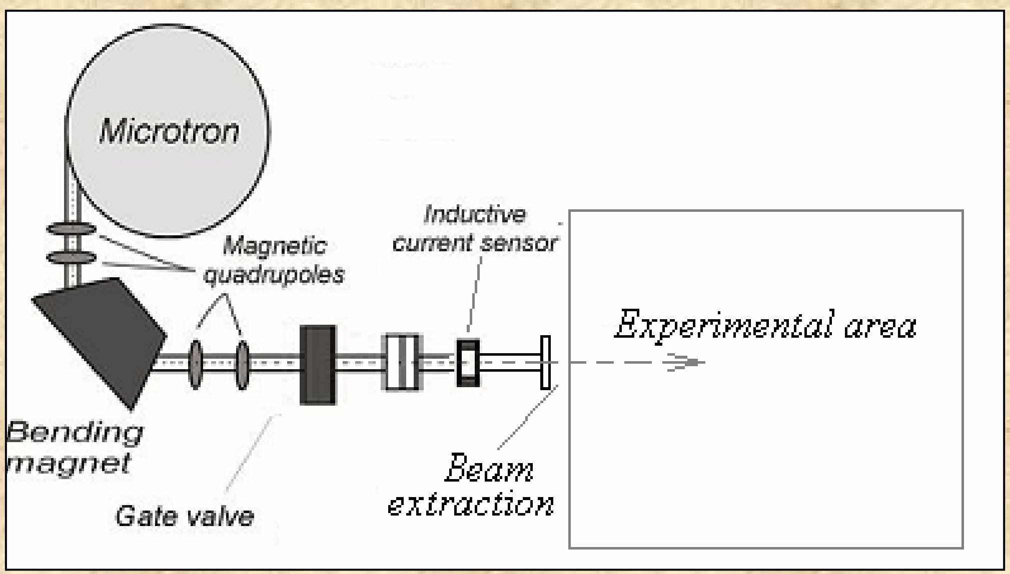
One may expect that for a "cylindrical" strip grating there may exist the "focusing" effect

The experimental verification of the model proposed allows to choose kind of grating (N, d, etc.) beam energy and impact parameter in order to receive the maximal SPR power at the fixed detector position.

So, from figures one can see that with decrease of distance to detector



Experimental scheme



Electron energy	**6.1 MeV**
Macropulse duration	**2 – 6 μs**
Pulse repetition rate	**1 – 8 Hz**
Bunch length σ (Gauss approximation)	**~1.3 mm**
Number of electrons per bunch	**$\sim 10^8$**
Number of bunches per macropulse	**$\sim 10^4$**
Beam size at the microtron output	**4×2 mm^2**
Emittance: **horizontal** **vertical**	 **$3\cdot10^{-2}$ mm×rad** **$1.5\cdot10^{-2}$ mm×rad**

Coherent Smith-Purcell radiation (CSPR):

Spectral-angular density of radiation:

$$\frac{dW_{CSPR}(\omega)}{\hbar d\omega d\Omega} = N_e\left[1 + N_e \cdot f(\omega)\right]\frac{dW_1(\lambda)}{\hbar d\omega d\Omega}$$

N_e - number of electrons per bunch,

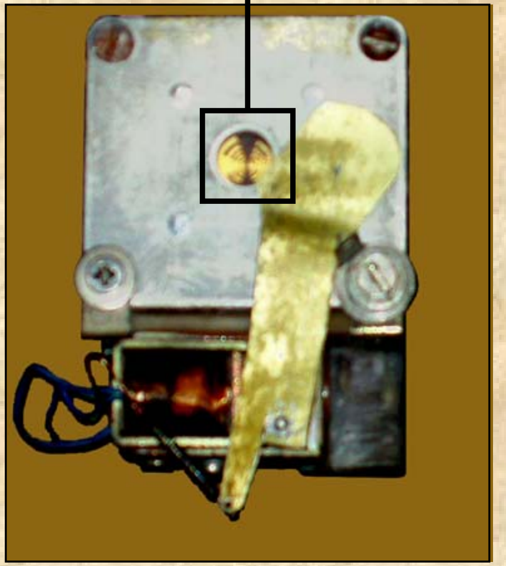
$\frac{dW_1(\lambda)}{\hbar d\omega d\Omega}$ - spectral-angular density of SPR from one electron,

$f(\omega)$ - “so-called” form-factor depending on radiation frequency, bunch shape and particles distribution functions in bunch.

One can find that for the case of $\lambda > \sigma_l$ ($\sigma_l -$ bunch length),

intensity of SPR increases ~N_e by times.

Detector

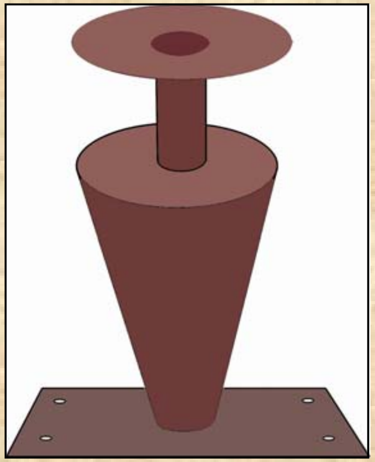


Semi-conductive Devices Institute production (Tomsk, RUSSIA)
www.niipp.ru

Based on the broadband antenna with the high frequency diode.

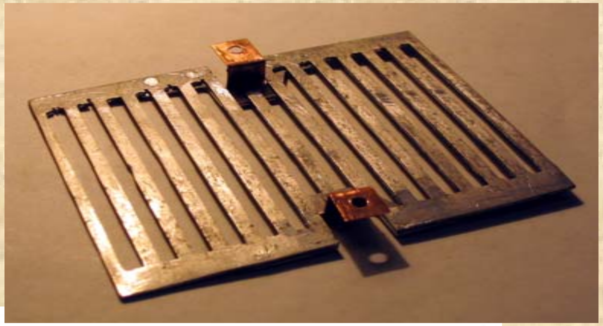
Detector operates at a room temperature.

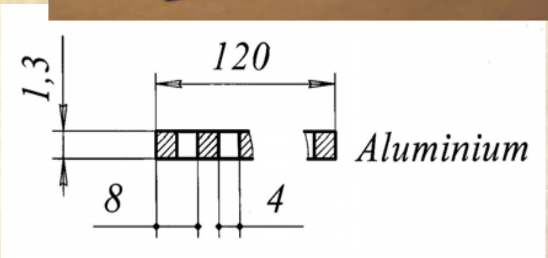
Wavelength region: λ= 3~20 mm
Sensitivity: $\approx$ 0.3V/mWatt

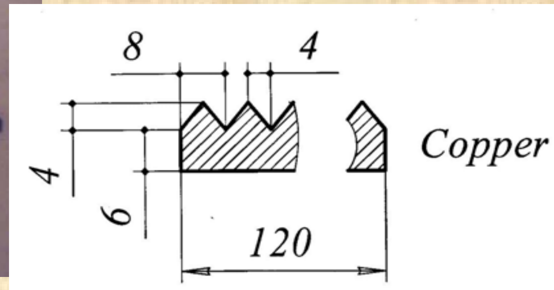


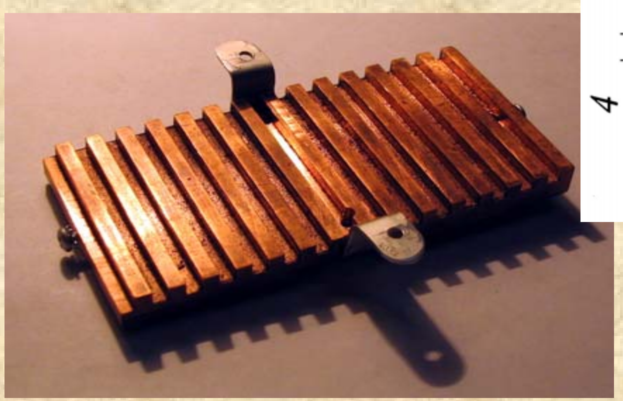
Wave-guide d=10 mm,
passes wavelengths λ <17 mm
[K. Hanke, DESY, CLIC Note 298, 19.04.1996]

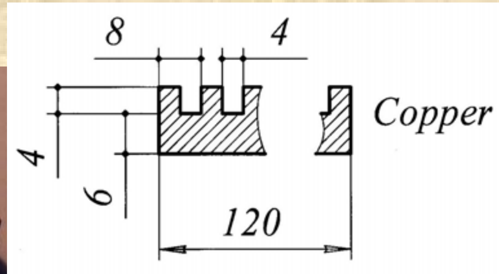
Targets

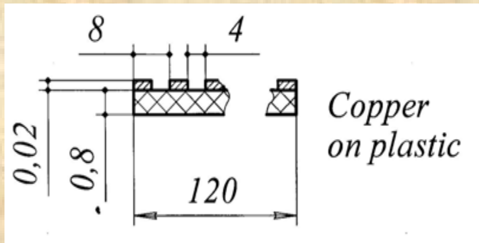



120
1,3
8
4
Aluminium

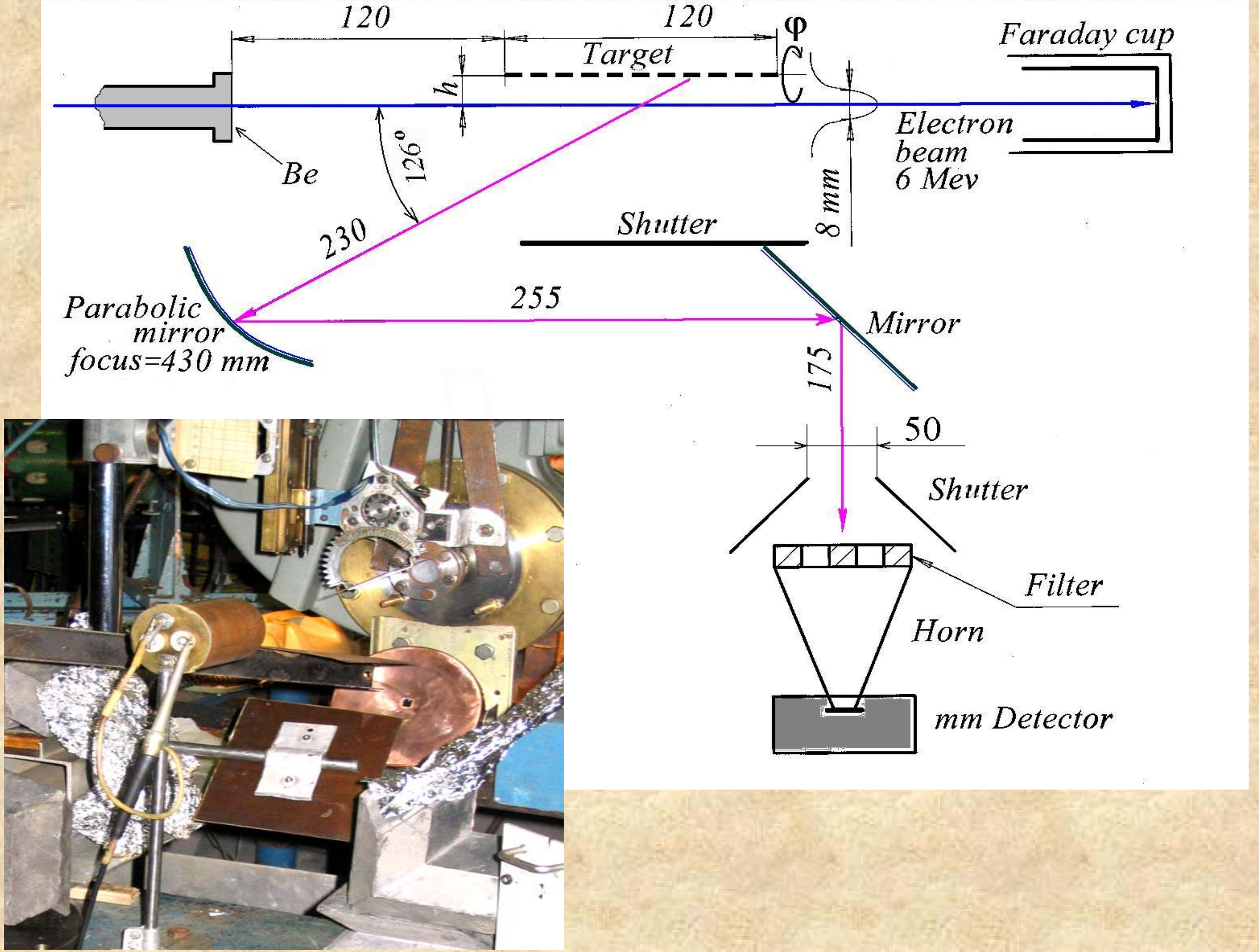

8
4
4
6
120
Copper



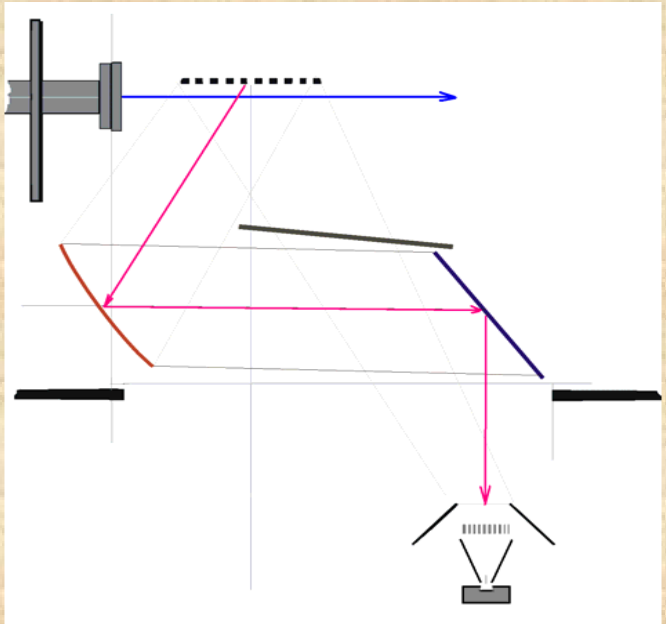

8
4
4
6
120
Copper

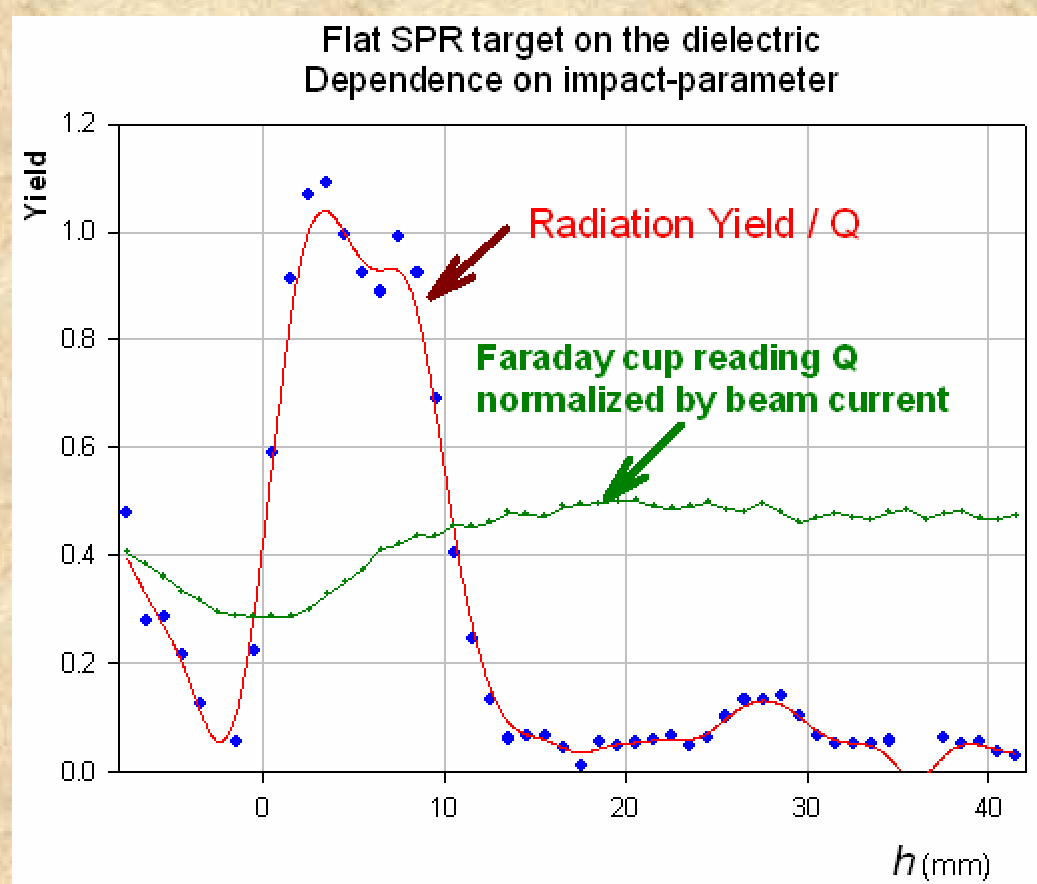

8
4
0,02
0,8
120
Copper
on plastic

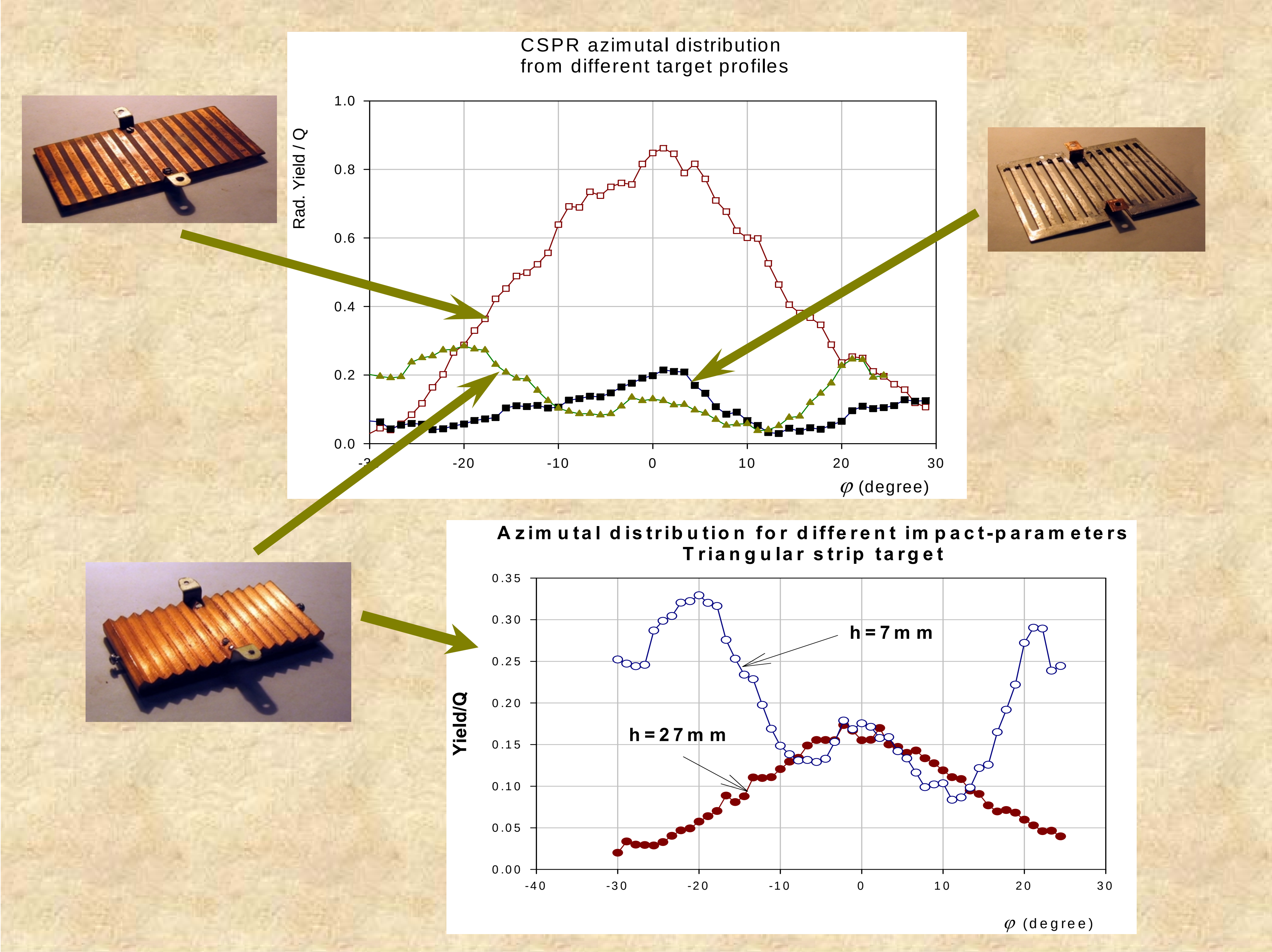
Scheme of experiment



Azimuthal CSPR distribution






CSPR azimutal distribution
from different target profiles
Rad. Yield / Q
1.0
0.8
0.6
0.4
0.2
0.0
-20
-10
0
10
20
30
φ (degree)
Azimutal distribution for different impact-parameters
Triangular strip target
Yield/Q
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
-40
-30
-20
-10
0
10
20
30
h=7mm
h=27mm
φ (degree)

Azimuthal CSPR distribution (theory)

Azimuthal dependences were calculated for 3 kinds of gratings:

1. Lamellar grating (modal expansion technique see in Y. Shibata et al. Phys. Rev., Vol.57, №1 (1998), 1061-1074),
2. Volume strip grating (see, for instance, G.Kube. NIM **B** 227 (2005),180-190),
3. Flat grating with vacuum gaps (A. Potylitsyn. Phys. Lett. **A** 238 (1998), 112-116).

Up to now there is no models for calculations of SPR characteristics from flat grating with dielectric gaps.

Azimuthal CSPR distribution (theory)

The intensities ratio of SPR from these gratings

Θ = 30º, Φ = 90º :

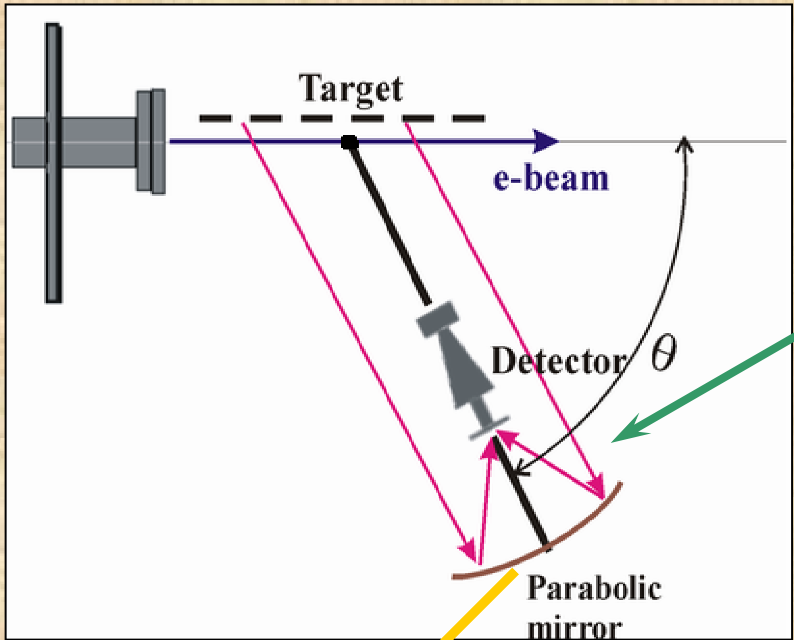
$$\frac{dW_{1\,flat}}{d(h\omega)d\Phi} : \frac{dW_{1Vol.(Above)}}{d(h\omega)d\Phi} : \frac{dW_{1Lam.}}{d(h\omega)d\Phi} \approx$$

1 : 0.093 : 0.0068

Thus, the most effective is the flat grating

B.N. Kalinin, D.V. Karlovets, A.S. Kostousov et. al. ***Comparison of Smith-Pursell radiation characteristics from gratings with different profiles //*** **Nucl. Instr. and Meth. in Phys. Res. B (2006) to be published.**

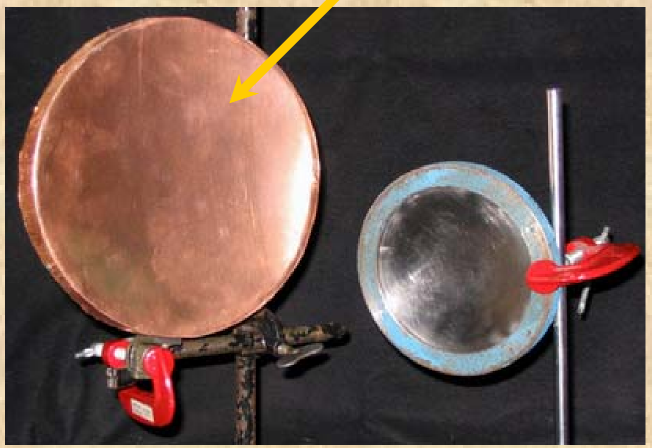
Setup for angular distributions measurements

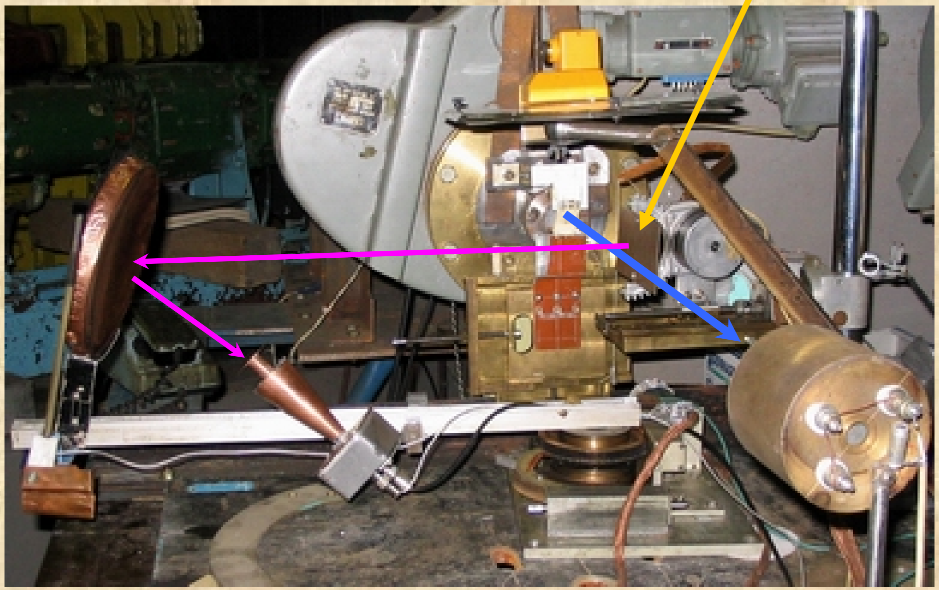


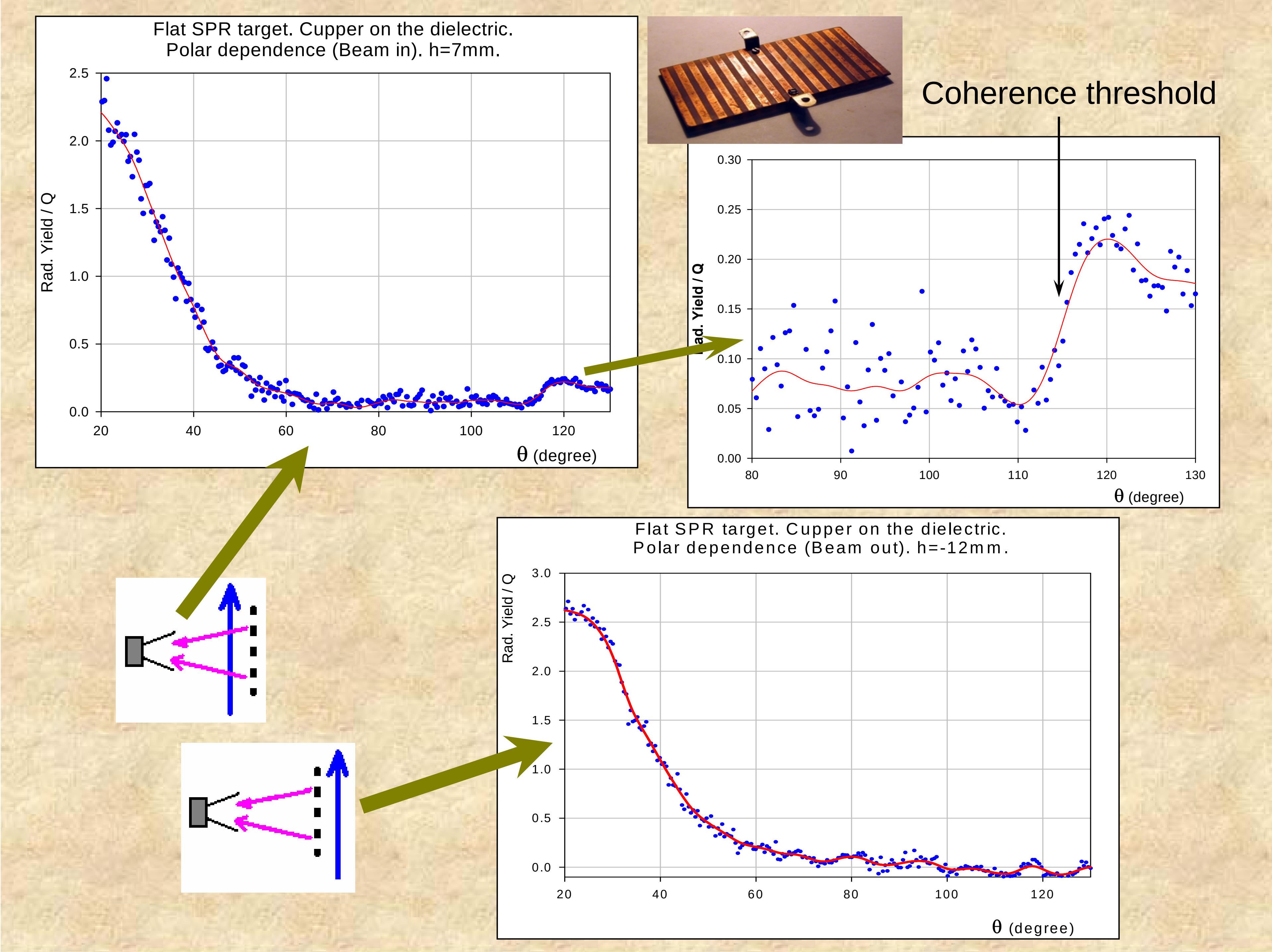
Aperture of the telescope

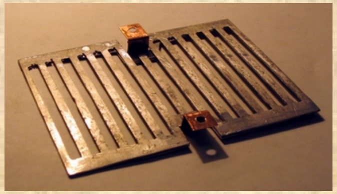
$$\Delta\theta = \frac{\phi_d}{2f} = \frac{10}{2\cdot 151} = 33 \ mrad$$

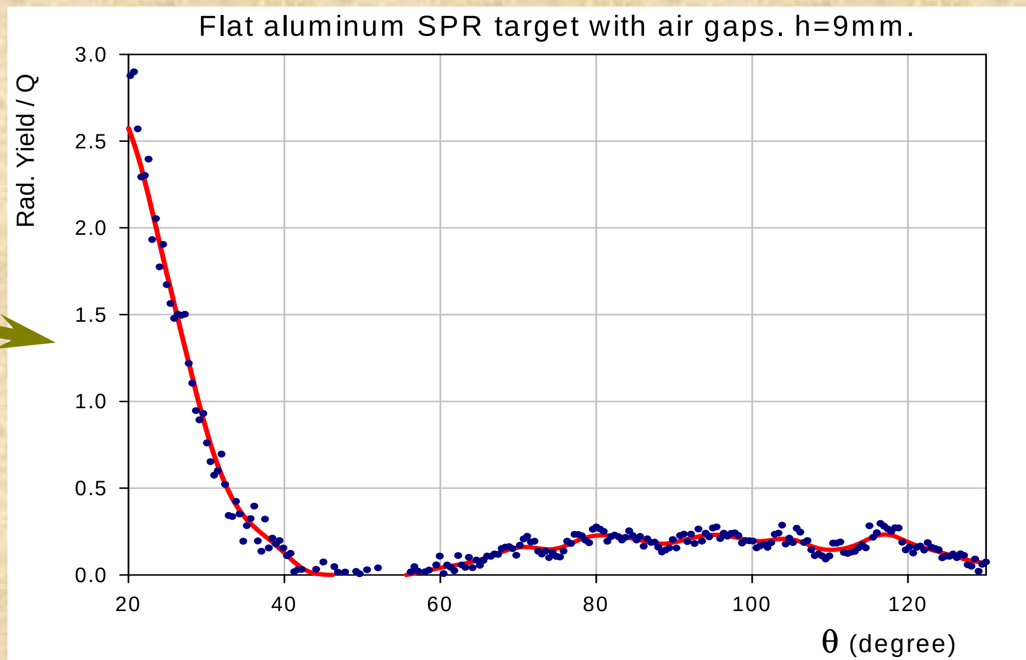
Target

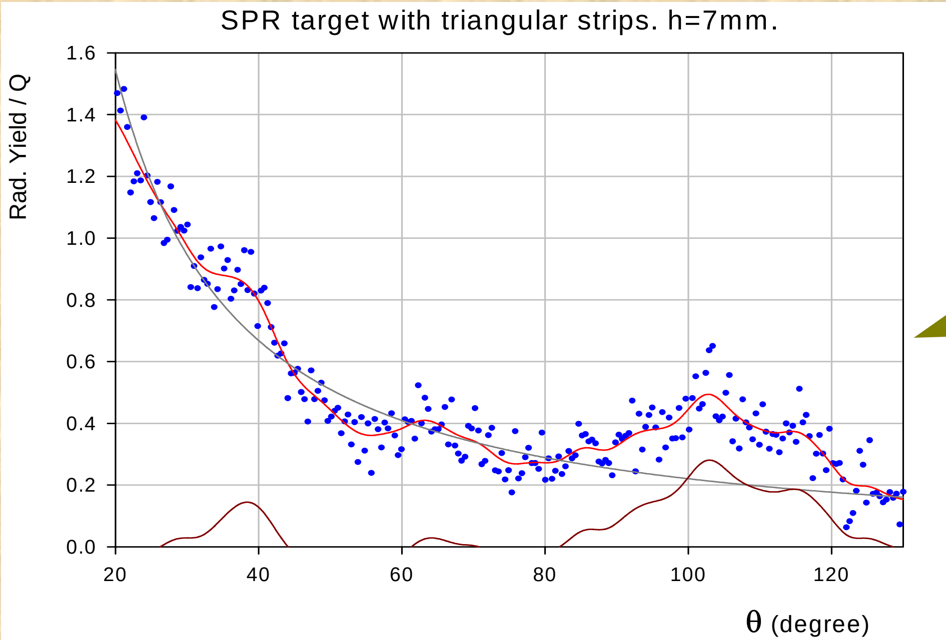





Flat SPR target. Cupper on the dielectric.
Polar dependence (Beam in). h=7mm.
Rad. Yield / Q
θ (degree)
Coherence threshold
Rad. Yield / Q
θ (degree)
Flat SPR target. Cupper on the dielectric.
Polar dependence (Beam out). h=-12mm.
Rad. Yield / Q
θ (degree)

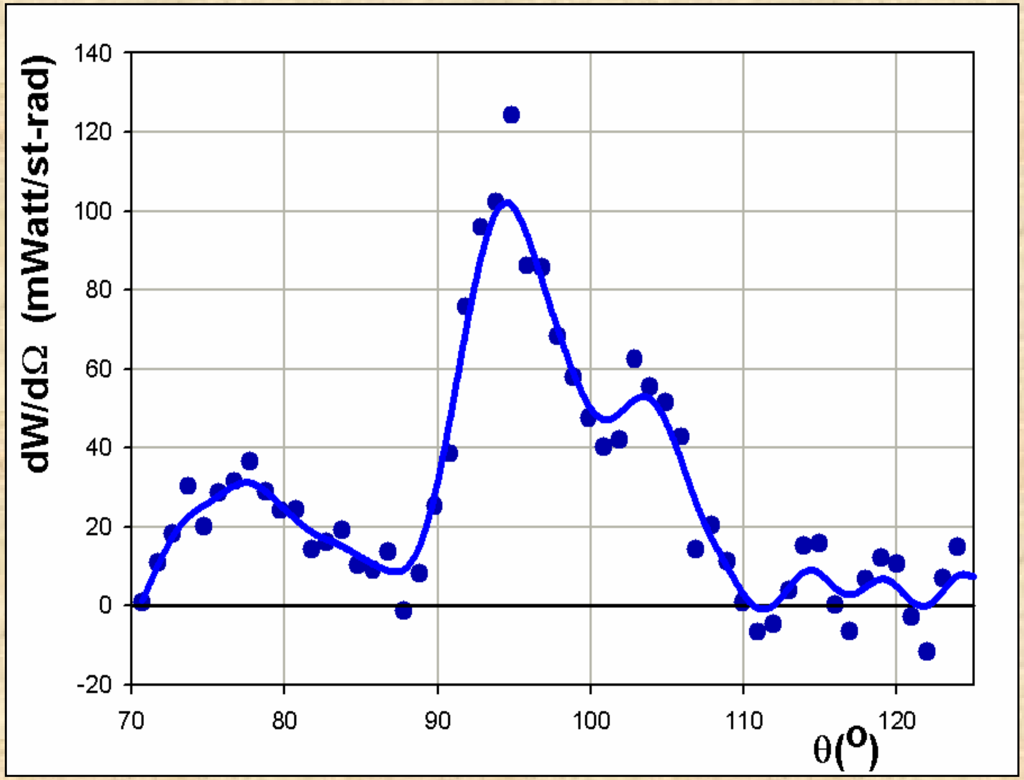



Flat aluminum SPR target with air gaps. h=9mm.
Rad. Yield / Q
3.0
2.5
2.0
1.5
1.0
0.5
0.0
20
40
60
80
100
120
θ (degree)


SPR target with triangular strips. h=7mm.
Rad. Yield / Q
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
20
40
60
80
100
120
θ (degree)

CSPR angular distribution

d=12 mm



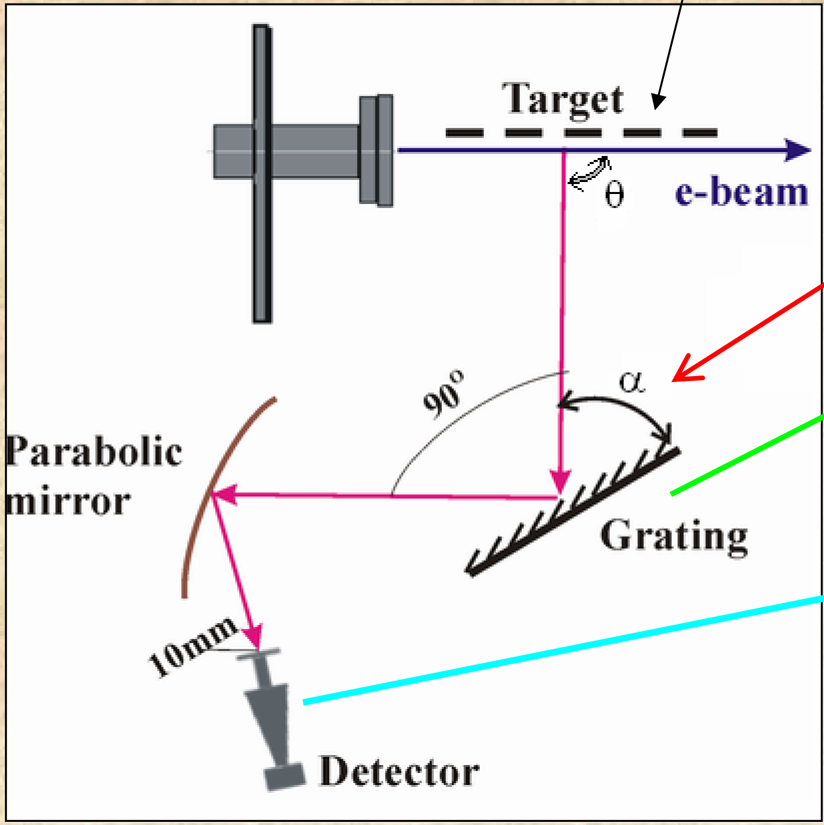
Theoretical estimations

$$P_{vdB} \approx 0.09 \quad \frac{mWatt}{sr}$$

$$P_{RDR} \approx 3000 \quad \frac{mWatt}{sr}$$

CSPR spectrum

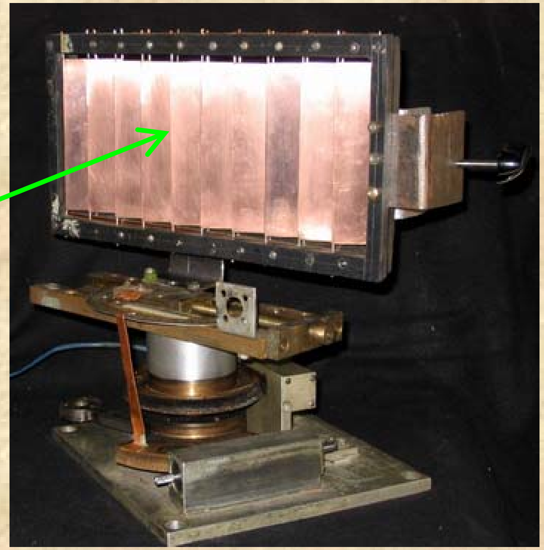
d=12mm, N=14, h=8 mm, θ = 100°



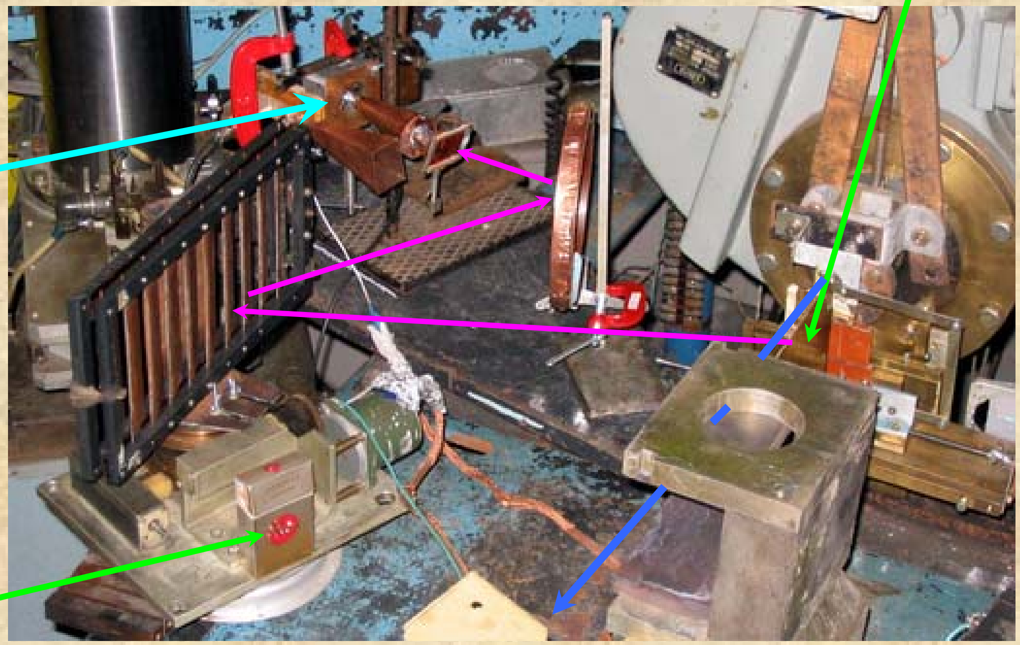
$0^\circ \le \alpha \le 90^\circ$

d=28mm

Grating

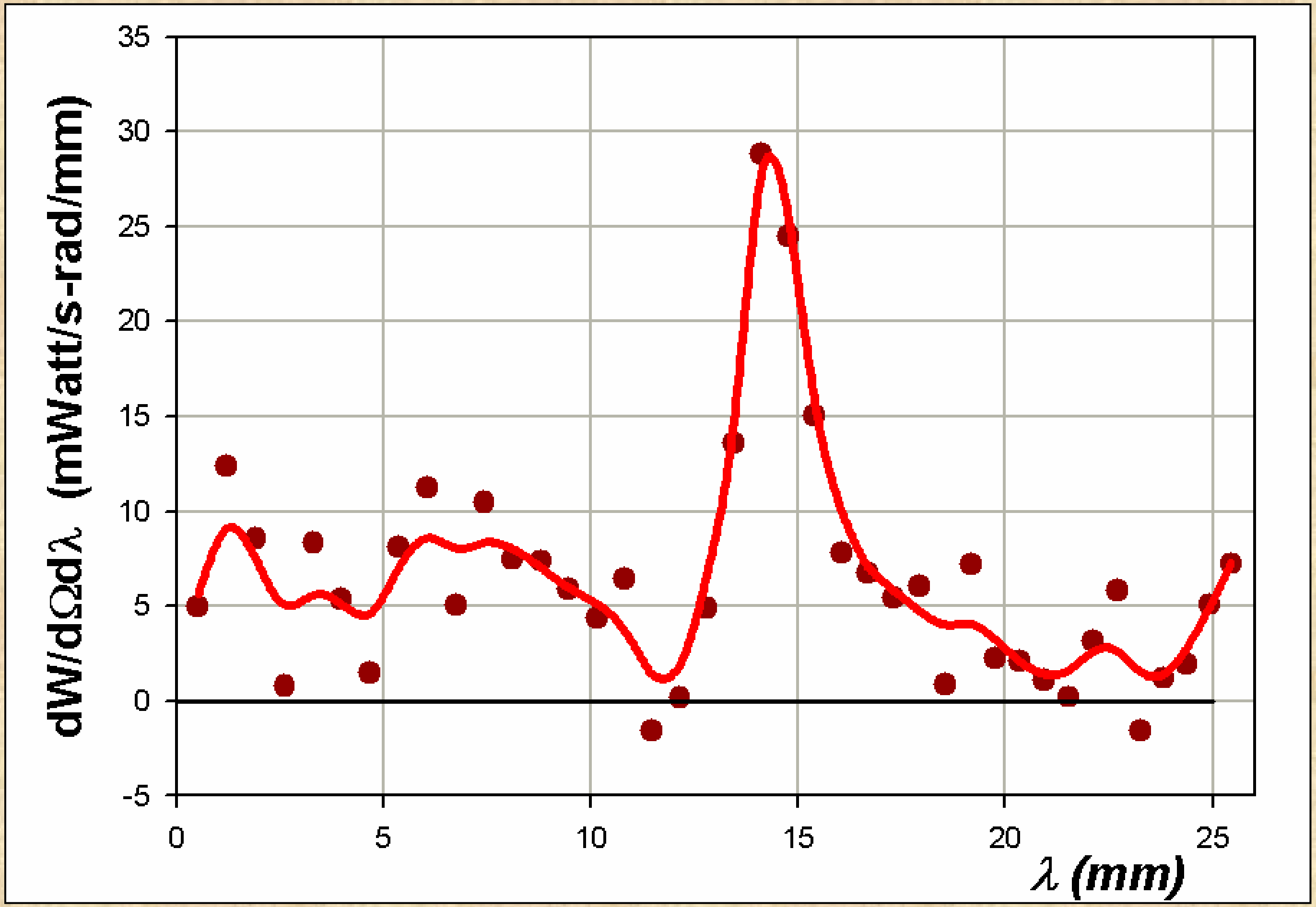


Target

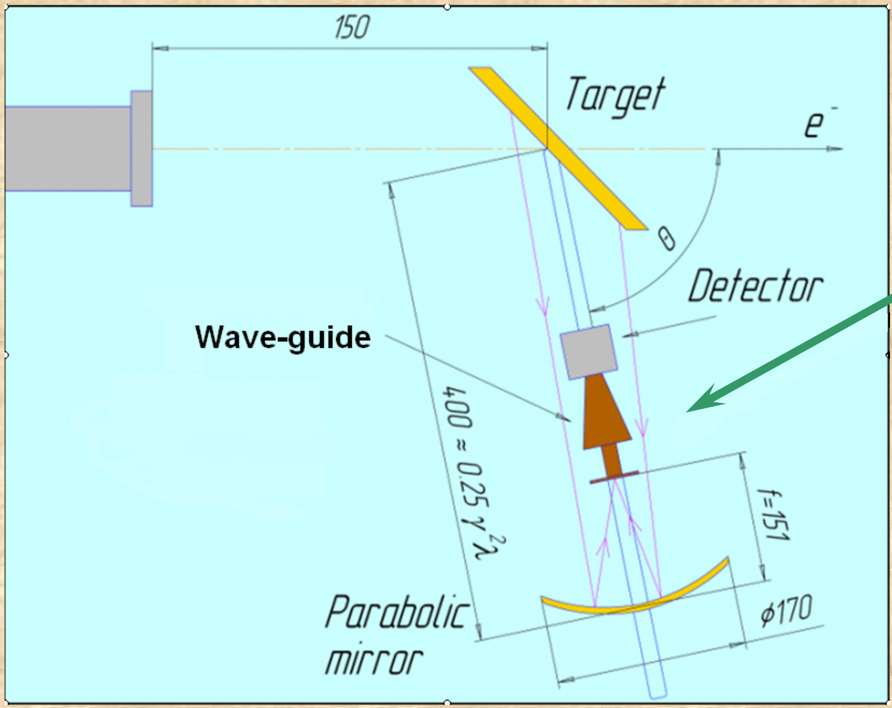


Cigarettes

CSPR spectrum

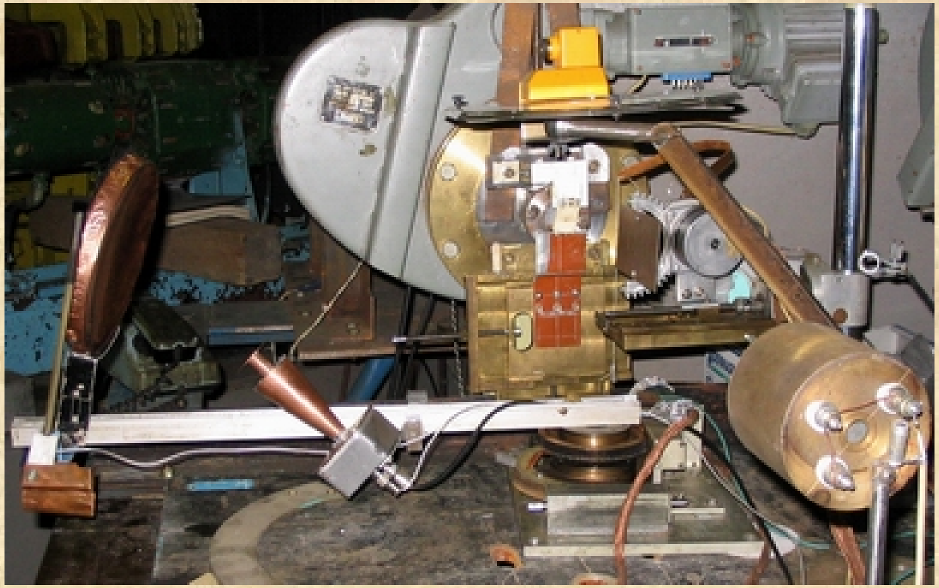


Setup for CTR angular distributions measurements

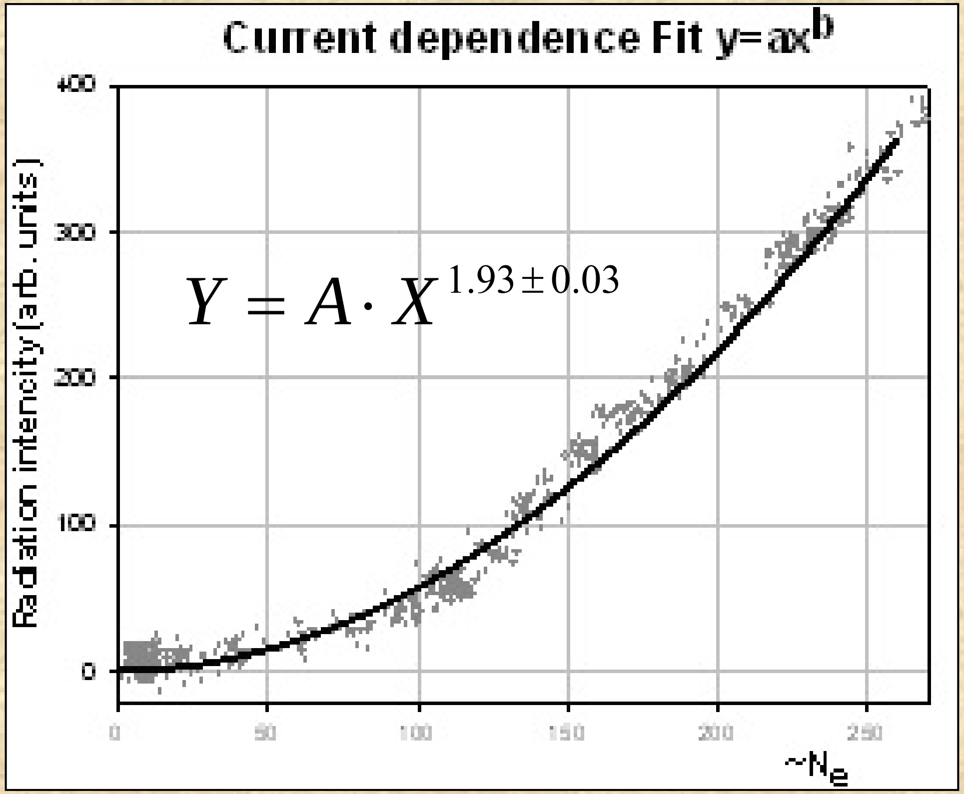


Aperture of the telescope

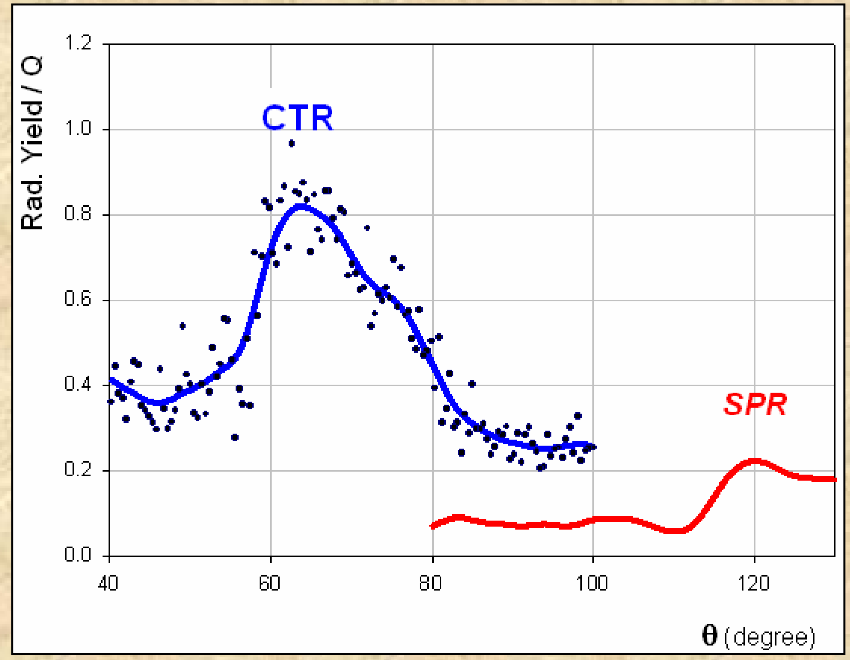
$$\Delta\theta = \frac{\phi_d}{2f} = \frac{10}{2\cdot151} = 33\ mrad$$



Coherence criterion



CTR & CSPR comparison



Maximal yield from different targets at $\varphi=0$

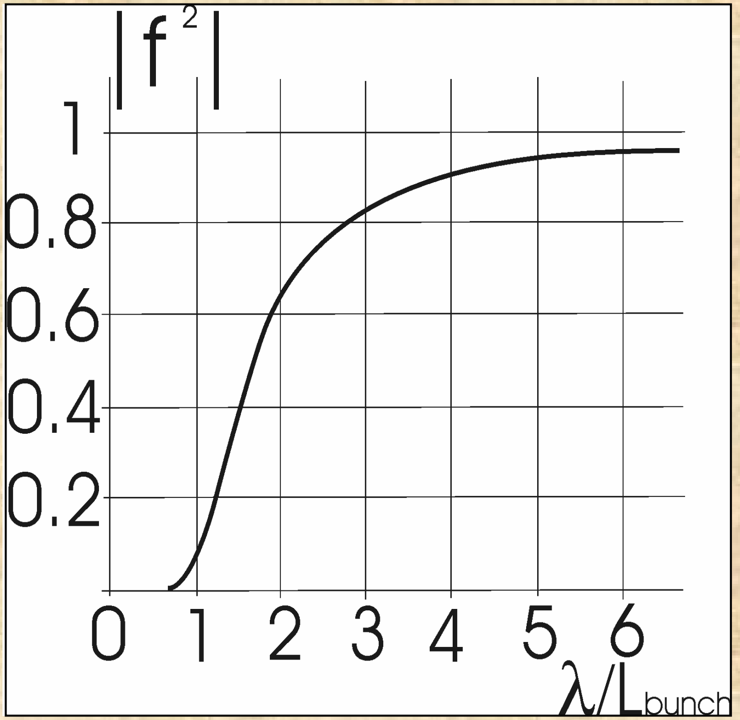
Target	Max. yield (arb.un.)
	0.87
	0.22
	0.14
	0.07
TR target	3.8

B.N. Kalinin, D.V. Karlovets, A.S. Kostousov et. al. *Comparison of Smith-Pursell radiation characteristics from gratings with different profiles* // Nucl. Instr. and Meth. in Phys. Res. B (2006) to be published.

Bunch length estimation

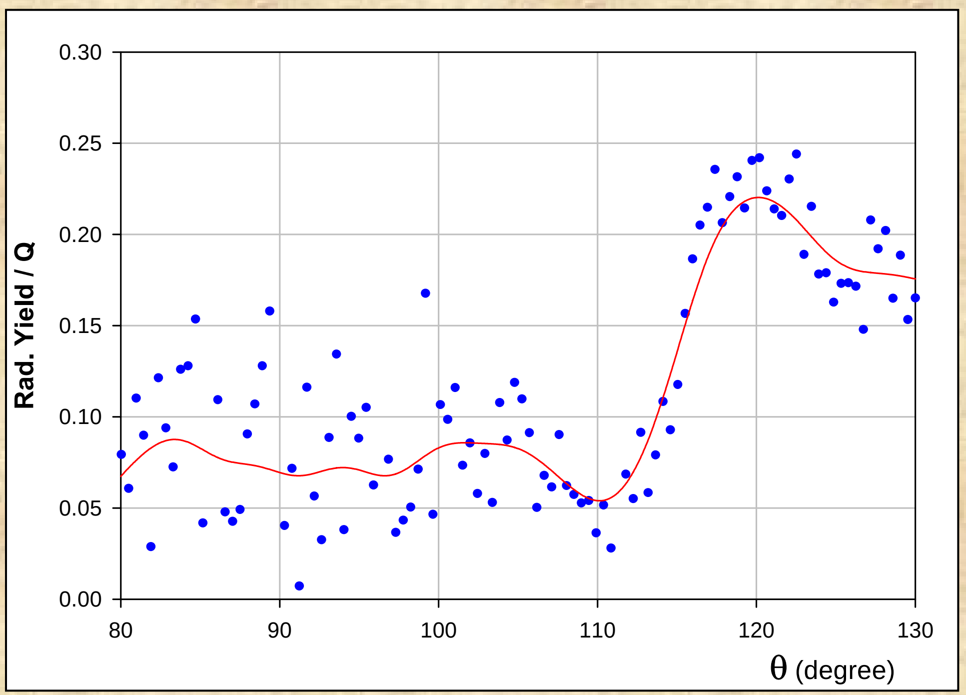
Form-factor

$$|f(\lambda)|^2 = Exp\{-\frac{4\pi^2\sigma^2}{\beta^2\lambda^2}\}$$



$$L_{bunch} = 6\sigma$$

Experiment



σ~1.13 mm

Conclusion

1. For moderately relativistic electron beam the flat target is most effective for CSPR generation (see upper table)
2. For fixed impact parameters the azimutal distribution of CSPR from a flat target has a maximum in the plane perpendicular grating
3. From angular distribution of CSPR it is possible to determine the bunch length using a broadband detector.
4. For small polar angles ($\theta<40^o$) we observed large contribution of coherent diffraction radiation from entrance and exit edges of target as whole.
5. Resonant diffraction radiation model theoretical estimations are in the better agreement with experimental results than van den Berg's model ones.