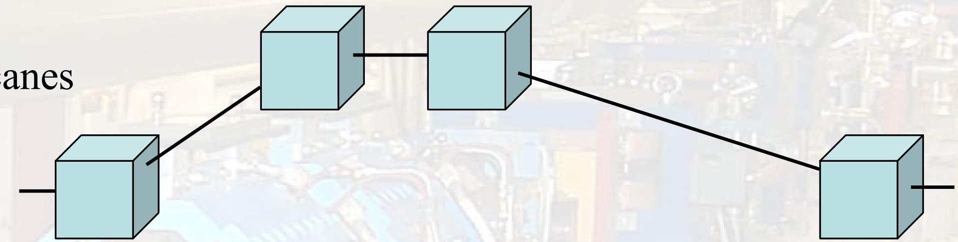
A Bunch Compressor for small Emittances and high Peak Currents at TTF2

- General remarks on Bunch Compression
- Coherent Synchrotron Radiation
- Simulation Codes
- Results of BC3 simulations
- CSR Modulation Gain

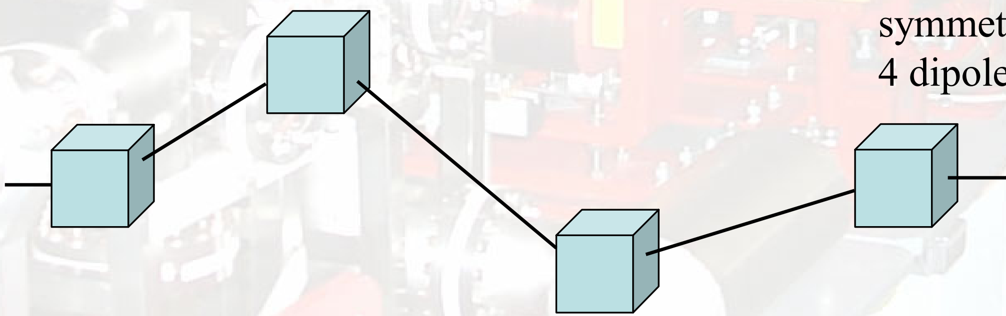
General Remarks on Bunch Compression

- There are several ways to compress a bunch
  - velocity bunching
  - magnetic chicanes
  - FODO-cell arcs
- I will only consider magnetic chicanes

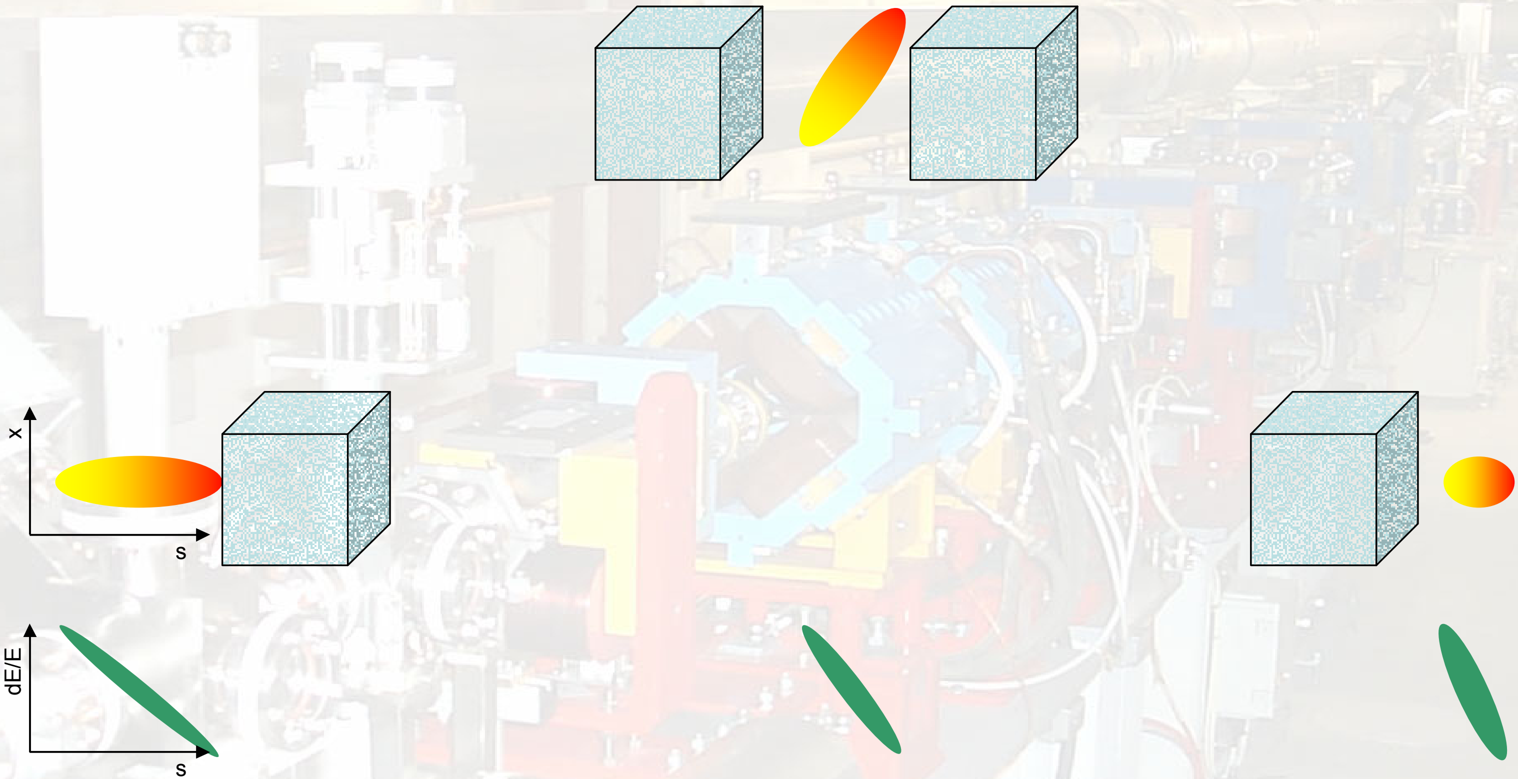


C-Chicanes
symmetric or asymmetric

S-Chicanes
symmetric or asymmetric
4 dipoles or 6 dipoles



How is a bunch compressed?



The change of bunch length is proportional to the relative energy spread $dE/E = \sqrt{\sigma_\gamma^2 + \sigma_\delta^2}$
R_{56} is an element of the first order transfer matrix
in magnetic chicanes R_{56}<0 always

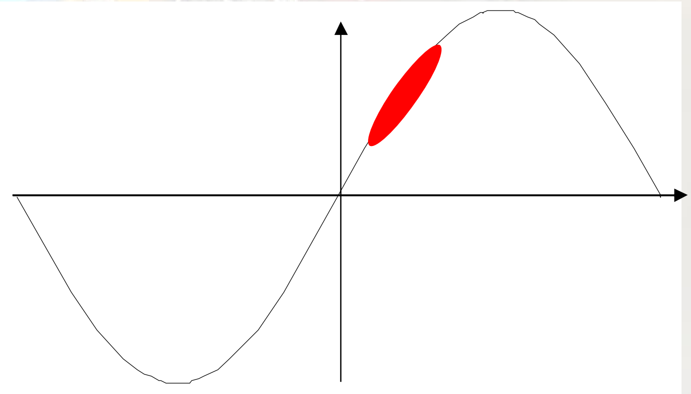
$$\sigma_{s,f} = \sqrt{\left(\sigma_{s,i} + R_{56}\,\sigma_\gamma\right)^2 + R_{56}^2\,\sigma_\delta^2}$$

if correlated energy spread σ_γ >> uncorrelated energy spread σ_δ

$$\sigma_{s,f} - \sigma_{s,i} \approx R_{56}\,\frac{dE}{E}$$

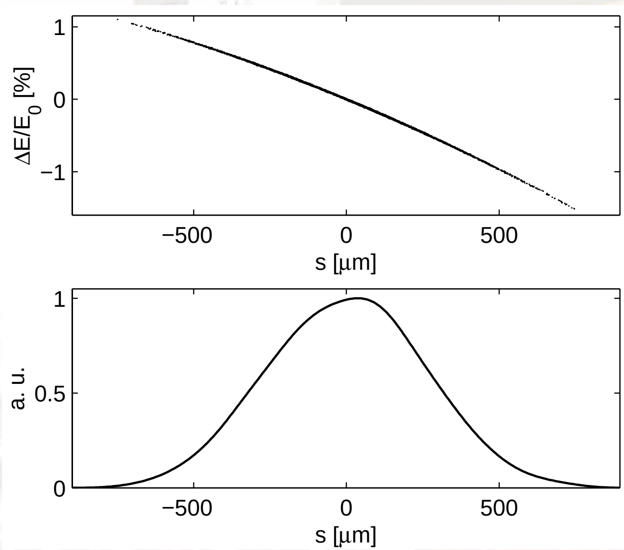
higher order terms, e.g. R_{566}, limit the bunch length

also non linear terms in the energy spread, e.g. RF curvature, limit the minimum bunch length

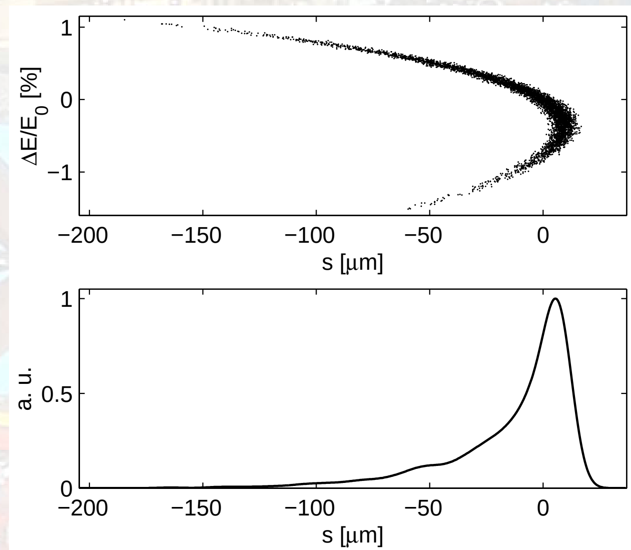


General Remarks on Bunch Compression

non linearities can produce a sharp spike in the profile
the width of the spike depends on the uncorrelated energy spread



before compression



after compression

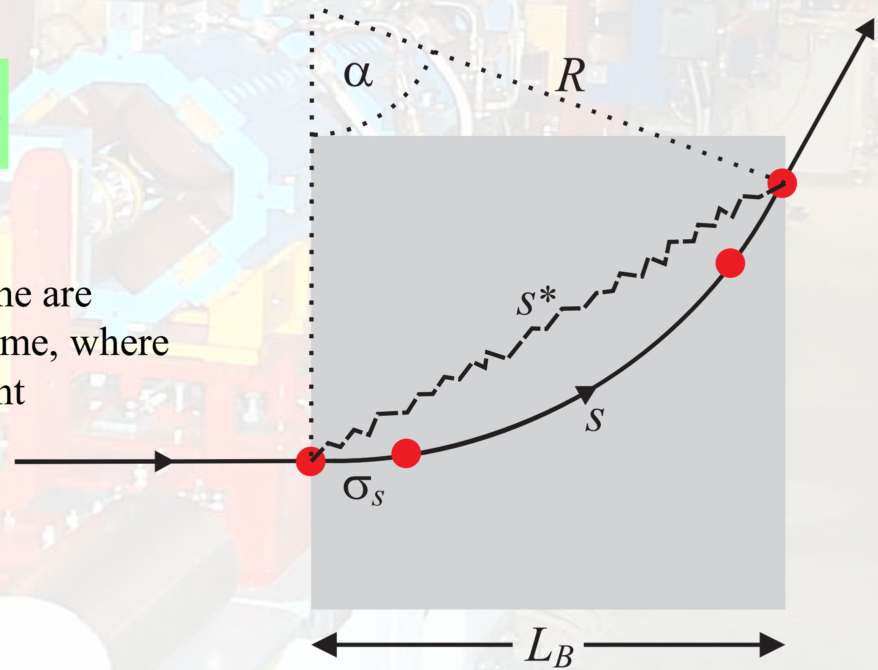
It is preferable to use short bunches!

Charged particles emit synchrotron radiation when passing a bending magnet.

Head and tail of a bunch of particles can interact if the slippage length is larger then the bunch length. The bunch can radiate coherently. (assume steady state, no transients)

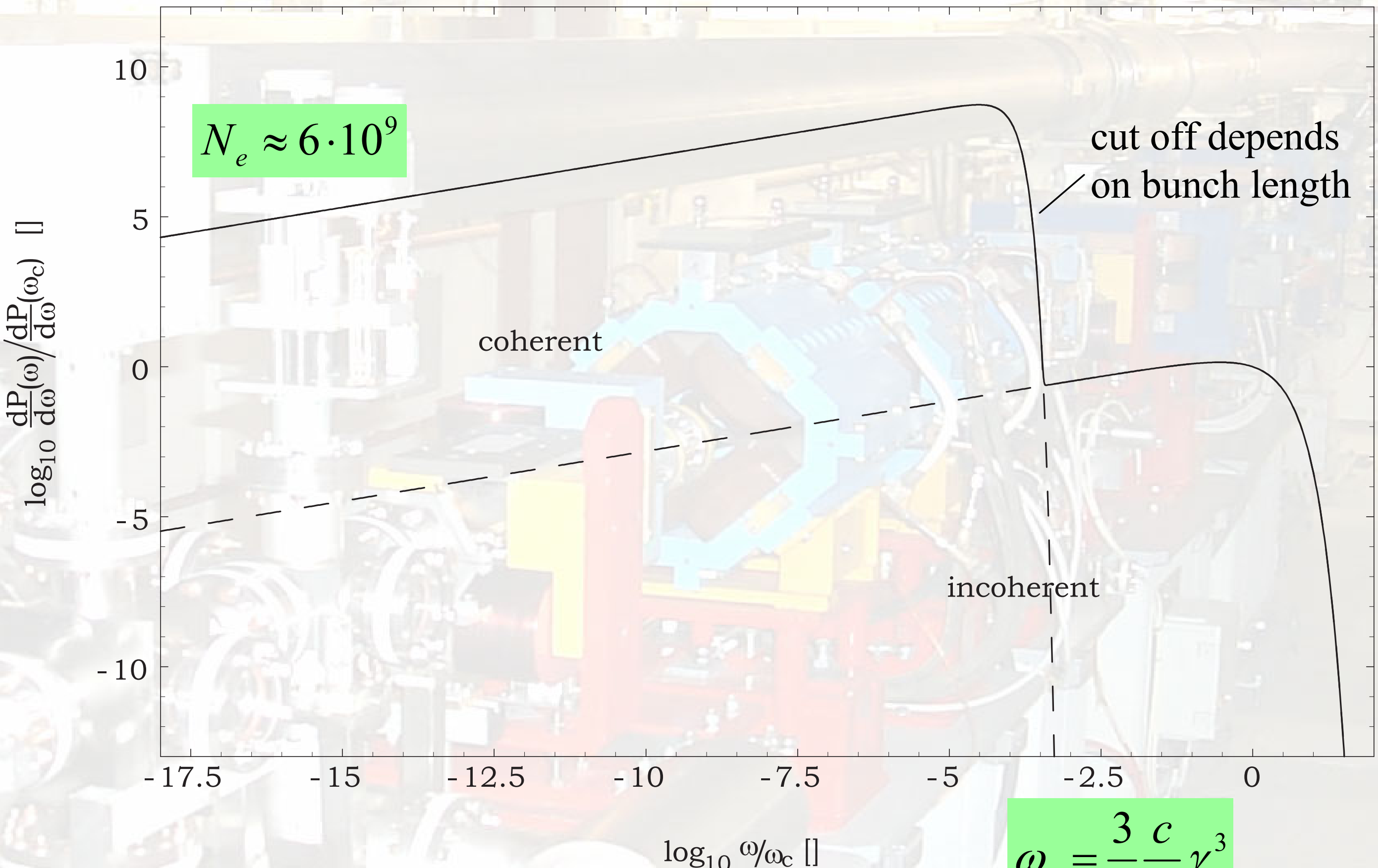
slippage length:

$$l_s = s - s^* \approx \frac{R\alpha^3}{24}$$



Incoherent and fully coherent regime are divided by a partially coherent regime, where only part of the spectrum is coherent

Coherent Synchrotron Radiation



$$\omega_c = \frac{3}{2}\frac{c}{R}\gamma^3$$

Three regimes can be distinguished:

Incoherent radiation

$$P_{i.c.} = \frac{1}{6\pi\,\varepsilon_0}\frac{N_e\, e_0^2\, c\,\gamma^4}{R^2}$$

scales with N_e — scales with energy

Partially coherent radiation

$$P_{p.c.} = \frac{\Gamma\left(\frac{5}{6}\right)}{6^{1/3}\,4\pi^{3/2}\,\varepsilon_0}\frac{N_e^2\, e_0^2\, c}{R^{2/3}\,\sigma_s^{4/3}}$$

scales with N_e^2 — scales with bunch length

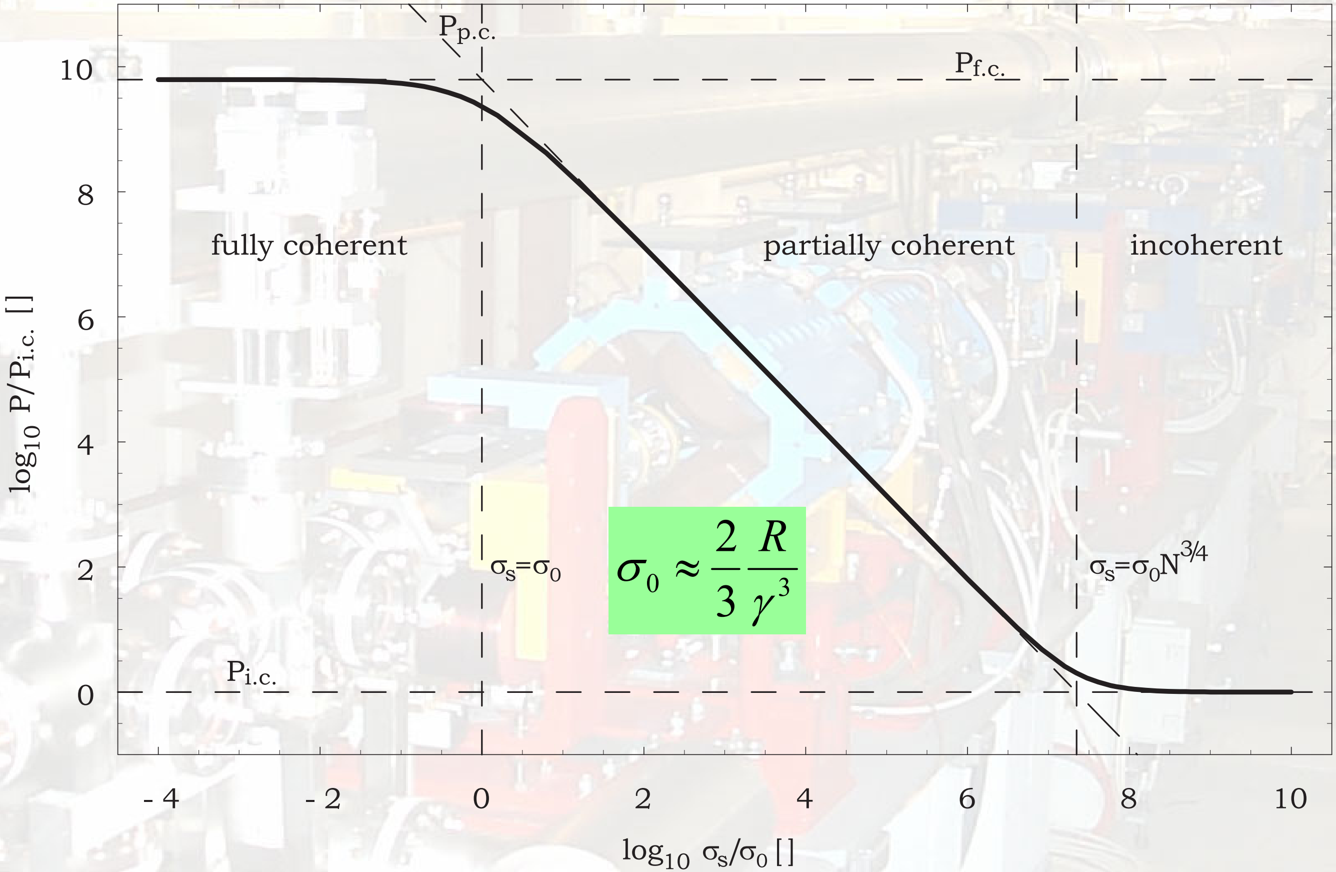
Fully coherent radiation

$$P_{f.c.} = \frac{1}{6\pi\,\varepsilon_0}\frac{N_e^2\, e_0^2\, c\,\gamma^4}{R^2}$$

scales with N_e^2 — scales with energy

Coherent Synchrotron Radiation

$$N_e \approx 6 \cdot 10^9$$



$$\sigma_0 \approx \frac{2}{3}\frac{R}{\gamma^3}$$

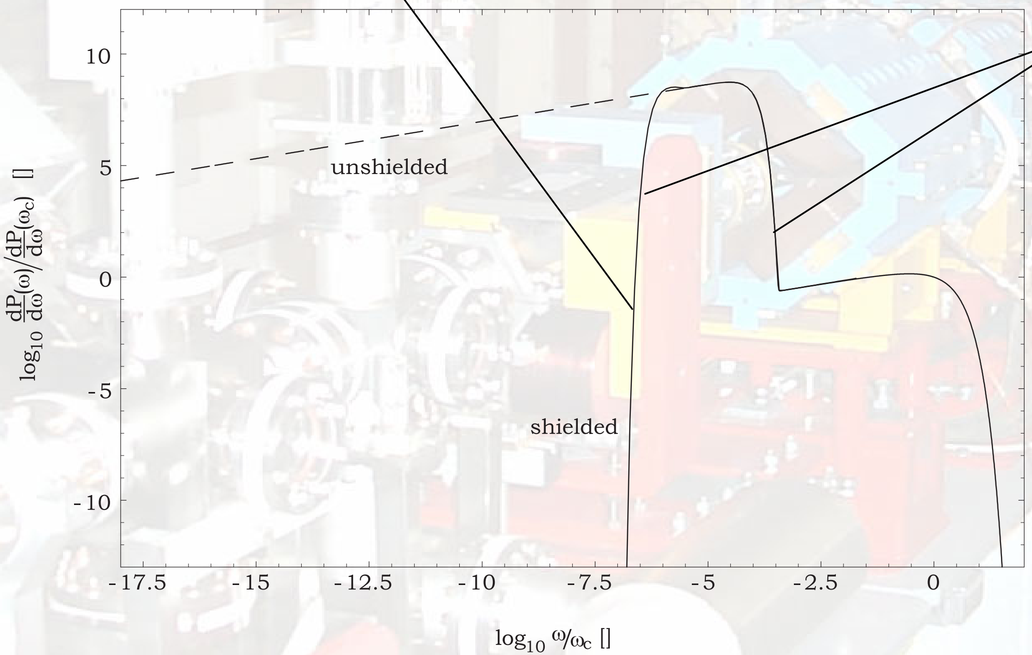
between two parallel conducting plates the low frequency part is cut off:

$$\omega << \sqrt{\frac{2}{3}\frac{c}{h}\left(\frac{\pi R}{h}\right)^{3/2}}$$

This can have a noticeable effect on the total radiation power if

$$\sigma_s >> \frac{h}{c}\sqrt{\frac{3h}{2\pi R}}$$

than distance of cut offs is small

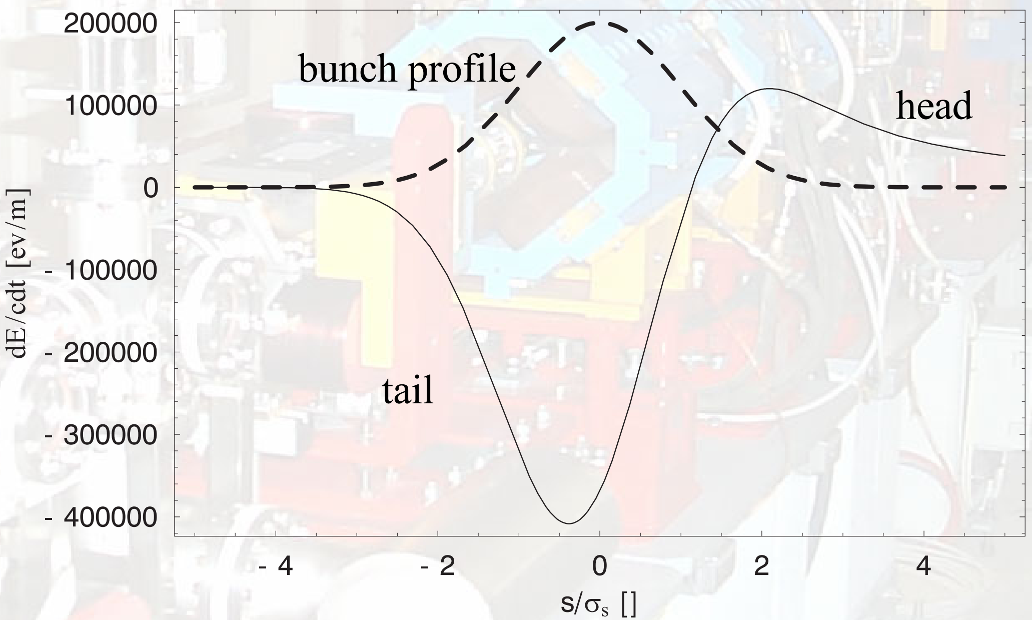


same spectrum as before but now in chamber of height *h*=10cm

CSR leads to an energy redistribution along the bunch.
The head gains energy and the tail loses energy

$$\frac{dE(s)}{cdt} = -\frac{2N_e e^2}{4\pi\,\varepsilon_0\sqrt{2\pi}\,3^{1/3}\,R^{2/3}\,\sigma_s^{4/3}} F_0\left(\tfrac{s}{\sigma_s}\right)$$

$$F_0(x) = \int_{-\infty}^{x} \frac{-x' e^{-x'^2/2}}{(x-x')^{1/3}}\,dx'$$



Due to the energy redistribution every particle gets a different kick in the bunch compressor chicane. Since the particles energy changes in each magnet also subsequent kicks for a single particle will differ along the chicane.

Every particle will have an individual offset and angle behind the chicane.

=> The projected emittance grows

$$\varepsilon_{proj.} = \gamma \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x\, x' \rangle^2}$$

Geometric considerations show, that behind an S-chicane the final x is smaller than behind a C-chicane, but the final x' is larger

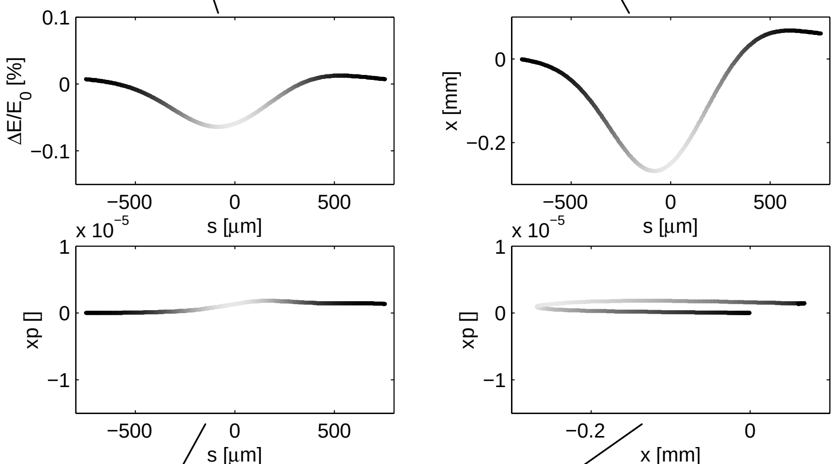
=> The projected emittance is smaller behind an S-chicane

Coherent Synchrotron Radiation

initially no unc. energy spread
no compression
gaussian profile

longitudinal phase space

s - x



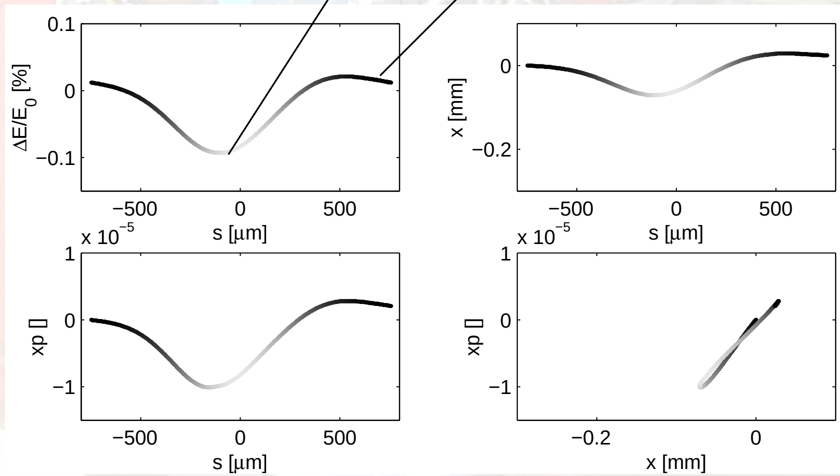
C-chicane

s - x'

transverse phase space

bright color=high charge

dark color=low charge

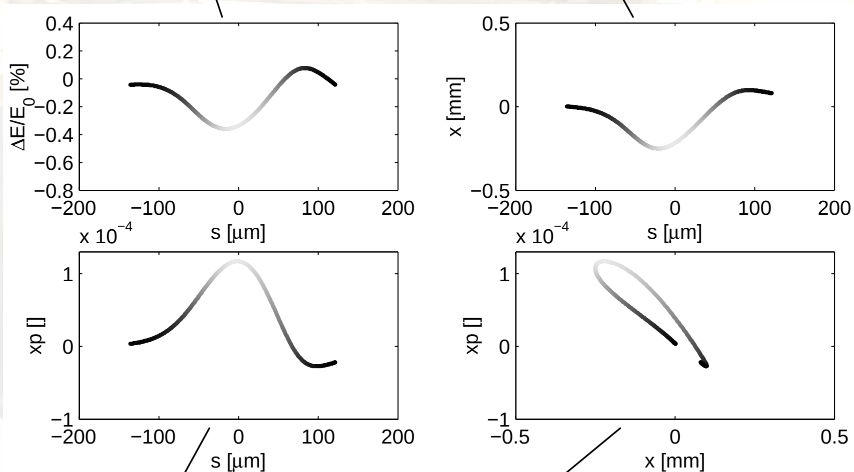


S-chicane

Coherent Synchrotron Radiation

longitudinal phase space

s - x



s - x'

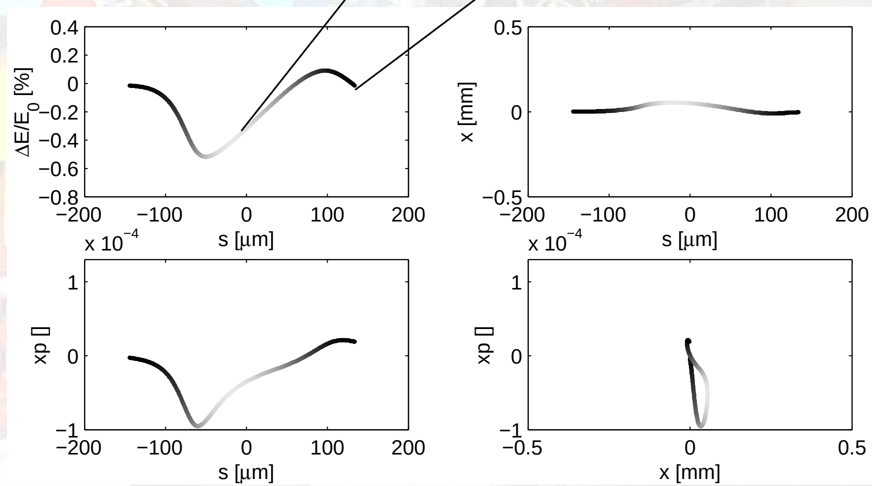
transverse phase space

initially no unc. energy spread
with compression
gaussian profile

C-chicane

bright color=high charge

dark color=low charge



linear part of energy spread is subtracted!

S-chicane

To simulate the CSR effect we would need to:

- track a 3D distribution made of 10^{10} particles
- through a beam line consisting of drifts, dipoles, quadrupoles, ...
- and take into account full 3D electromagnetic fields in conducting vacuum chambers of arbitrary cross section (shielding)

this is far out of reach, so we have to:

- reduce the number of particles as far as reasonable
- simulate only part of beam line where CSR is expected, i.e. dipoles and drifts behind them
- simplify the field calculation
- usually neglect shielding

different approaches a possible:

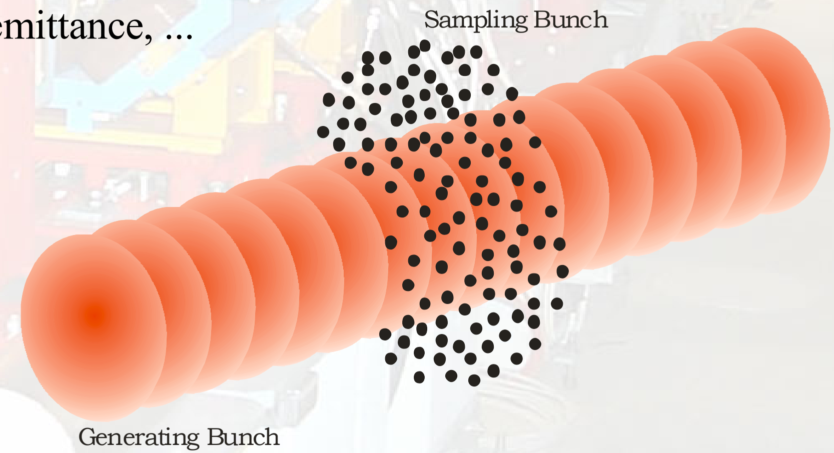
- use very simple field calculation and complicated beam => 1D method (e.g. in ELEGANT, CSRTrack)
- use rather complicated field calculation and rather complicated beam => 2D method (e.g. in CSRTrack)
- use complicated field calculation and very simple beam => 3D method (e.g. in TraFiC4 and CSRTrack)

ELEGANT describes bunch as a 3D distribution of point-like particles

- very many particles are needed to suppress noise

TraFiC4 and CSRTrack use gaussian charge distributions to describe bunch:

- 3D particle distribution is represented by a generating bunch and a sampling bunch
- generating bunch consists of 3D sub bunches, i.e. gaussian charge distributions
- they emit fields and are tracked self consistent within these fields
- 3D sampling bunch consists of point-like particles
- they are tracked in fields of gen. bunch but do not emit fields
- sampling bunch is used to calculate emittance, ...

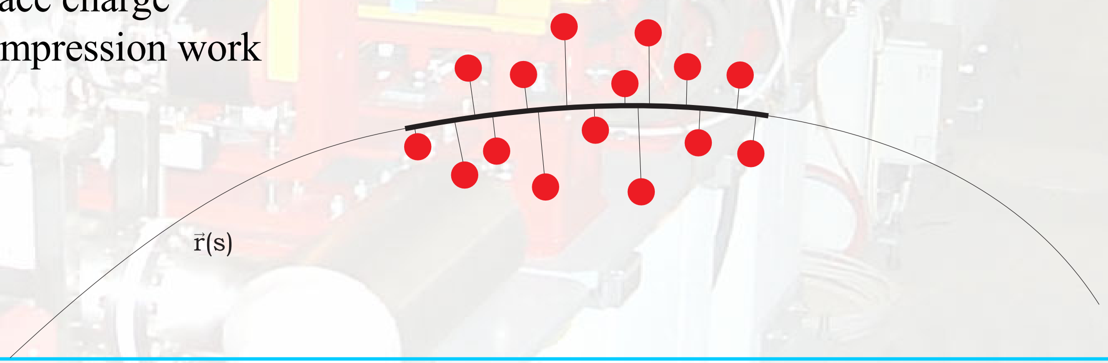


1D method / projected method:

- project 3D particle distribution on a reference trajectory
- calculate longitudinal profile
- smooth profile (if point-like particles are used, e.g. in ELEGANT)
- use analytical formulas to get the longitudinal field
- apply field to each particle (i.e. apply energy kick)

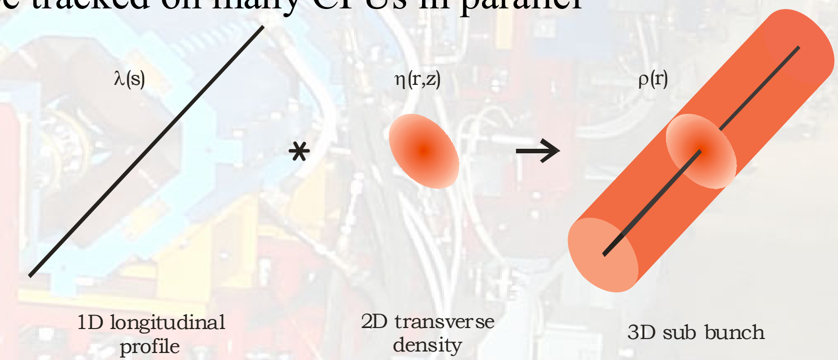
- very fast
- calculation effort scales only with the number of particles
- some 100000 particles can be tracked on a single CPU

- no transverse field
- transverse dependence of longitudinal field neglected
- no space charge
- no compression work



3D method / convolution method:

- convolution method splits each sub bunch in a 1D longitudinal profile and a 2D transverse density function
- fields are convolution of 1D field and transverse density
- approx. fields split in analytical part and 1D integration
- effort scales with N^2
- some 1000 sub bunches can be tracked on many CPUs in parallel



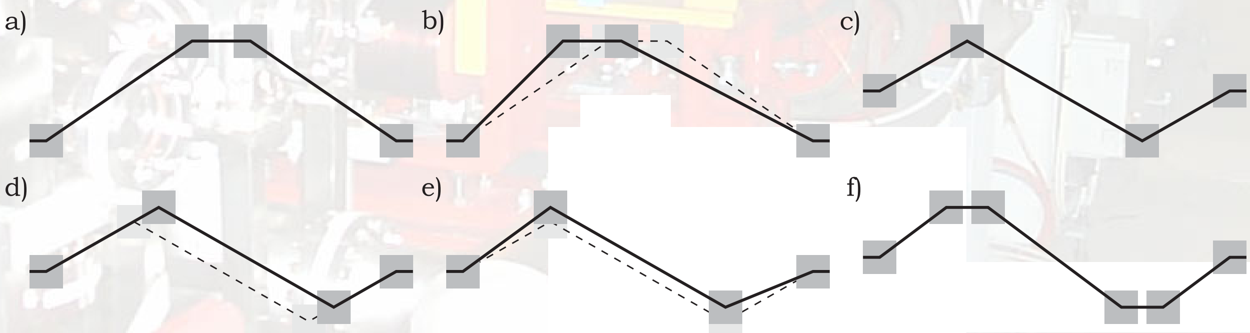
2D method / Green's function method:

- greens function method solves fields on 2D mesh
- interpolate mesh to get field at positions of particles
- also scales with N^2, but each step is a lot faster
- some 1000-10000 sub bunches can be tracked on a single CPU

At TTF2 we need a bunch compressor that:

- compresses 1nC bunches of 250µm length to 50µm
- i.e. from a peak current of 500A to 2500A
- has a nominal R_{56}=5cm
- is flexible R_{56}=2.5cm - 10cm
- works at 450MeV
- produces only a small emittance growth (slice and projected)

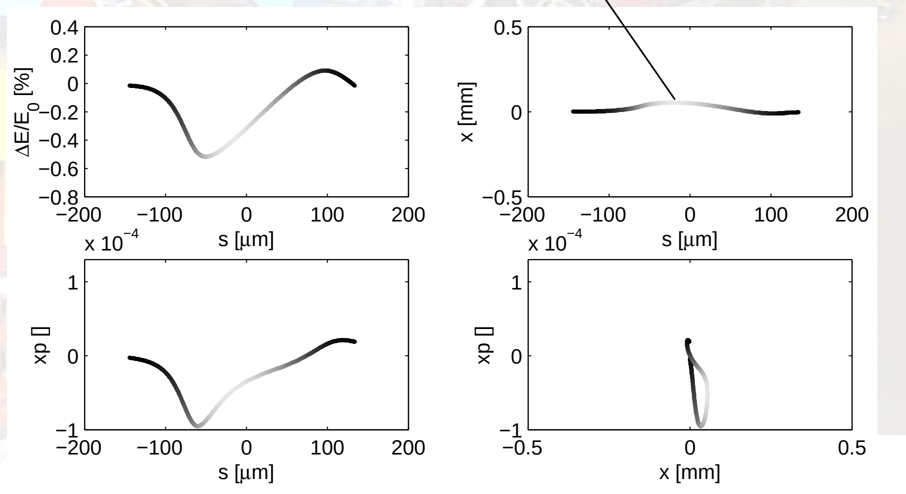
different layouts are compared:



Why use asymmetric S-chicane case d (same angle in all magnets, but drifts changed)?

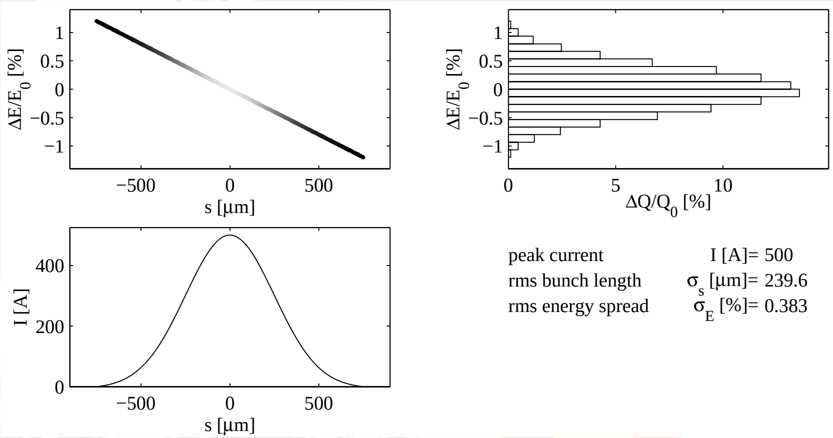
geometric considerations show
that the final offset can be lowered
by shifting inner magnets

transverse emittance is than smaller!



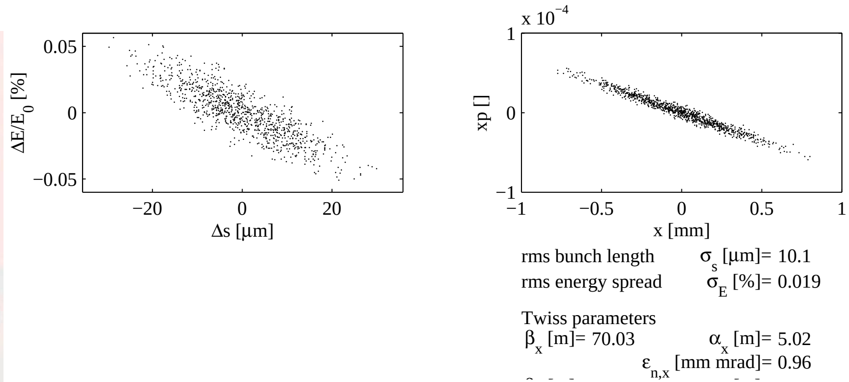
Comparison of various C- and S-chicanes (symmetric and asymmetric)

Initial distributions



generating bunch:

601 sub bunches
total charge 1nC
peak current 500A
bunch length 250µm

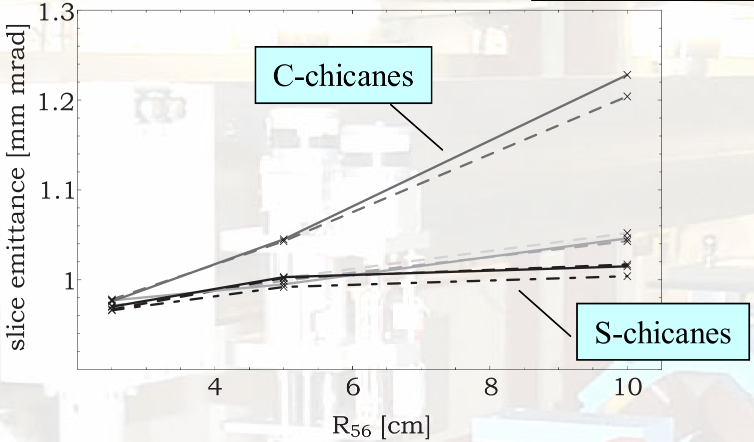


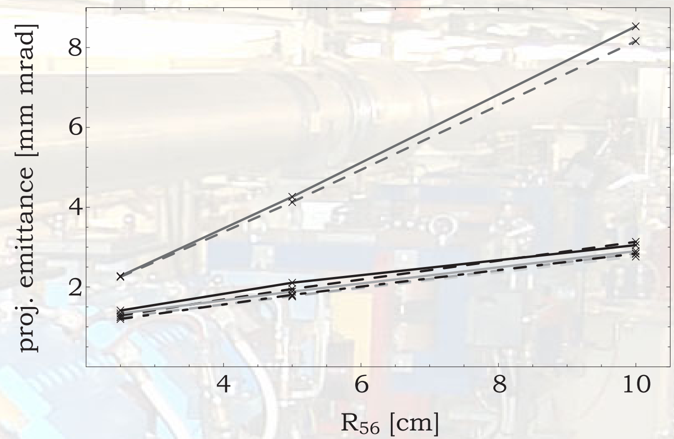
sampling bunch:

1000 particles in a slice
unc. energy spread $1\cdot10^{-4}$

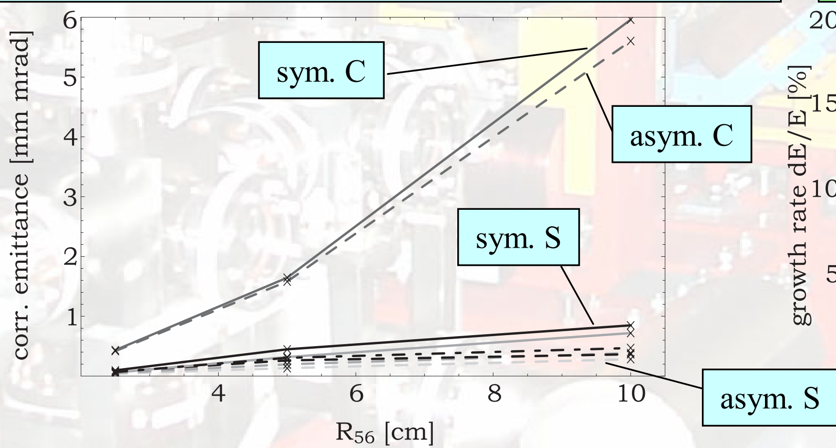
BC3 Simulations

Emittance grows stronger in C-chicanes than in S-chicanes

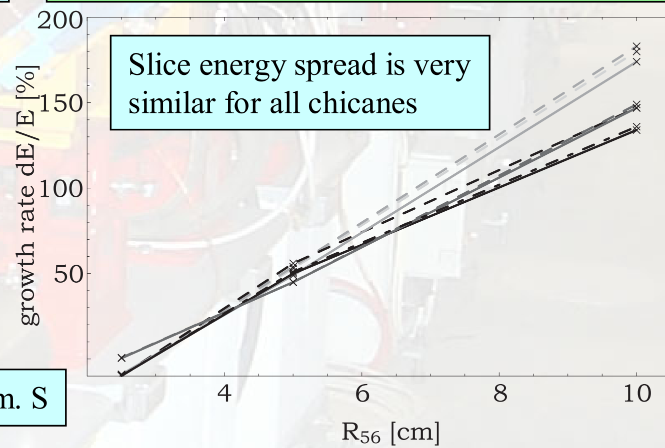




Asymmetric layouts only slightly better than symmetric layouts



symmetric 6-bend S-chicane good choice for BC3!



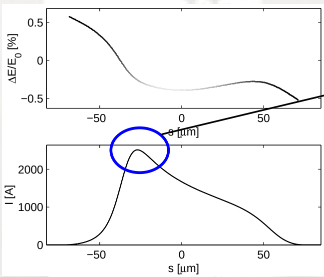
What happens if the charge is varied?

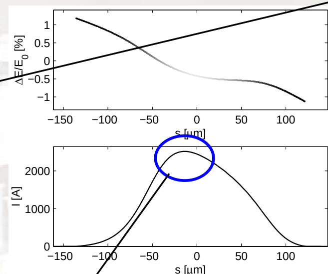
- constant peak current
- bunch length adjusted
- R56=5cm
- energy spread slope dE/ds=-16
- compression factor 5
- initially gaussian distribution
- 500A initial peak current

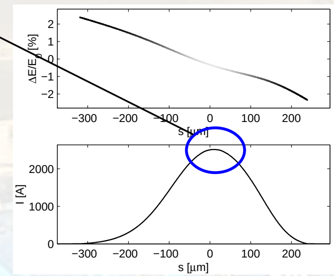
fixed R_{56}, fixed energy spread slope, peak current constant!

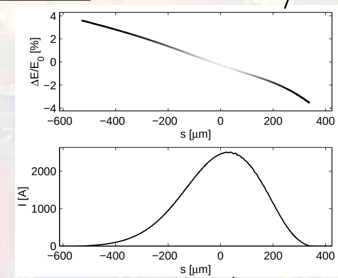
final longitudinal phase space

a steeper rise and fall of the initial profile produces a stronger deformation of the final profile









final longitudinal profile

symmetric C-chicane

Any sharp edges in the initial bunch profile will disturb the compressed profile!

0.5nC, 125µm

1nC, 250µm

2nC, 500µm

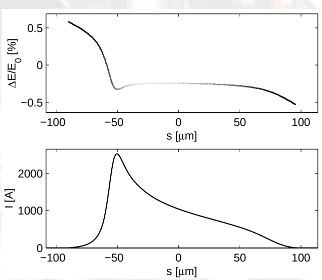
charge

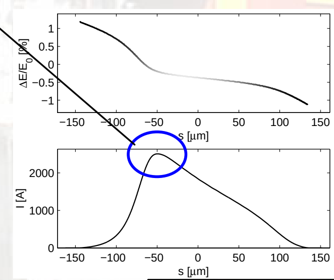
3nC, 750µm

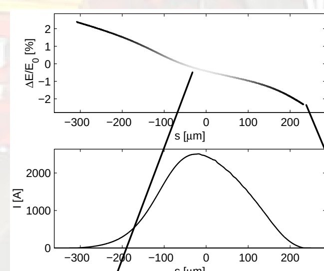
initial bunch length

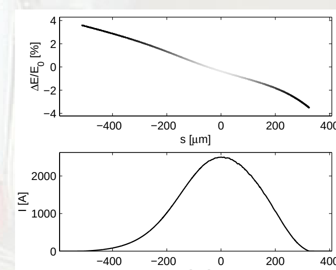
S-chicanes produce a sharper spike than C-chicanes

symmetric 6-bend S-chicane





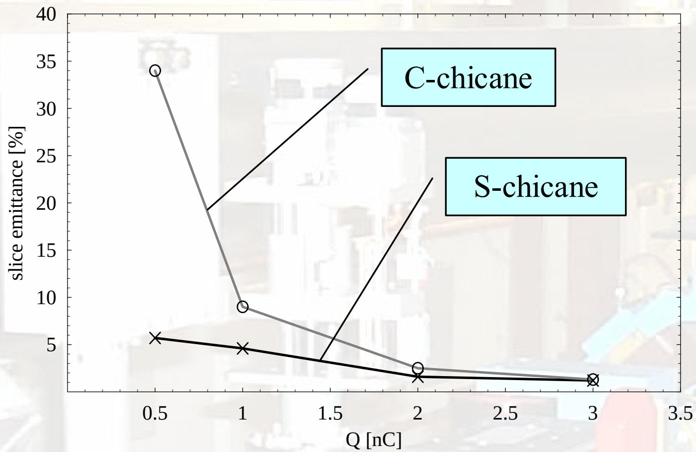


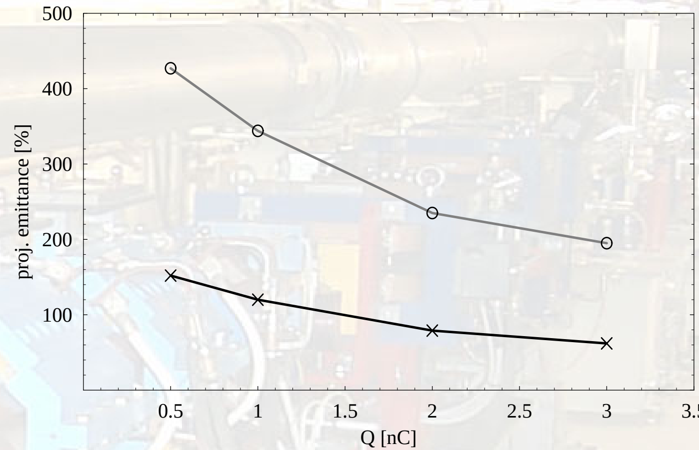


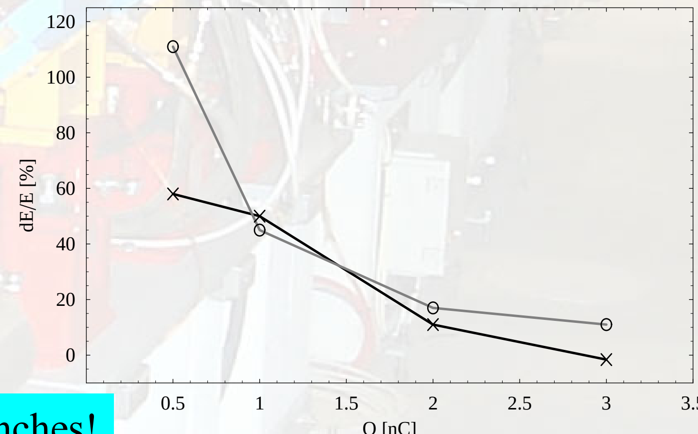
bright color=high charge

dark color=low charge

BC3 Simulations







o C-chicane

× S-chicane

0.5nC, 125µm

1nC, 250µm

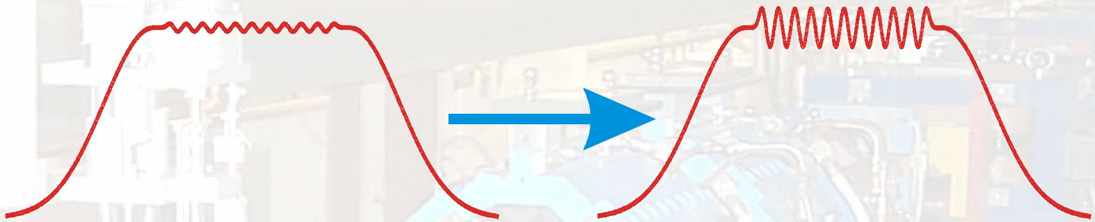
2nC, 500µm

3nC, 750µm

peak current 500A → 2500A

It is preferable to use long bunches!

Small density or energy modulations can be amplified due to CSR fields when the bunch passes a bunch compressor chicane:



- an initial density modulation is converted into an energy modulation due to CSR
- an energy modulation is converted into a density modulation due to dispersion

Since the particles energy is modulated the emittance will grow!

theory by Saldin et al.: (no compression, no emittance!)

strong dependence on k=2π/λ,
λ –> 0 => G –> ∞

gain for a beam without uncorrelated energy spread

$$G = \frac{2\Gamma^2(2/3)}{3^{5/3}} \left(\frac{I_0}{\gamma_0 I_A} \right)^2 \frac{k^{8/3} \left| R_{56} \right|^2 L_d^2}{R^{4/3}}$$

gain for a beam with uncorrelated energy spread

$$G = \frac{2\Gamma^2(2/3)}{3^{5/3}} g_0^2 \, f(\hat{k}) \qquad g_0 = \frac{I_0}{\sigma_\gamma I_A} \left(\frac{\gamma_0}{\sigma_\gamma} \right)^{1/3} \frac{L_d}{\left(R^2 \left| R_{56} \right| \right)^{1/3}}$$

maximum at $\hat{k}_{max}$=2.15
=> G_{max}=1.16 g_0^2

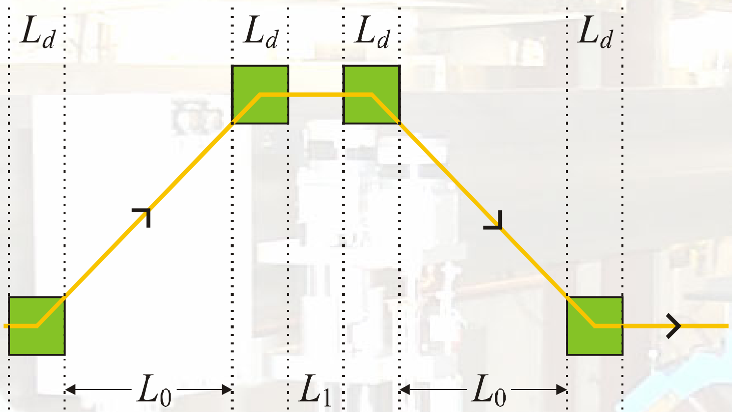
suppression of the gain for small
wavelengths, λ –> 0 => G –> 0

$$\hat{k} = \frac{\sigma_\gamma}{\gamma_0} \left| R_{56} \right| k$$

$$f(\hat{k}) = 3\hat{k}^{2/3} \exp\left(-\hat{k}^2/2 \right) \left[1 + \frac{\sqrt{\pi}}{2} \frac{\hat{k}^2 - 2}{\hat{k}} \exp\left(\hat{k}^2/4 \right) \operatorname{erf}\left(\hat{k}/2 \right) \right]$$

Also emittance suppresses gain!

Since this model does not include emittance and compression the formulas by Stupakov et al. are used for the plots

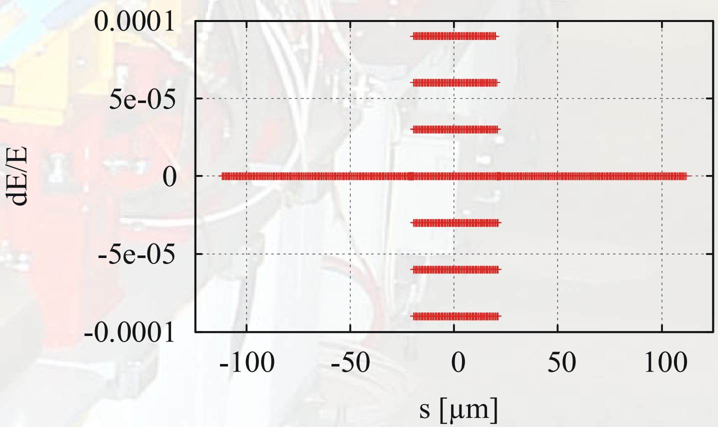


Beam parameters

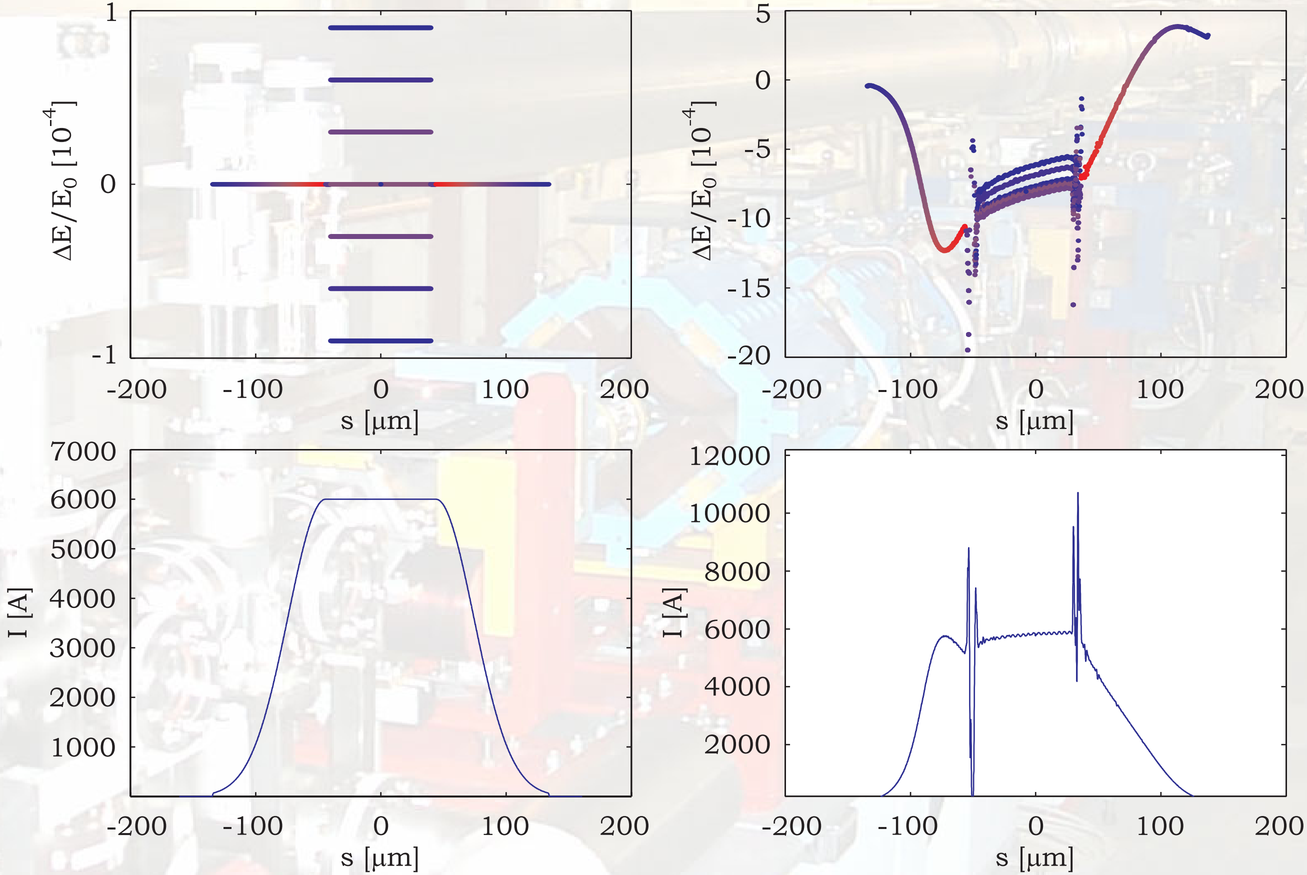
nominal energy	E_0	5.0 GeV
flat top current	I_0	6 kA
rel. modulation depth	η	10^{-4}
uncorr. rms energy spread	σ_δ	$0 / 3 \cdot 10^{-5}$
corr. rms energy spread → no compression	σ_γ	0
norm. emittance	$\varepsilon_{x,y}$	0 / 1 mm mrad

Benchmark Chicane parameters

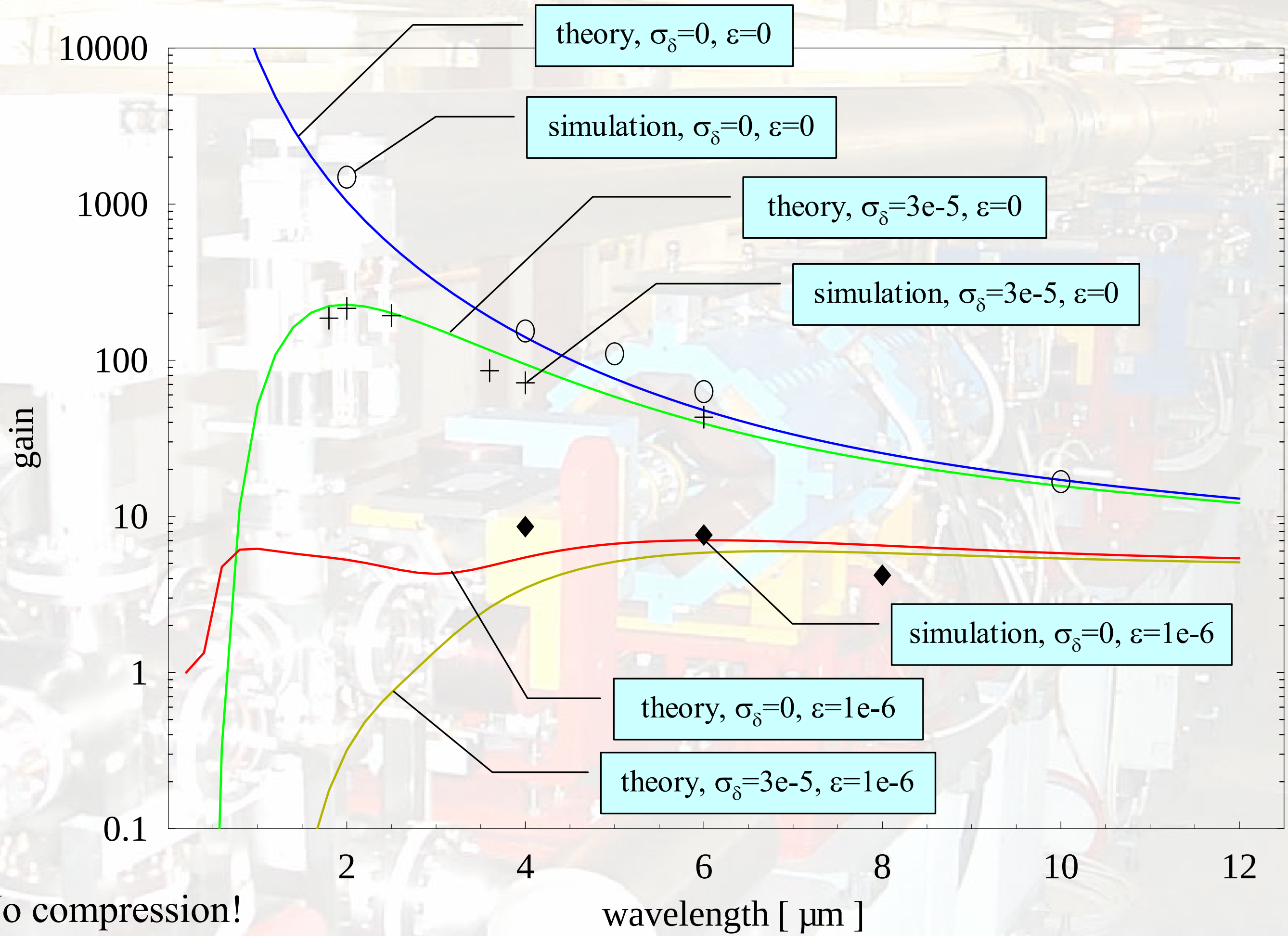
dipole length	L_d	0.5 m
length of 1st and 3rd drift	L_0	5.0 m
length of 2nd drift	L_1	1.0 m
bend radius	R	10.35 m
bending angle	Φ	2.77 deg
momentum compaction	R_{56}	-25 mm



initial: final:

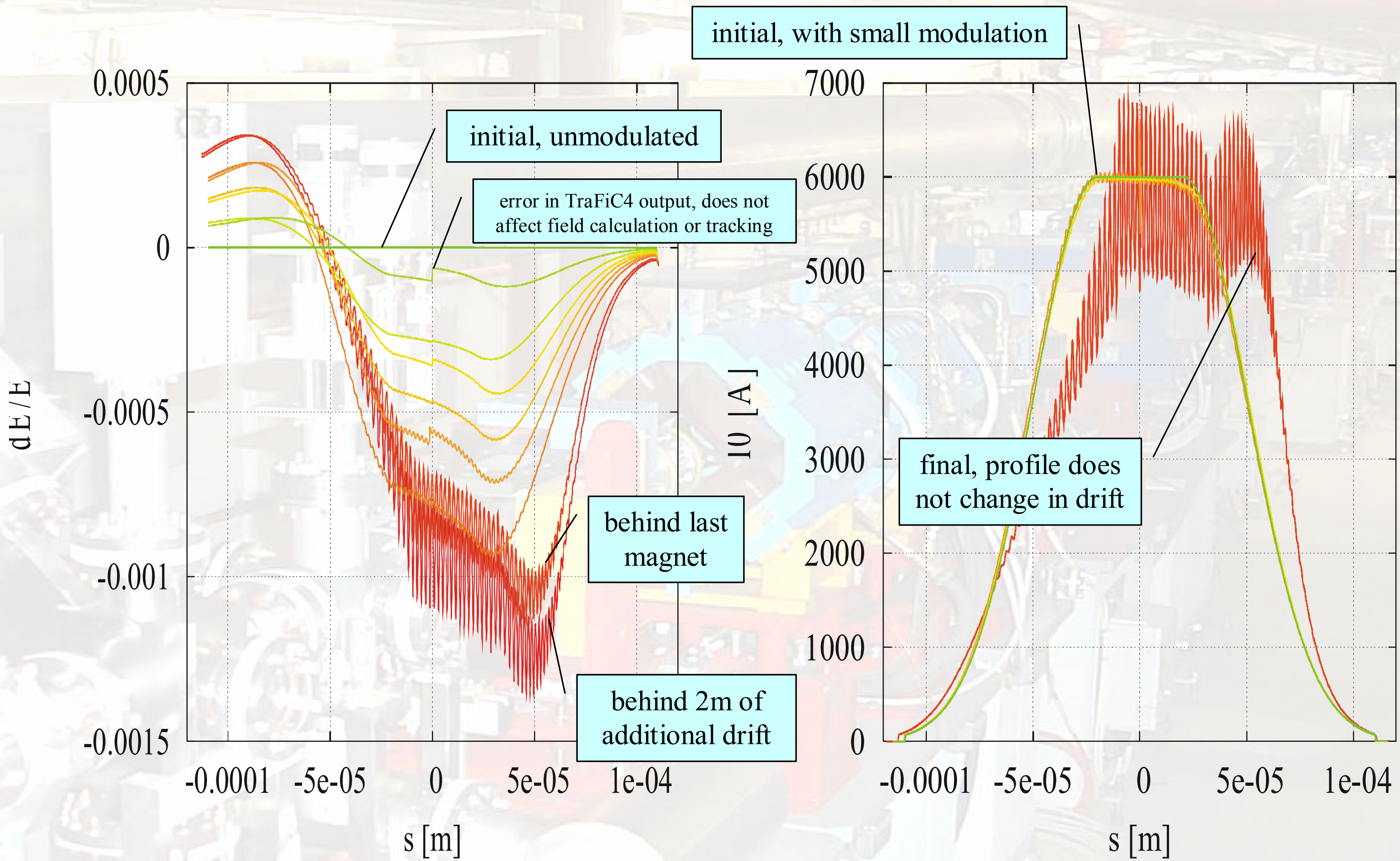


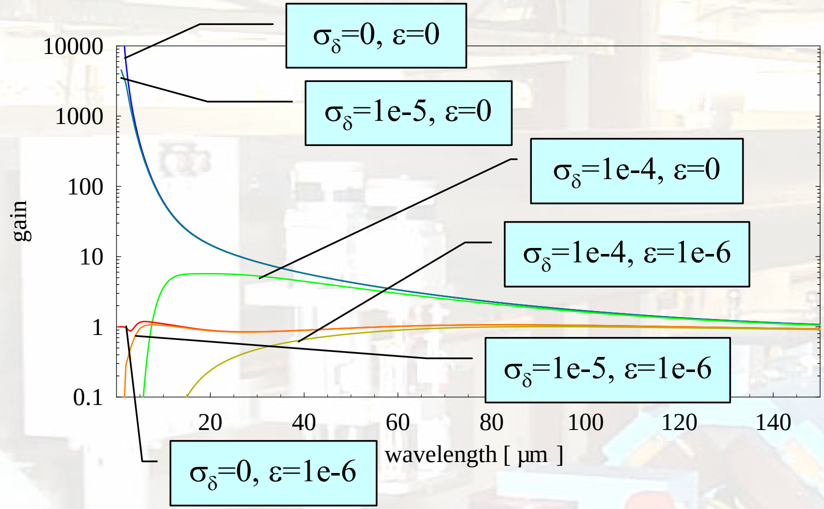
Modulation Gain due to CSR



No compression!

Modulation Gain due to CSR



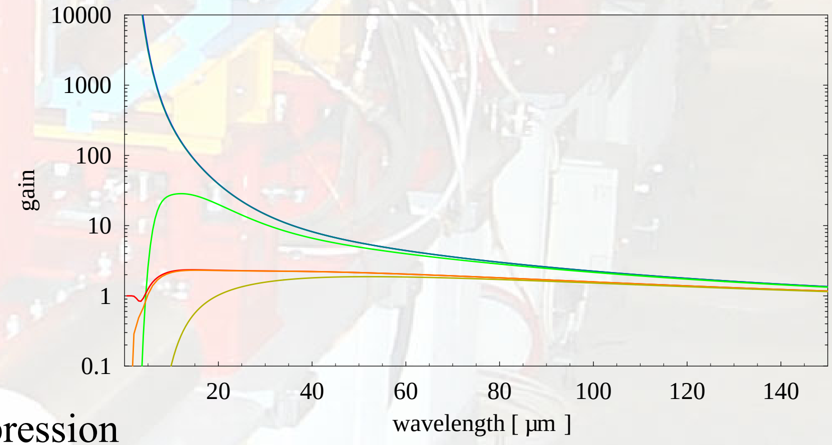


C-chicane, no compression

R56	0.05m
peak current	500A
unc. energy spread	0 / 1e-5 / 1e-4
emittance	0 / 1e-6

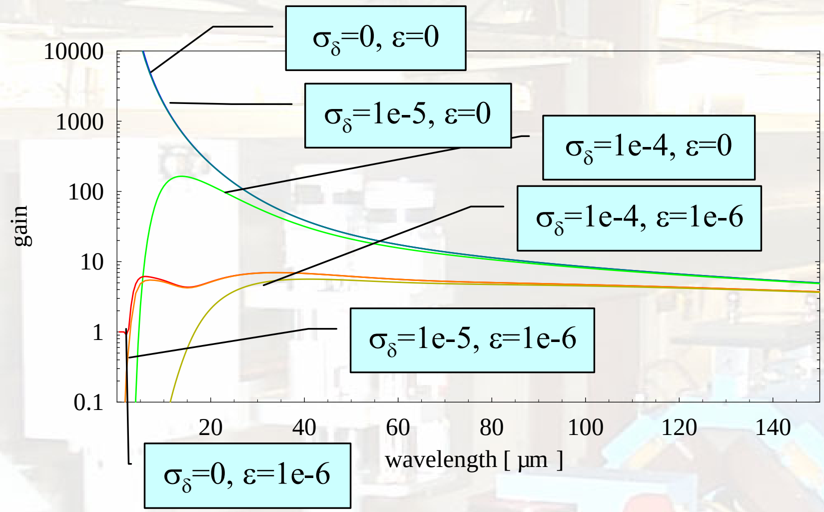
Attention:

Theory makes assumptions:
- infinite bunch length
- radiation in steady state



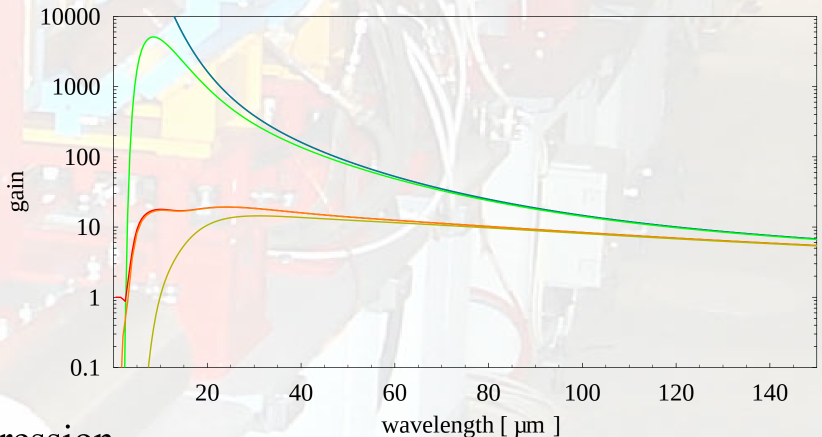
S-chicane, no compression

Modulation Gain due to CSR



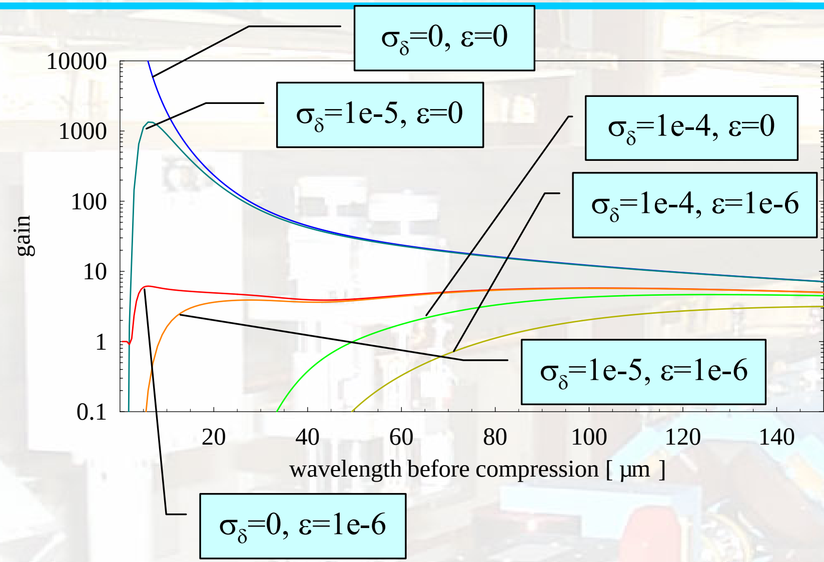
C-chicane, no compression

R56	0.05m
peak current	2500A
unc. energy spread	0 / 1e-5 / 1e-4
emittance	0 / 1e-6



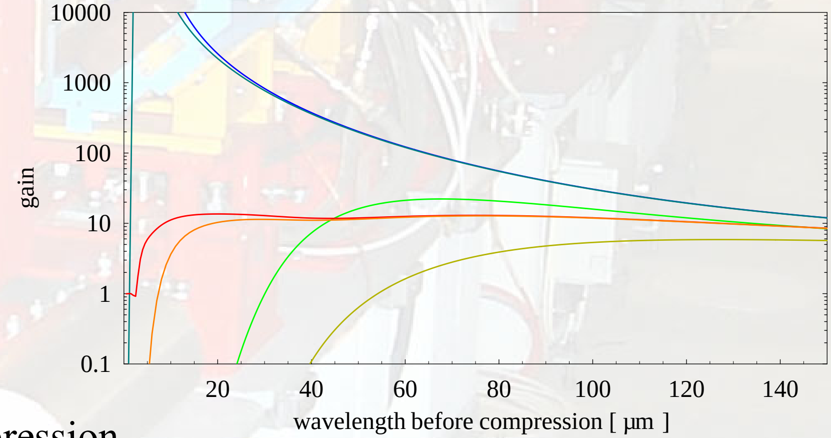
S-chicane, no compression

Modulation Gain due to CSR



C-chicane, with compression

compression factor 5	
R56	0.05m
ini. peak current	500A
unc. energy spread	0 / 1e-5 / 1e-4
emittance	0 / 1e-6

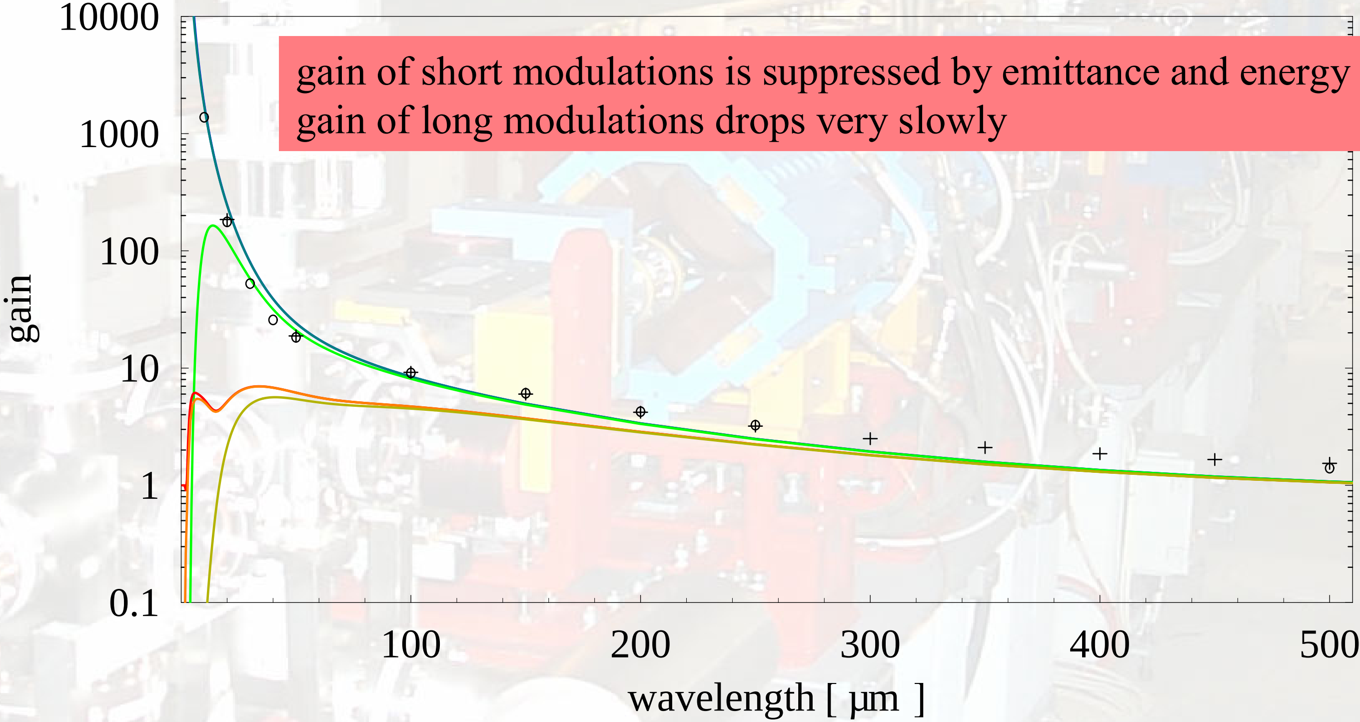


S-chicane, with compression

C-chicane, no compression
no emittance, no energy spr.
peak current 2500A

o gain of 10 periods on flat top
\+ peak gain of modulated 250µm Gauss

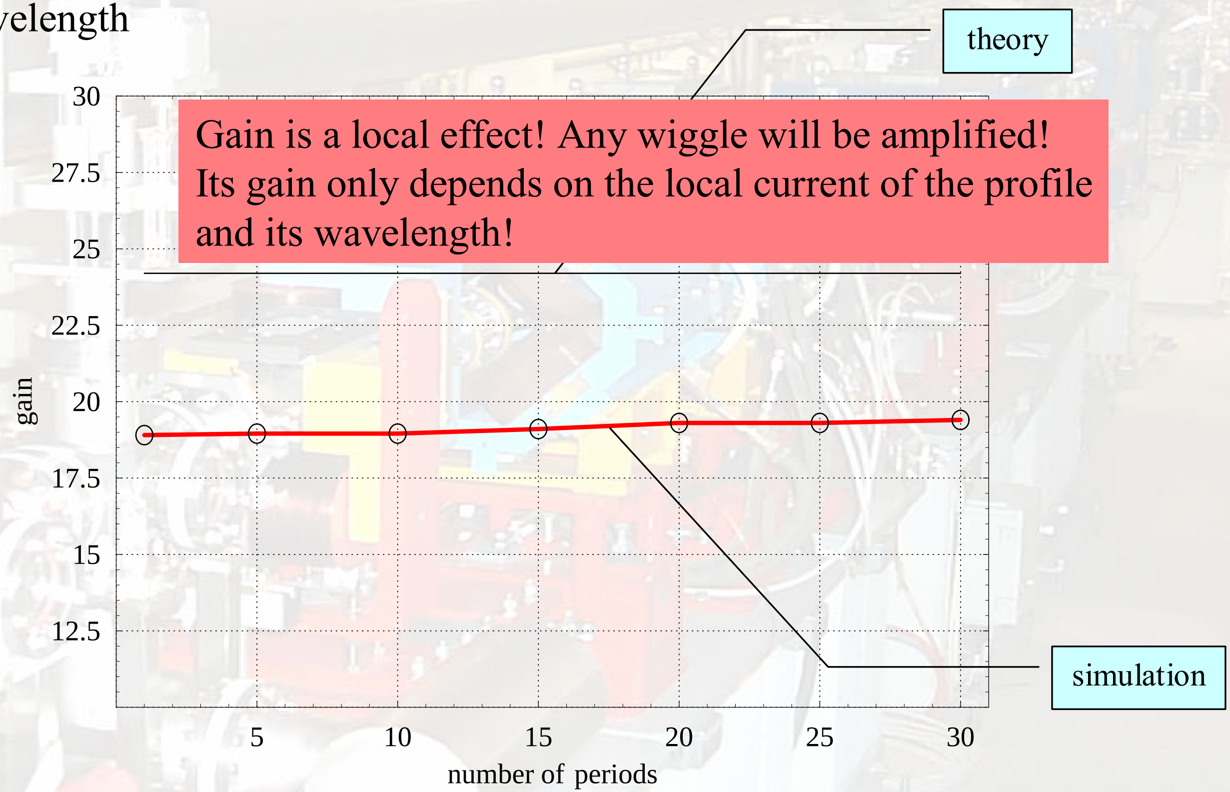
gain of short modulations is suppressed by emittance and energy spread
gain of long modulations drops very slowly



Modulation Gain due to CSR

C-chicane, no compression
no emittance, no energy spr.
peak current 2500A
50µm wavelength

Gain vs. number of modulation periods on flat top



Gain is a local effect! Any wiggle will be amplified!
Its gain only depends on the local current of the profile and its wavelength!

Conclusion

- when electron bunches pass magnetic chicanes strong CSR affects the particle dynamics and emittance is diluted
- the emittance behind an S-chicane is smaller then behind a C-chicane
- a symmetric 6-bend S-chicane is a good choice for BC3
- fast changes in charge density strongly disturb the bunch profile
- CSR leads to an amplification of energy and density modulations and dilutes emittance
- the gain calculated from analytical formulas and simulation data matches very good
- the gain in an S-chicane is a little bit (2-3 times) higher than in a C-chicane
- for short wavelength modulations the gain is suppressed by uncorrelated energy spread and emittance
- towards long wavelengths the gain drops very slowly
- gain is independent of the number of periods
- for realistic beam parameters the gain in BC3 will be small (<10) but not negligible