BPMs, Linear Optics & Coupling

@RHIC

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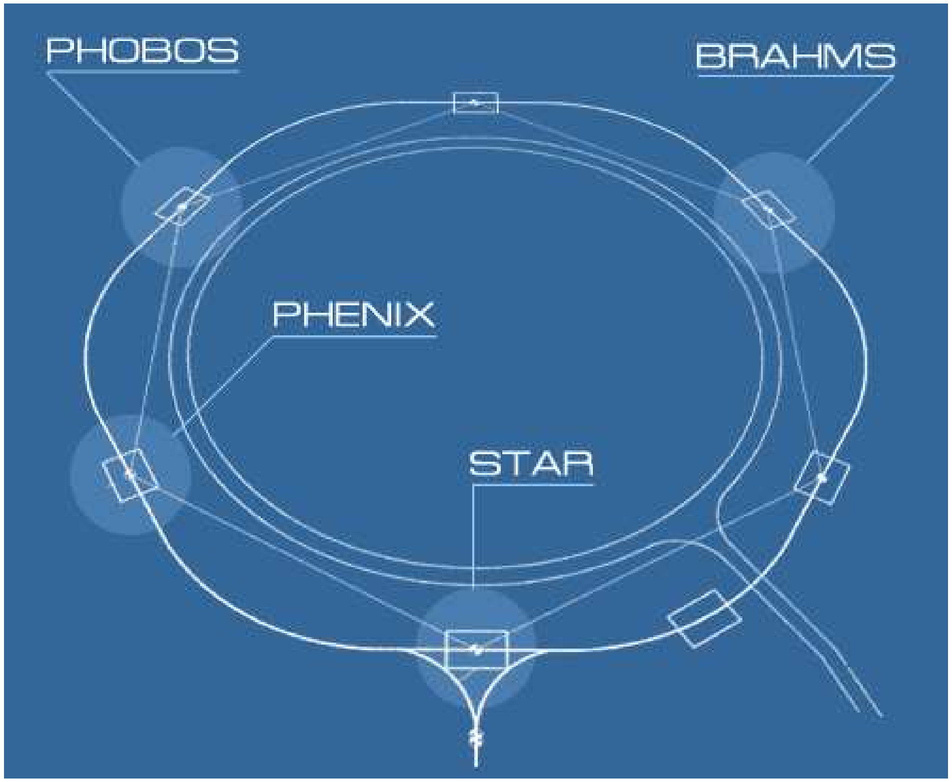
RHIC BPM System

There are 164 BPMs per plane per ring:

- 72 dual-plane BPMs distributed through the IR's
- 176 single-plane BPMs distributed in the arcs



Each BPM channel is devised to acquire 1024 turn-by-turn positions.

Physical Base Decomposition

Turn-by-turn BPM data matrix can be decomposed:

$$B = \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 & \cdots \\ b_1^2 & \ddots & & \\ b_1^3 & & & \\ \vdots & & & \end{pmatrix}_{n\times m} = \underbrace{\begin{bmatrix} q_1^1 & \cdots & q_d^1 \\ \vdots & \ddots & \\ \vdots & & \\ q_1^t & & \end{bmatrix}}_{Q} \underbrace{\begin{bmatrix} f_1^1 & \cdots & f_d^1 \\ \vdots & \ddots & \\ f_1^d & & \end{bmatrix}^T}_{F^T} + N$$

$d \longrightarrow$ rank of the matrix

$[q_1 \ldots q_d] \longrightarrow$ temporal series, $[f_1 \ldots f_d] \longrightarrow$ spatial series

Principle Component Analysis:

Find the leading variable $\Rightarrow$

$$var(Bf) = max\left[\frac{f^T(B^TB)f}{f^Tf}\right]$$

where $f = [f_1, \ldots, f_m]^T$

Singular Value Decomposition:

$$B = U\Sigma V^T = \sum_i \sigma_i u_i v_i^T$$

Σ - Singular Values
U - Temporal Vector
V - Spatial Vector

Perturbative View (J. Irwin, C. X. Wang et al.)

Taylor expanding 'b' over all physical variables:

$$\begin{aligned} b - \langle b \rangle &= \sum_v \frac{\partial b}{\partial v}\bigg|_{v=\overline{v}} (\Delta v - \langle \Delta v \rangle) \\ &+ \frac{1}{2} \sum_{v_1, v_2} \frac{\partial^2 b}{\partial v_2 \partial v_1}\bigg|_{v=\overline{v}} (\Delta v_1 \Delta v_2 - \langle \Delta v_1 \Delta v_2 \rangle) \\ &+ \ldots \end{aligned}$$

Treating 1^{st} and 2^{nd} order terms the same (higher-order negligible):

$$b - \langle b \rangle = \sum_{(q)} q f_q \tag{1}$$

where $q = \frac{\Delta v - \langle \Delta v \rangle}{std(\Delta v)}$ or $\frac{\Delta v_1 \Delta v_2 - \langle \Delta v_1 \Delta v_2 \rangle}{std(\Delta v_1 \Delta v_2)}$

$f_q = \frac{\partial b}{\partial v}\Big|_v std(\Delta v)$ or $\frac{\partial^2 b}{\partial v_2 \partial v_1}\Big|_{v_1 v_2} std(\Delta v_1 \Delta v_2)$

Betatron Motion - 1D

SVD - 1D

Assume only betatron motion (no coupling):

$$x(m) \;=\; \sqrt{2J\beta_m}\,\cos(\phi_t + \psi_m)$$

$$B = U\Sigma V^T$$

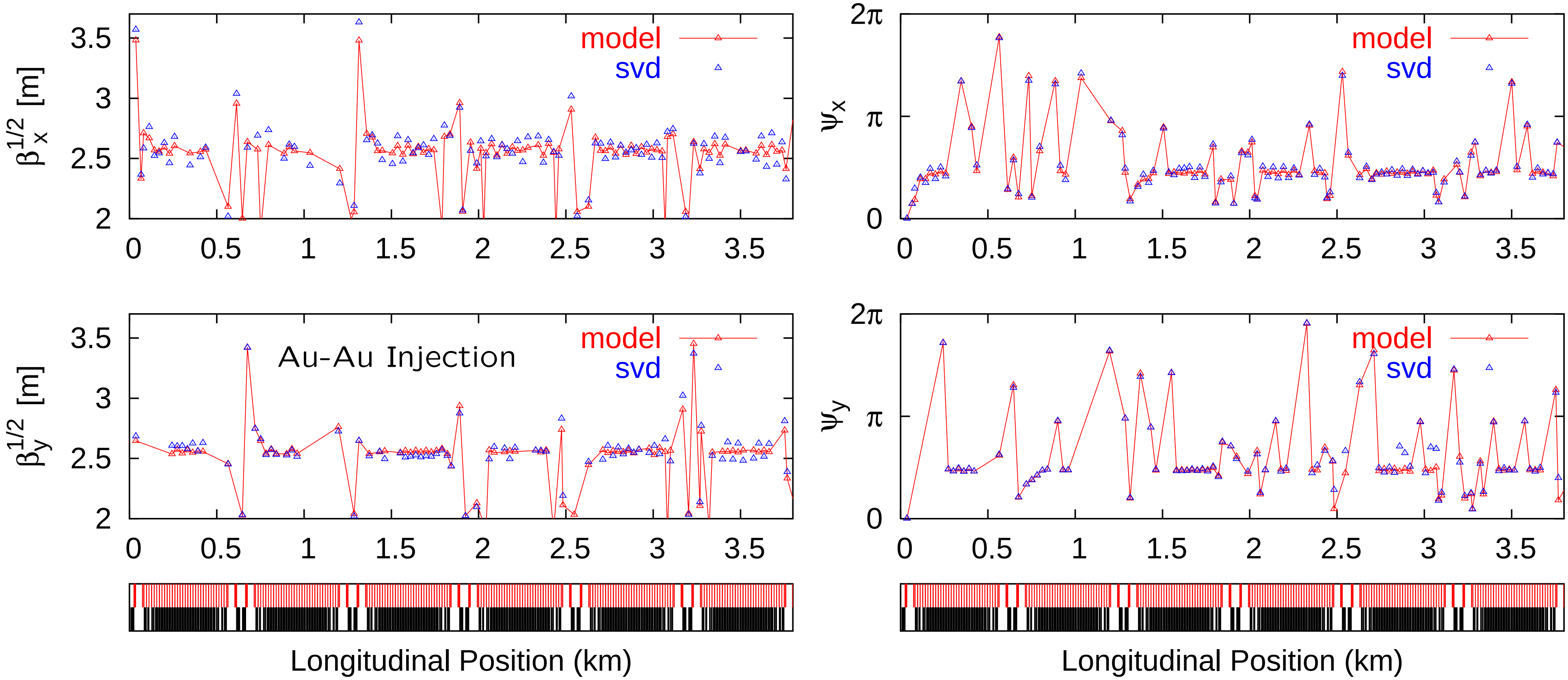
$$\begin{bmatrix} u_1^+ & u_1^- & \dots \\ \vdots & \vdots & \\ \vdots & \vdots & \\ u_t^+ & u_t^- & \dots \end{bmatrix} \begin{bmatrix} \sigma_+ & 0 & \dots & \dots \\ 0 & \sigma_- & & \\ \vdots & & \ddots & \\ \vdots & & & \sigma_m \end{bmatrix} \begin{bmatrix} v_1^+ & v_1^- & \dots \\ \vdots & \vdots & \\ \vdots & \vdots & \\ v_m^+ & v_m^- & \dots \end{bmatrix}^T$$

$$u_+ \;=\; \sqrt{\frac{2J_t}{T\langle J\rangle}}\cos(\phi_t - \phi_0) \qquad v_+ \;=\; \frac{1}{\sqrt{\lambda_+}}\left[\sqrt{\langle J\rangle\beta_m}\cos(\phi_0 + \psi_m)\right]$$

$$u_- \;=\; -\sqrt{\frac{2J_t}{T\langle J\rangle}}\sin(\phi_t - \phi_0) \qquad v_- \;=\; \frac{1}{\sqrt{\lambda_-}}\left[\sqrt{\langle J\rangle\beta_m}\sin(\phi_0 + \psi_m)\right]$$

RHIC Optics Measurements

$$\psi = \tan^{-1}\left(\frac{\sigma_- v_-}{\sigma_+ v_+}\right); \qquad \beta = \langle J\rangle^{-1}(\sigma_+^2 v_+^2 + \sigma_-^2 v_-^2)$$

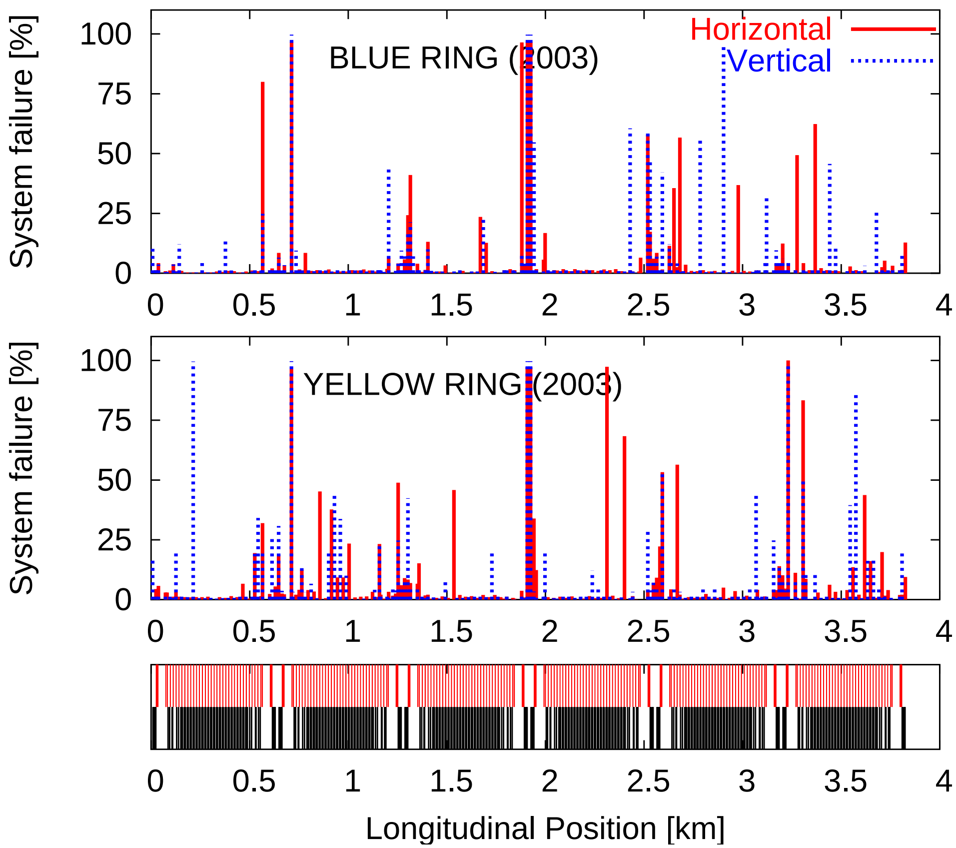


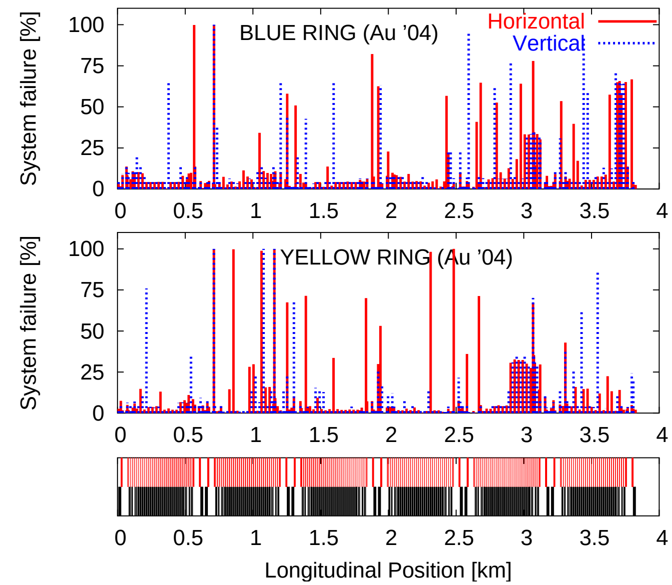
C.X. Wang, V. Sajaev, C.Y. Yao, Phys. Rev. ST Accel. Beams 6,104001(2003).

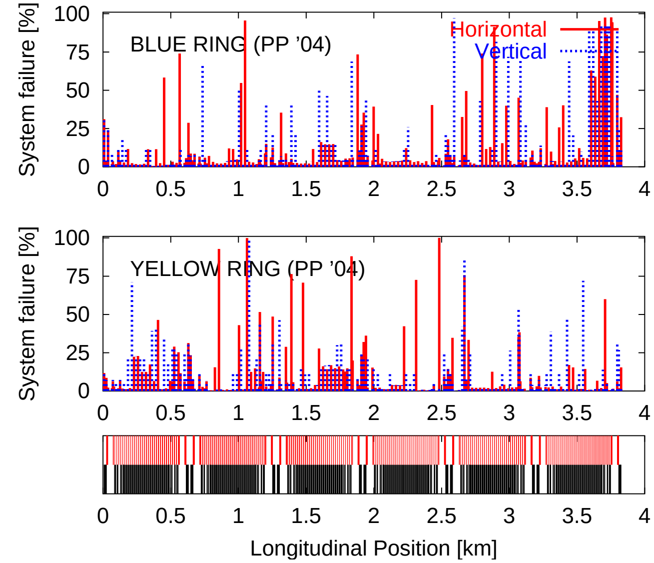
Faulty BPMs !!!

Hardware Cut

BPM is tagged faulty due to obvious electronic failures by some hardware thresholds

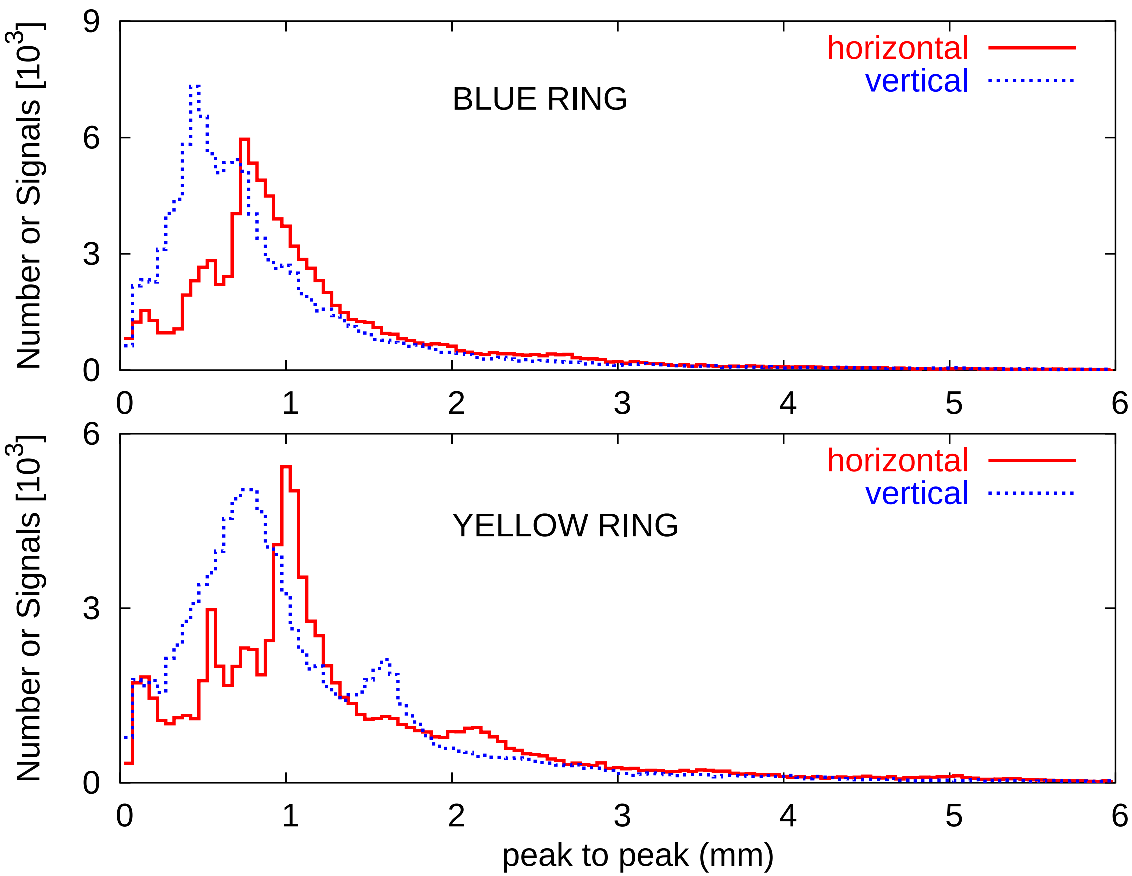




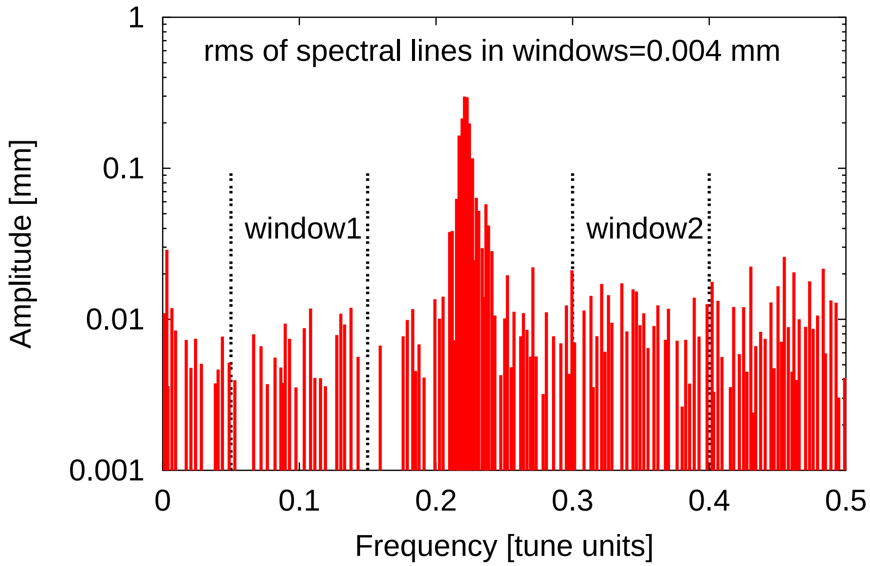


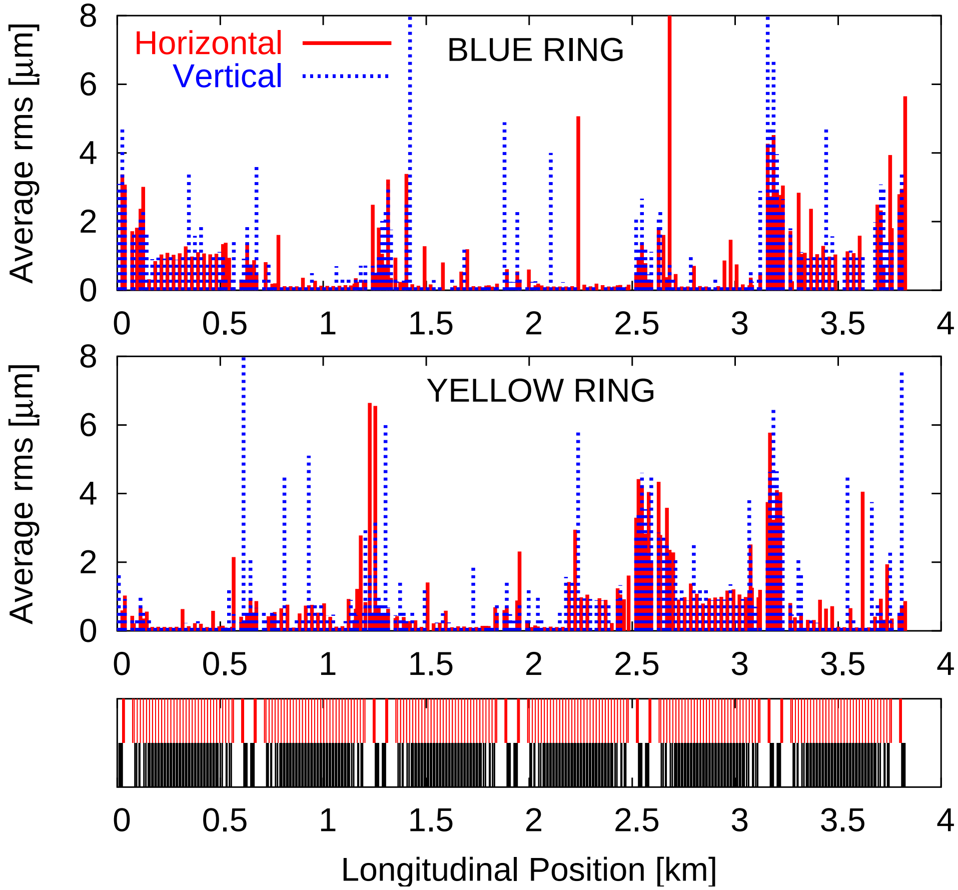
Peak to Peak Cut

Numerical techniques used to identify faulty bpms become less sensitive when signal to noise to ratio is small



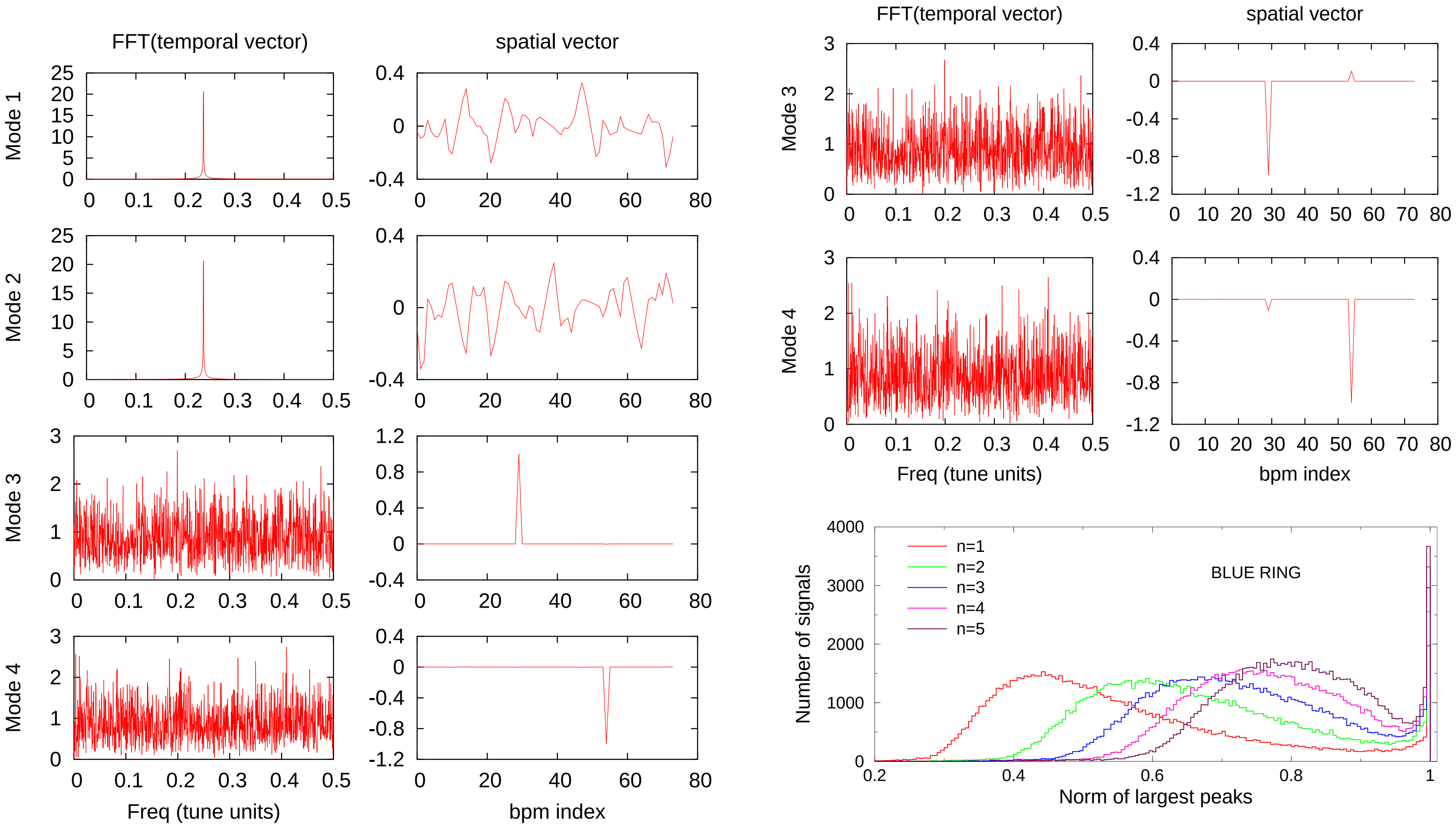
FFT Technique





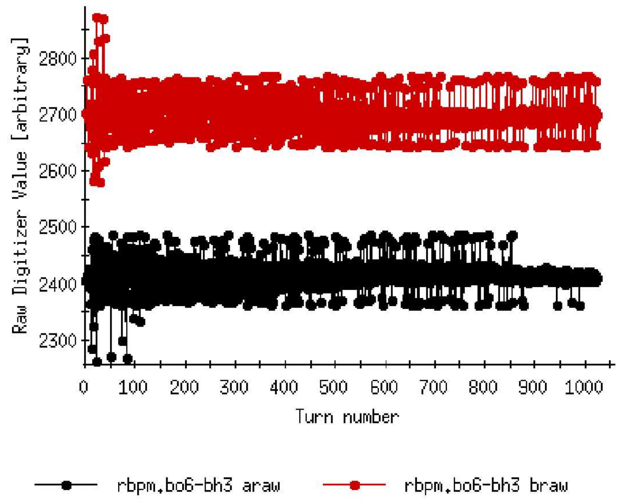
The rms of the background is use as the observable to determine the threshold for a faulty BPM

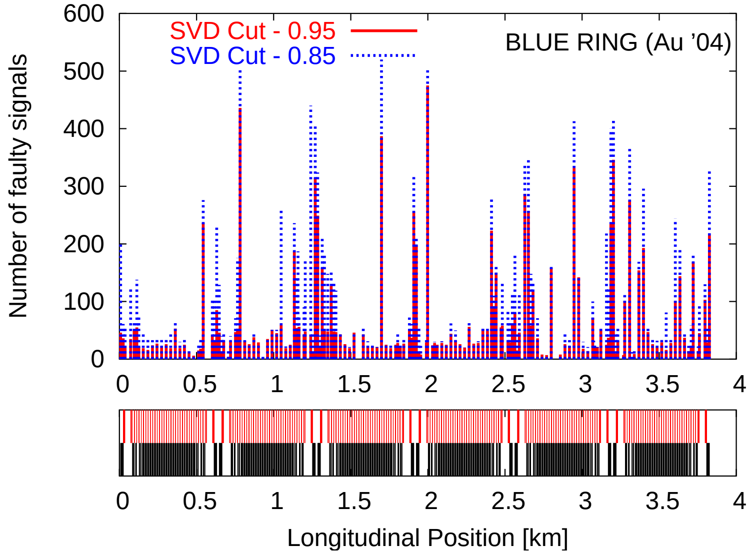
SVD Technique

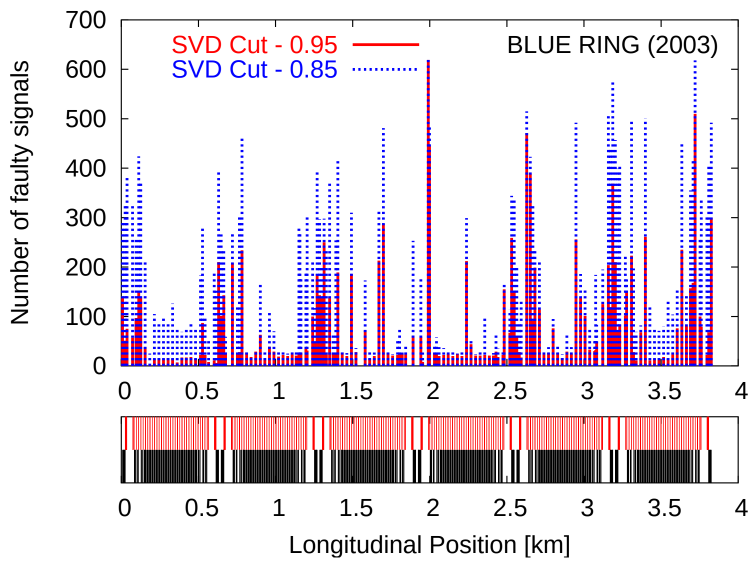


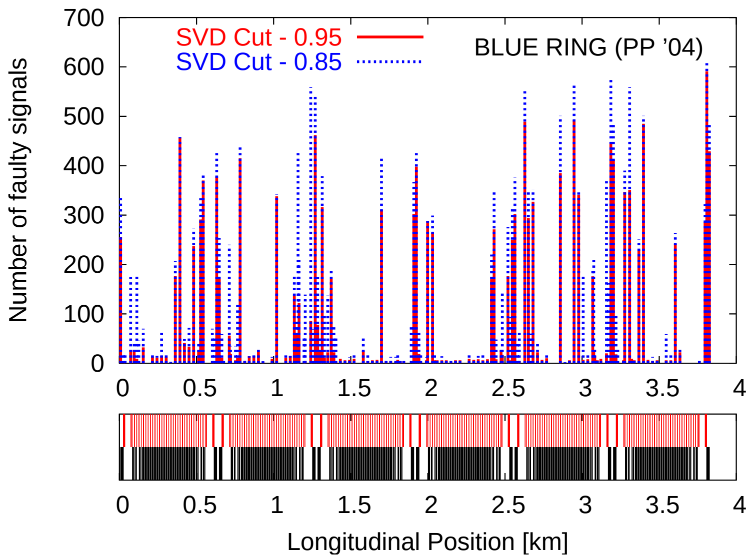
Sum of the largest peaks of the spatial vectors are used as observables

“Hairs” & Improvement









Linear Coupling

Local & Global

- Formalism
- Local coupling
- Ideas on optimizing global coupling

Coupling Formalisms & Equivalence

Hamiltonian Formalism

$$\hat{x} - i\hat{p}_x^h = \sqrt{2I_x}e^{i\psi_x} - 2if_{1001}\sqrt{2I_y}e^{i\psi_y} - 2if_{1010}\sqrt{2I_y}e^{-i\psi_y},$$

$$\hat{y} - i\hat{p}_y^h = \sqrt{2I_y}e^{i\psi_y} - 2if^*_{1001}\sqrt{2I_x}e^{i\psi_x} - 2if_{1010}\sqrt{2I_x}e^{-i\psi_x} \;,$$

$$f(s)_{\substack{1001\\1010}} = -\frac{1}{4(1-e^{2\pi i(Q_x \mp Q_y)})} \times \sum_l k_l \sqrt{\beta_x^l \beta_y^l} e^{i(\Delta\phi_x^{sl} \mp \Delta\phi_y^{sl})}$$

Matrix Formalism

$$\mathbf{T} = \begin{pmatrix} \mathbf{M} & \mathbf{m} \\ \mathbf{n} & \mathbf{N} \end{pmatrix} = \mathbf{VUV^{-1}}$$

$$\mathbf{U} = \begin{pmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \end{pmatrix} \;, \quad \mathbf{V} = \begin{pmatrix} \gamma\mathbf{I} & \mathbf{C} \\ -\mathbf{C}^+ & \gamma\mathbf{I} \end{pmatrix}$$

$$\begin{pmatrix} \hat{x} \\ \hat{p}_x \\ \hat{y} \\ \hat{p}_y \end{pmatrix} = \begin{pmatrix} \gamma & 0 & \overline{C}_{11} & \overline{C}_{12} \\ 0 & \gamma & \overline{C}_{21} & \overline{C}_{22} \\ -\overline{C}_{22} & \overline{C}_{12} & \gamma & 0 \\ \overline{C}_{21} & -\overline{C}_{11} & 0 & \gamma \end{pmatrix} \begin{pmatrix} A_x \cos\psi_x \\ A_x \sin\psi_x \\ A_y \cos\psi_y \\ A_y \sin\psi_y \end{pmatrix}$$

$$f_{\substack{1001\\1010}} = \frac{1}{4\gamma}(\pm\overline{C}_{12} - \overline{C}_{21} + i\overline{C}_{11} \pm i\overline{C}_{22})$$

$$\frac{|\overline{C}|}{4\gamma^2} = |f_{1001}|^2 - |f_{1010}|^2$$

$$\Delta Q_{min} \approx \frac{2\gamma}{\pi}\left(\frac{\cos\nu_x - \cos\nu_y}{\sin\nu_x + \sin\nu_y}\right)\sqrt{|\overline{C}|}$$

SVD & $|\overline{C}|$ Matrix

$$x = \sqrt{2J_a\beta_a}\gamma\cos(\phi_a+\psi_a) + \sqrt{2J_b\beta_a}c_b\cos(\phi_b+\psi_b+\Delta\psi_b)$$

$\gamma = \sqrt{1-|\overline{C}|}$, $\quad c_b = \sqrt{\overline{C}_{11}^2+\overline{C}_{12}^2}$, $\quad \Delta\psi_b = tan^{-1}(\overline{C}_{12}/\overline{C}_{11})$

$$\mathbf{B} = \mathbf{U\Sigma V^T}$$ NOT EIGENMODES

Rotated SVD Matrix:

$$O^T\Sigma V^T = \begin{pmatrix} \cdots & \sqrt{\bar{J}_a\beta_a}\gamma\cos(\psi_a-\psi_a^0) & \cdots \\ \cdots & \sqrt{\bar{J}_a\beta_a}\gamma\sin(\psi_a-\psi_a^0) & \cdots \\ \cdots & \sqrt{\bar{J}_b\beta_a}c_b\cos(\psi_b+\Delta\psi_b-\tilde{\psi}_b^0) & \cdots \\ \cdots & \sqrt{\bar{J}_b\beta_a}c_b\sin(\psi_b+\Delta\psi_b-\tilde{\psi}_b^0) & \cdots \end{pmatrix}$$

$$\frac{\bar{C}_{12}}{\gamma} = \text{sgn}(\sin\Delta\psi_a)\sqrt{\frac{\tilde{A}_a\tilde{A}_b}{A_aA_b}\sin\Delta\psi_a\sin\Delta\psi_b}$$

$$\frac{\overline{C}_{11}}{\gamma} = \frac{\overline{C}_{12}}{\gamma}\cot\Delta\psi_b \quad ; \quad \frac{\overline{C}_{22}}{\gamma} = -\frac{\overline{C}_{12}}{\gamma}\cot\Delta\psi_a$$

$|\overline{C}|$ Matrix

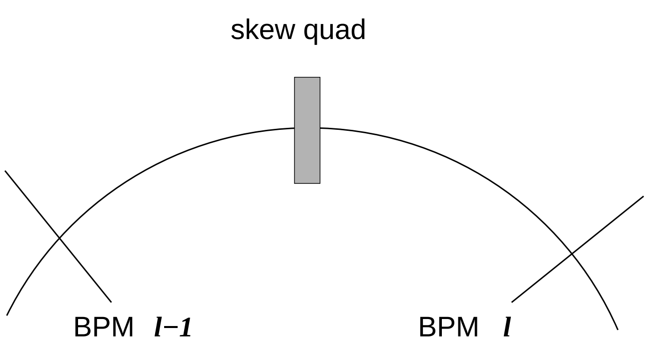
Propogation of $\overline{C}$ Matrix w/o skew quads:

$$\overline{C}_{\mathbf{2}} = \mathbf{R}_x(\phi_x)\overline{C}_{\mathbf{1}}\mathbf{R}_y^{-1}(\phi_y)$$

$$\overline{C}_{21}^{(1)} = \frac{-\overline{C}_{11}^{(1)}\cos\phi_a\sin\phi_b + \overline{C}_{12}^{(1)}\cos\phi_a\sin\phi_b + \overline{C}_{22}^{(1)}\sin\phi_a\cos\phi_b - \overline{C}_{12}^{(2)}}{(\sin\phi_a\sin\phi_b)}$$

The $|\overline{C}|$ is discontinuous thro' Skew Quad

$$\overline{C}_{\mathbf{2}} = \overline{C}_{\mathbf{1}} - \overline{k}$$



$$\bar{k} = -\frac{|C^{(2)}| - |C^{(1)}|}{C_{12}^{skew}}$$

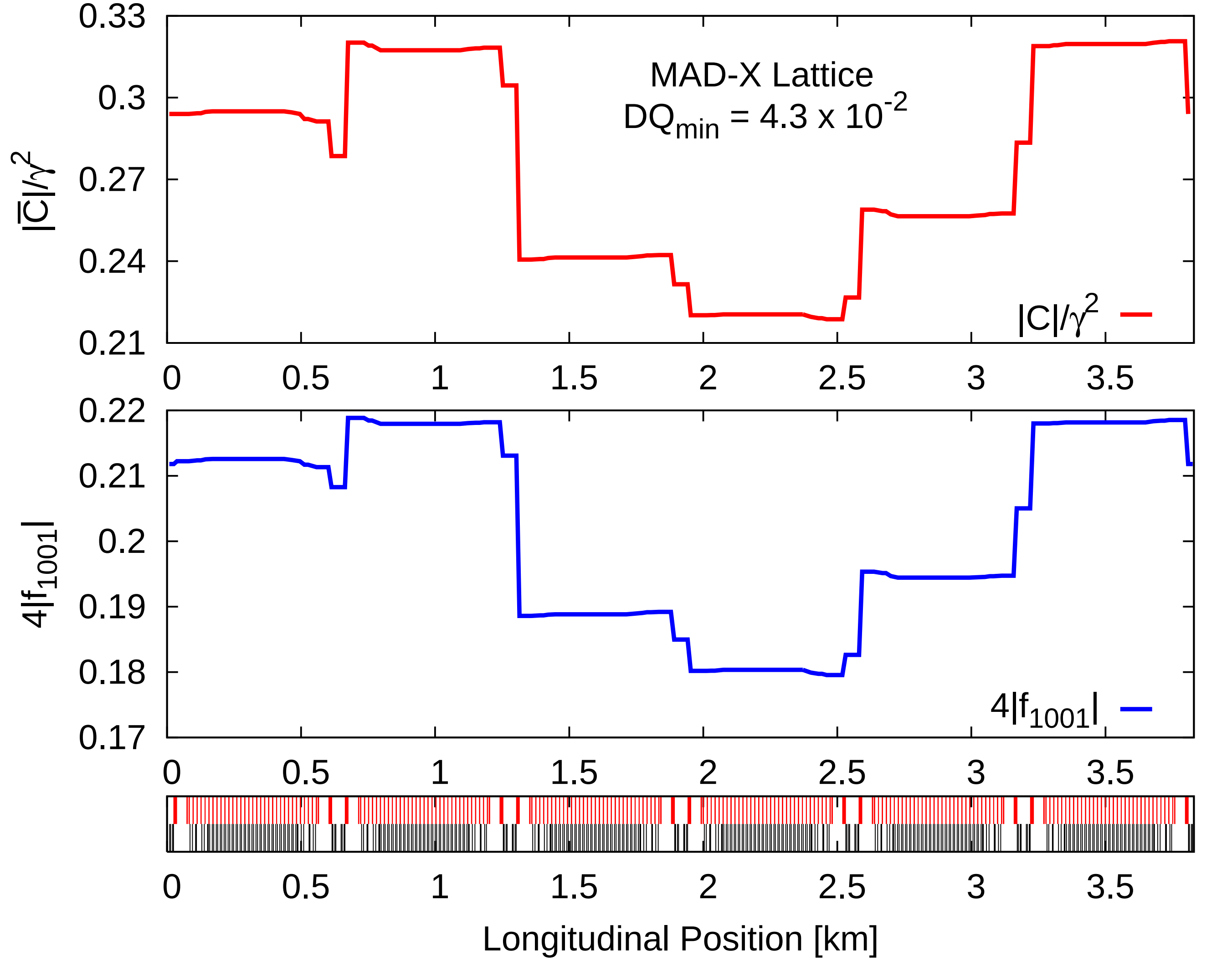
Approximate RHIC Model

RHIC has the following relevant sources of coupling:

- Skew quadrupolar errors in IR magnets
- Tilts in the triplet quadrupoles
- IR skew quadrupolar correctors
- Sextupolar feed-down at the chromaticity sextupoles and at all the dipoles

MAD-X models containing the first three items of the list have been constructed at injection and store energies.

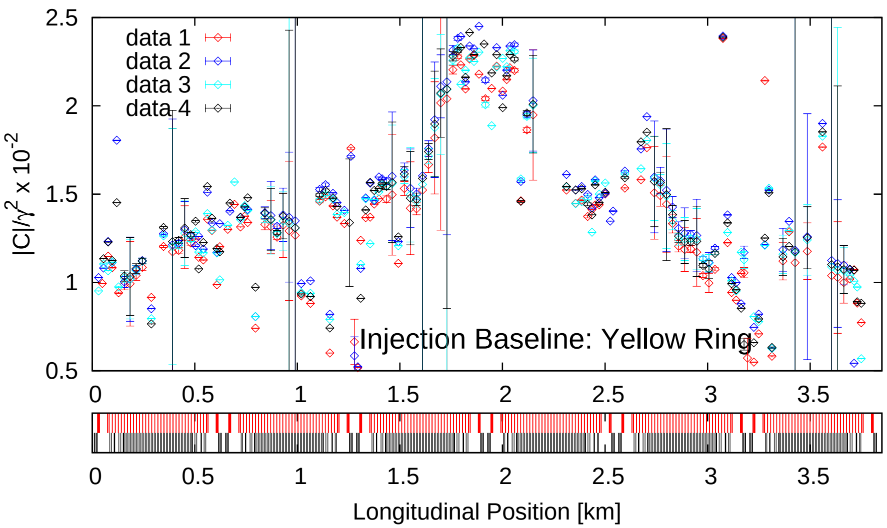
RHIC Lattice (MAD-X Tracking)

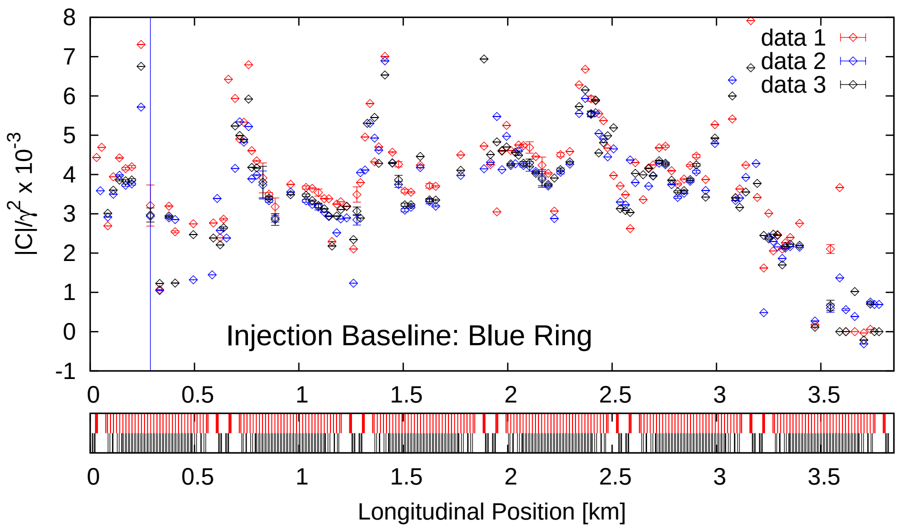


Local IR skew correctors powered to generate coupling sources

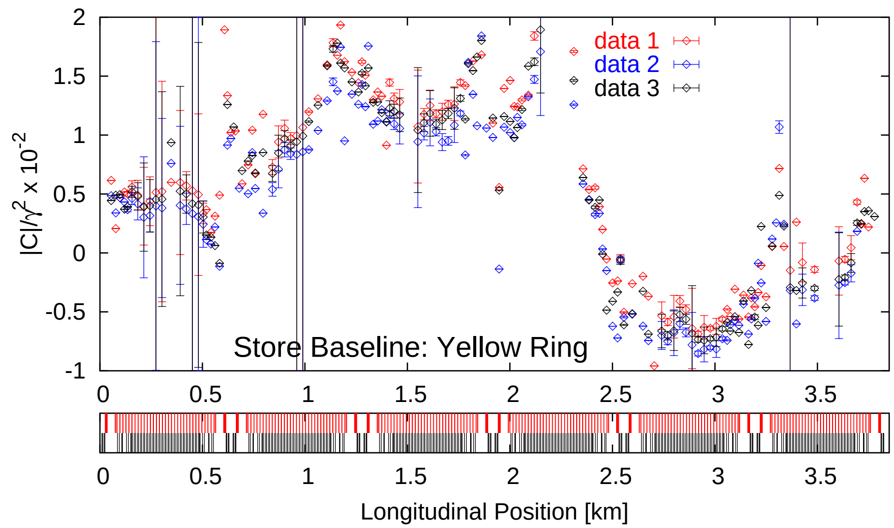
Run 2005: Cu-Cu (AC Dipole Data)

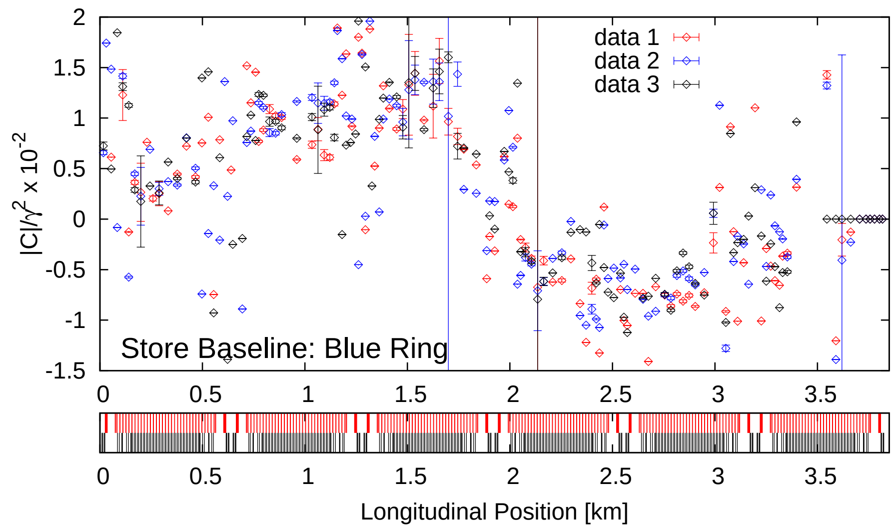
Injection



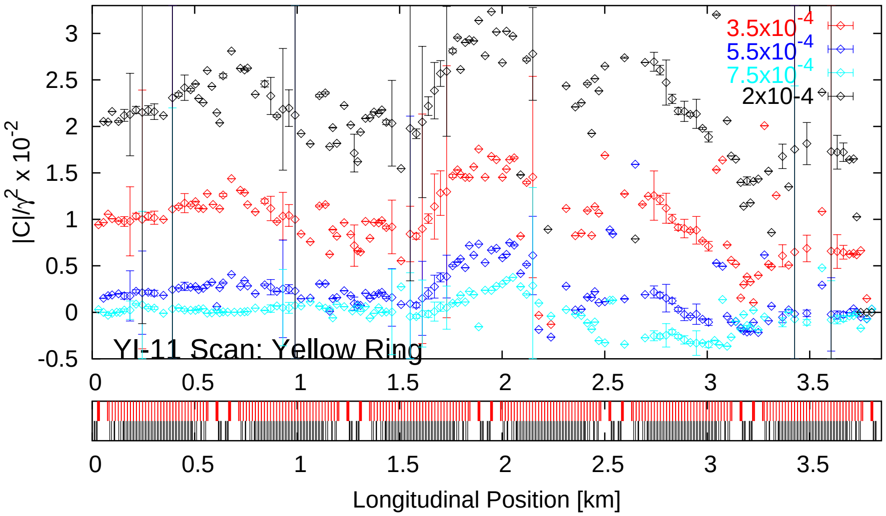


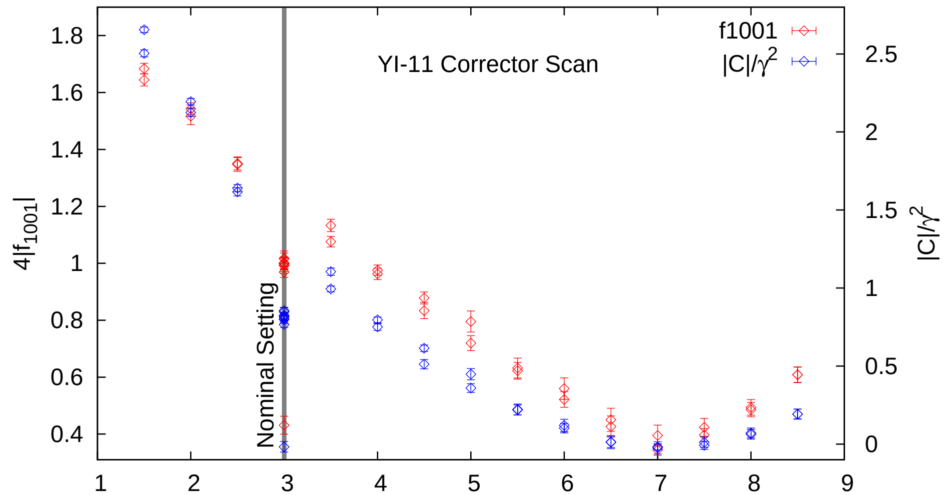
Store

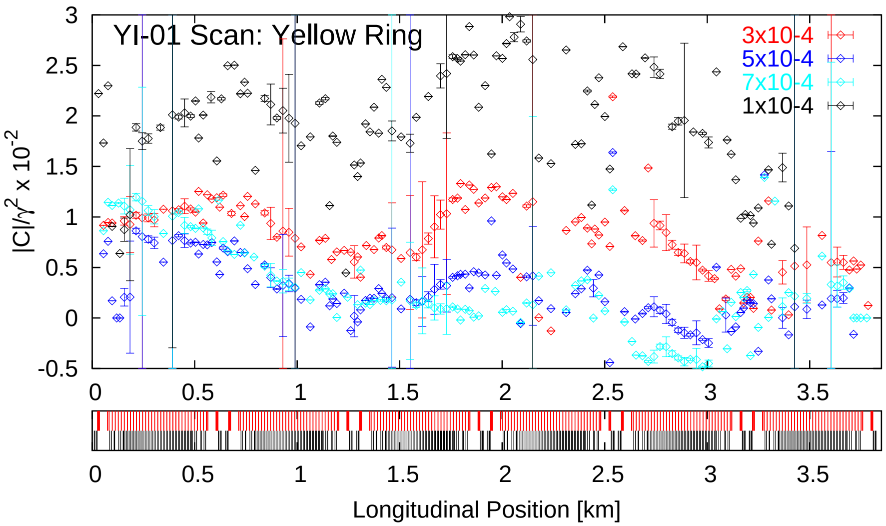


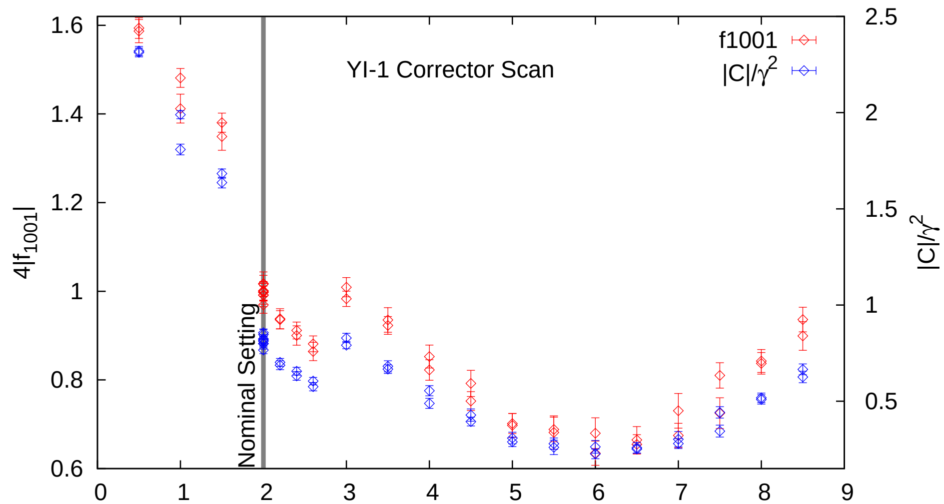


IR Corrector Scan: Injection



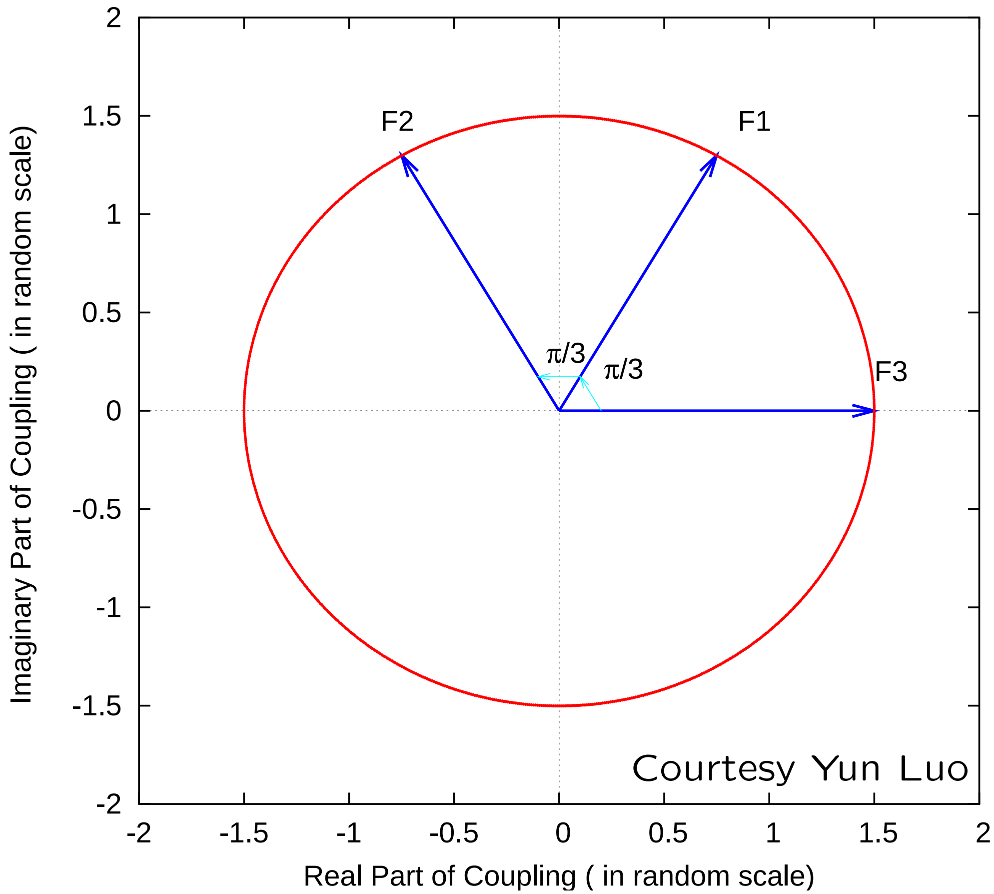






Global Coupling

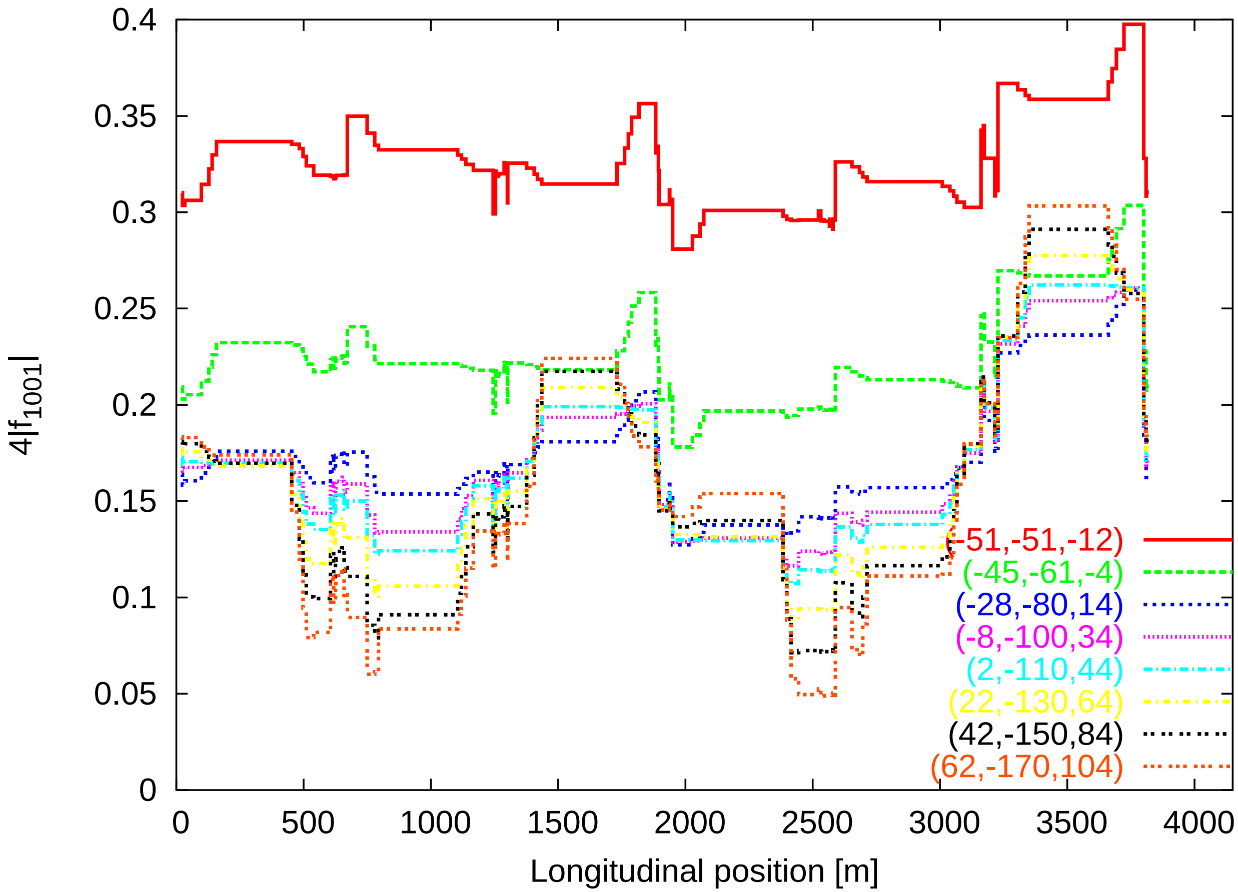
Global Coupling - Three Family Scheme



All settings of the form (F1$-\Delta$,F2$+\Delta$, F3$+\Delta$) have the same ΔQ_{min}

MAD-X RHIC Model

Global coupling compensated ($\Delta Q_{min} = 1 \times 10^{-3}$)



The numbers in brackets represent the strength of the families in units of $10^{-5}m^{-1}$.