Characterization of radiating apertures using Multiple Multipole Method

And

Modeling and Optimization of a Spiral Antenna for Ground Penetrating Radar Applications

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Part I

Outline

1. Introduction
2. Multiple Multipole Method to characterize radiating apertures
3. Problem statement
4. Solution approaches:
    - SVD-Angle approach
    - Rank Revealing QR approach
5. Simulation results
6. Conclusions

Motivation

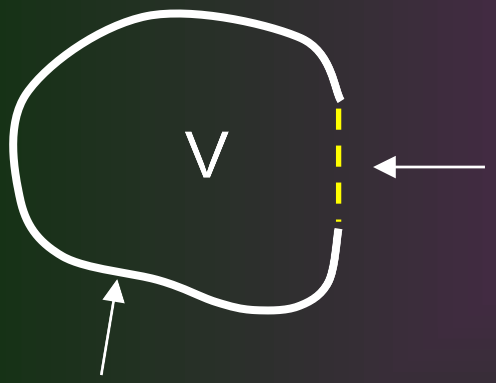
- Advantages of Multiple Multipole Technique:
  - Highly flexible.
  - Does not require time consuming convergence tests, provides possibility for residual error estimation.
  - The radiation conditions are automatically satisfied (no approximation like infinite flange, absorbing boundaries)
  - Accurate estimation of the back-scattered field and near field.

- Crucial point
  - Using appropriate number of multipoles at appropriate positions.

1. Introduction

The multiple multipole method is a semi-analytic method for electromagnetic field computation.

It is a frequency domain technique.



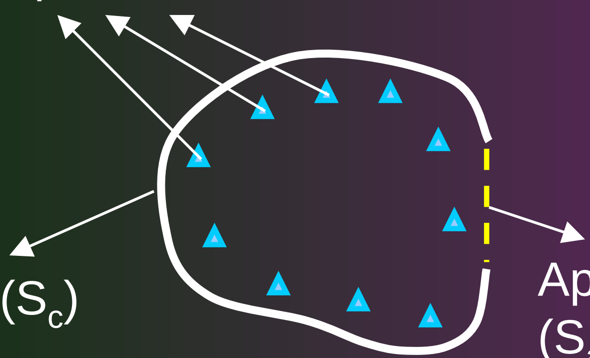
The space outside the radiating object is source free.

The field on the aperture is expressed in terms of aperture waveguide modes.

Our aim is to compute the Y matrix description for the aperture.

2. MM Method to Characterize Radiating Apertures

TE (TM) multipoles => oscillating magnetic (electric) surface charge distributions residing on the surface of an infinitesimal sphere emitting electromagnetic field as spherical wave.



The electromagnetic fields radiated by the multipoles located within the volume V are used to represent the field outside V

Sum on multipole locations — Sum on multipole orders — Unknowns — elementary multipole field

External field

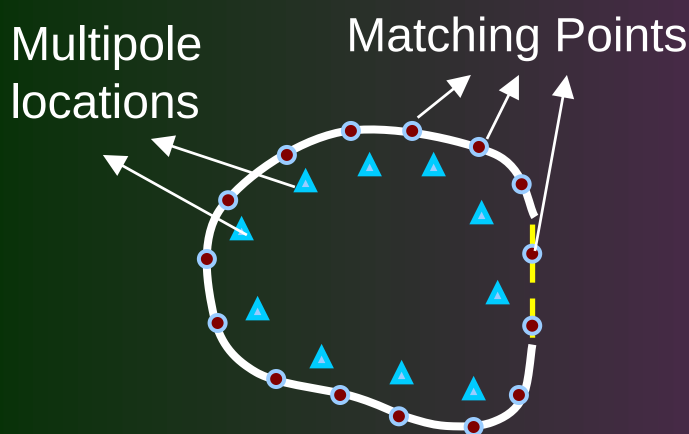
$$\mathbf{E}^{(ext)}(\mathbf{r}) = \sum_k \sum_l Q_{kl}\, \mathbf{E}_l^{(mult)}(\mathbf{r}-\mathbf{r}_k)$$

$$\mathbf{H}^{(ext)}(\mathbf{r}) = \sum_k \sum_l Q_{kl}\, \mathbf{H}_l^{(mult)}(\mathbf{r}-\mathbf{r}_k)$$

multipole locations

2. MM Method to Characterize Radiating Apertures (Contd.)

The unknown coefficients (Q_{kl}) should be chosen so that the total field match the boundary conditions on S_c and S_a



The field on some matching points $\mathbf{r}_m$ (on the surface of the volume) are enforced in order to find out the expansion coefficients Q_{kl}

Enforcing tangential E-field on (S_c+S_a)

$$\mathbf{E}(\mathbf{r}_m) = \sum_k \sum_l Q_{kl}\, \mathbf{E}_l^{(mult)}(\mathbf{r}_m - \mathbf{r}_k) \quad \longrightarrow \quad \mathbf{v} = [C]\,\mathbf{Q}$$

Enforcing tangential H-field on (S_a)

$$\mathbf{H}(\mathbf{r}_m) = \sum_k \sum_l Q_{kl}\, \mathbf{H}_l^{(mult)}(\mathbf{r}_m - \mathbf{r}_k) \quad \longrightarrow \quad \mathbf{i} = [\tilde{C}]\,\mathbf{Q}$$

Vector containing E and H fields at matching points

Vector containing the unknowns

Matrices containing the multipole field components at the matching points

2. MM Method to Characterize Radiating Apertures (Contd.)

Modal expansion of the aperture field on the field matching points (r_m) gives

$$\mathbf{E}(\mathbf{r_m}) = \sum_{\mathbf{n}} V_n \mathbf{e}_n(\mathbf{r_m})$$

$$\mathbf{H}(\mathbf{r_m}) = \sum_{\mathbf{n}} I_n \mathbf{h}_n(\mathbf{r_m})$$

Matrix form

$$\mathbf{v} = [\mathrm{D}] \cdot \mathbf{V}$$

$$\mathbf{i} = [\tilde{\mathrm{D}}] \cdot \mathbf{I}$$

Modal voltages and currents

Matrices containing field components of aperture waveguide modes

Equating these fields to the multipole fields

$$[C]\, \mathbf{Q} = [\mathrm{D}] \cdot \mathbf{V}$$

$$[\tilde{C}]\, \mathbf{Q} = [\tilde{\mathrm{D}}] \cdot \mathbf{I}$$

$$\mathbf{Q} = \left([C]^{\mathrm{H}} \cdot [C]\right)^{-1} \cdot [C]^{\mathrm{H}} \cdot [\mathrm{D}] \cdot \mathbf{V}$$

Inversion

$[C]$ must be well conditioned

3. Problem Statement

We need a dense distribution of multipoles, at the same time having a well conditioned *C* matrix.

The redundant multipoles should be removed.

$$[C] = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ c_{31} & c_{32} & & c_{3n} \\ \vdots & & & \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix}$$

Field component of all multipoles at a particular point

Field component of a particular multipole on all matching points

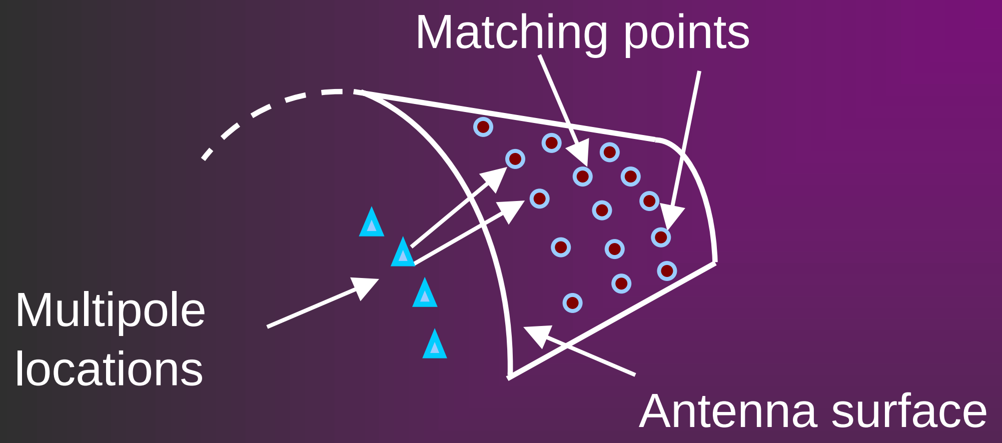
n = total number of multipoles

m = 3 x number of matching points

m>n => Over-determined system

3. Problem Statement (contd.)

Effect of redundant multipoles:



The field produced by distant multipoles at the adjacent matching points are almost identical.

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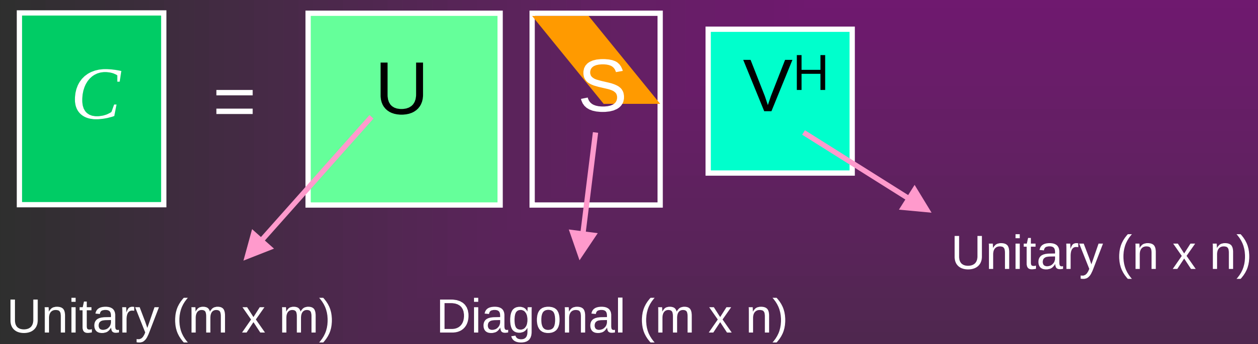
linearly dependent or nearly dependent columns leading to ill conditioned or rank deficit *C*-matrix.

4. Solution approaches

SVD-Angle approach:

a. Estimation of the linear dependency among the columns of C matrix using the Singular Value Decomposition (SVD).



Zero or very small singular values depict rank deficiency or ill conditioning i.e. Redundant multipoles.

Number of Singular values which are less than a threshold t_{svd} is equal to the number of redundant columns (multipoles) (with respect to the corresponding threshold t_{svd}).

4. Solution approaches (contd.)

b. Identification of the redundant multipoles:

The angle matrix A is calculated as,

$$A_{ij} = \cos^{-1}\left(\frac{\left|C_j^{*} \cdot C_i\right|}{\left\|C_i\right\| \cdot \left\|C_j\right\|}\right)$$

C_i = i-th column of C matrix
i,j = 1,2 .... n

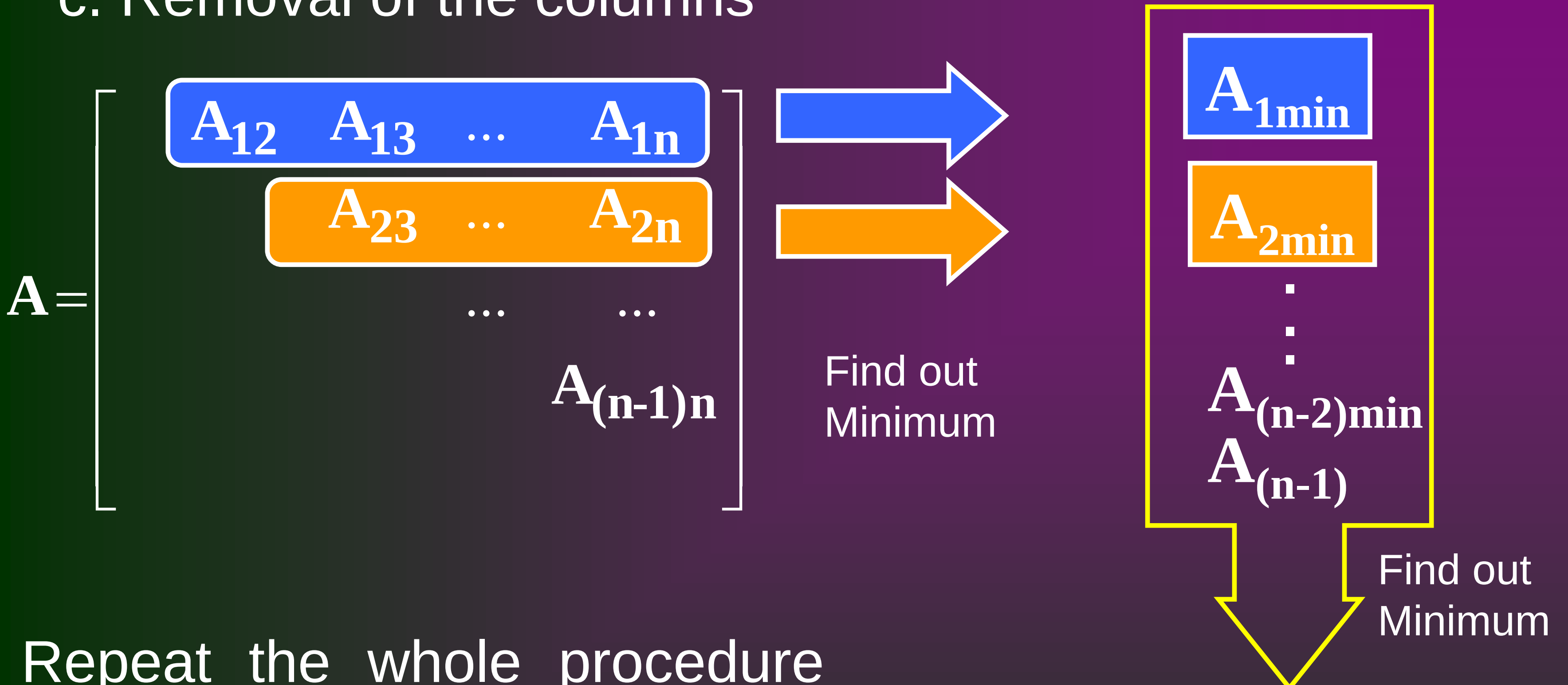
The elements of matrix A provide a measure of the linear dependency among the columns of C, hence the redundancy among the multipoles.

A_{ij} is upper triangular and the diagonal elements are zero.

The redundant multipoles are identified according to the values of A.

4. Solution approaches (contd.)

c. Removal of the columns



Repeat the whole procedure till the number of deleted columns equals the number of linearly dependent columns detected previously by SVD.

Note the corresponding column numbers and remove the column from C which has smaller normamong them.

4. Solution approaches (contd.)

Rank Revealing QR factorization approach:

The matrix C can be decomposed in a rank revealing QR factorization as:

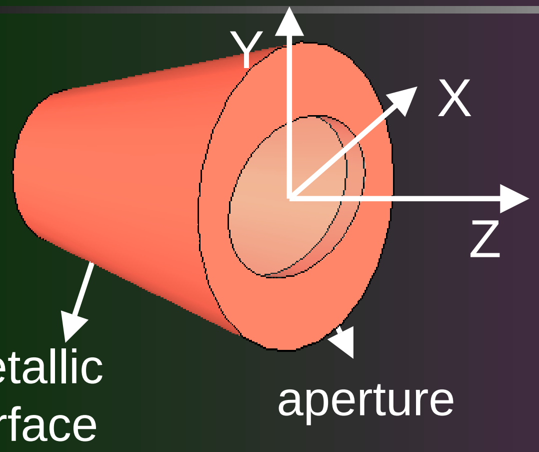
$$CP = QR = Q\begin{pmatrix} R_{11} & R_{12} \\ 0 & R_{22} \end{pmatrix}$$

- P: Permutation matrix
- Q: Matrix with orthogonal columns
- R: Upper triangular matrix
- R_{11}: Well conditioned triangular submatrix
- R_{22}: Triangular matrix having very small norm

The threshold t_{rrqr} is used as the upper bound of the condition number of R_{11}.

From the permutation matrix P, we can get the columns treated as linearly dependent during the decomposition.

5. Simulation results

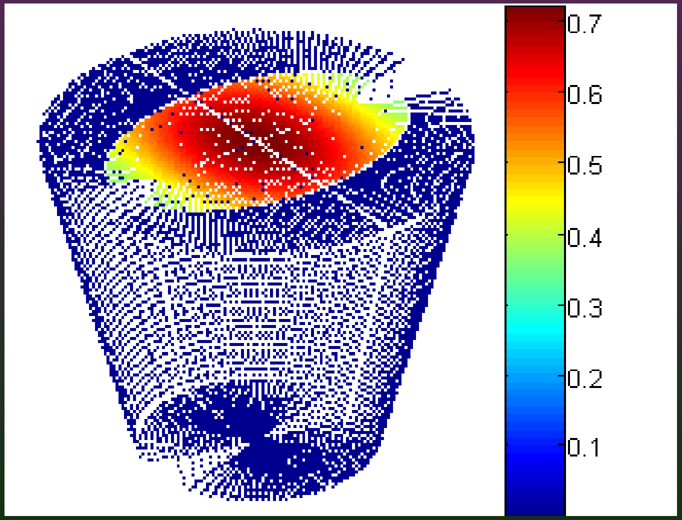


Illumination with the fundamental mode only

EM symmetry (x-axis electric wall, y axis magnetic wall)

Schematic diagram of the simulated structure

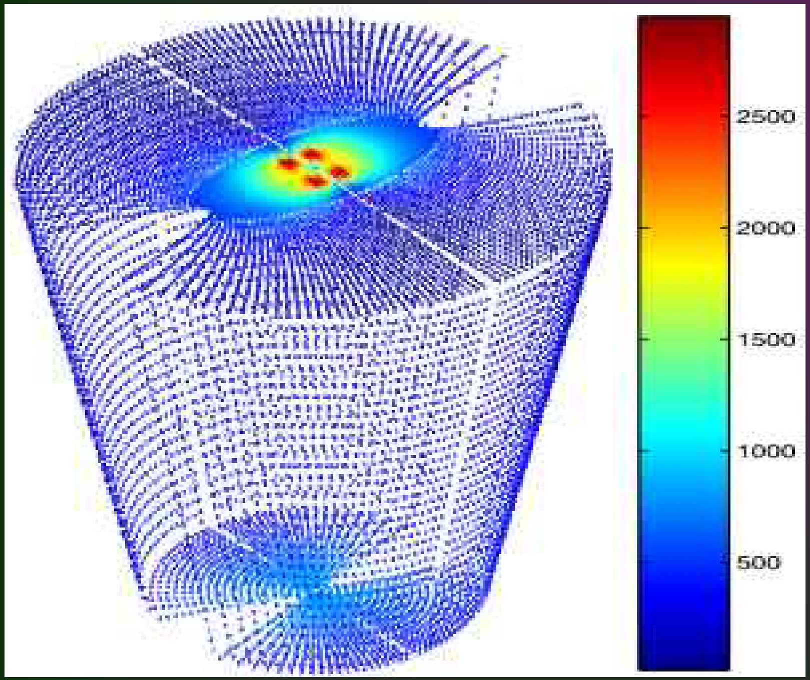
The boundary condition of the problem (electric field amplitude)



Color bar represents the electric field strength

5. Simulation results (contd.)

Aperture matching points = 400, Matching points on metallic surface = 1205, Multipole locations = 199, Total number of multipoles = 1194



Calculated E-field amplitude without multipole reduction

The *C* matrix is severely ill-conditioned

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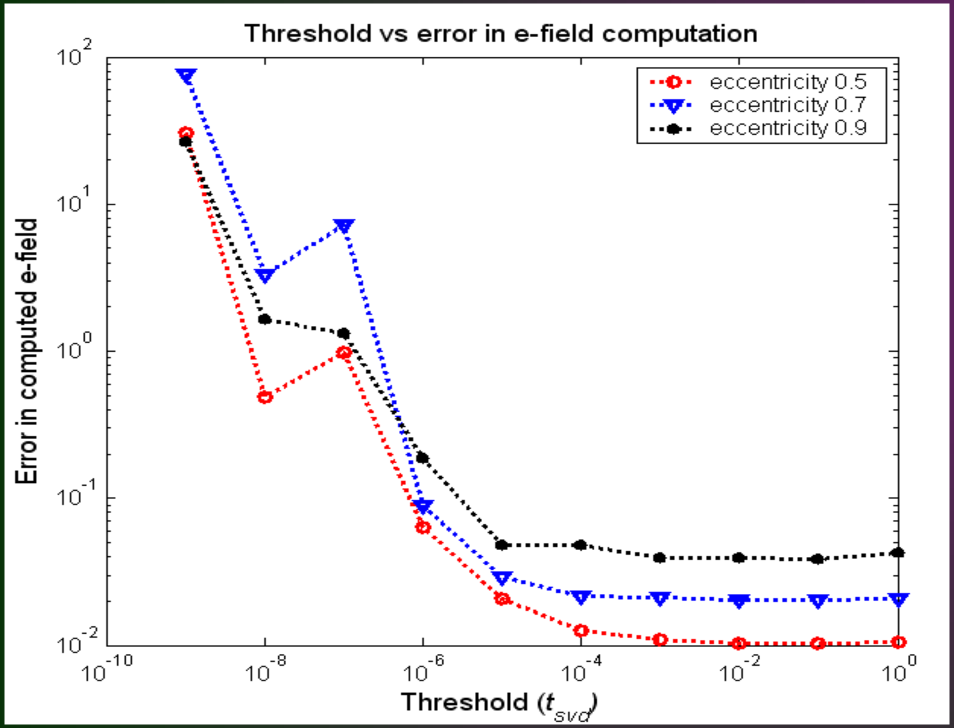
Completely wrong field calculation

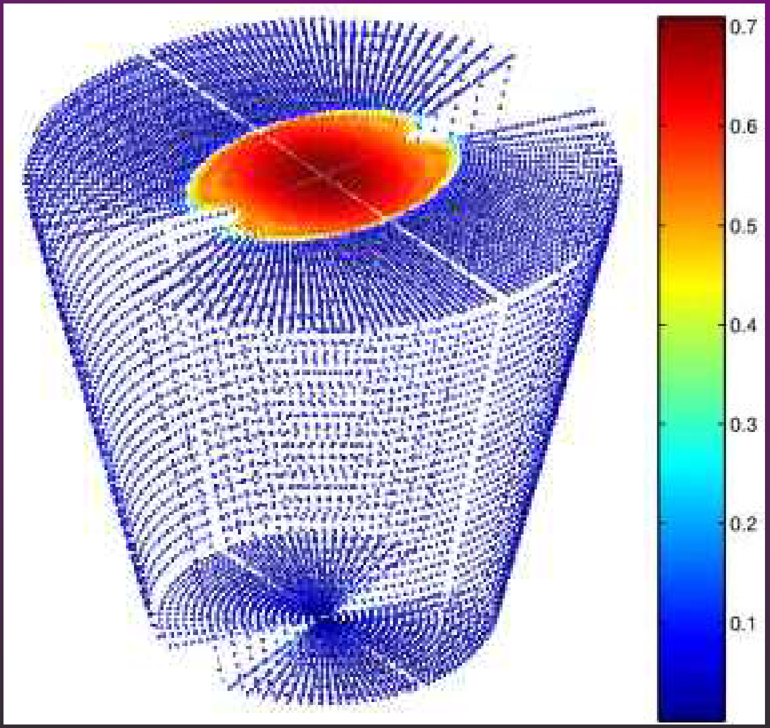
Average normalized error at the aperture ~ 0.99 (out of 1.0)!!

5. Simulation results (contd.)

SVD-Angle approach:



Error in the electric field calculation for different thresholds (t_{svd})



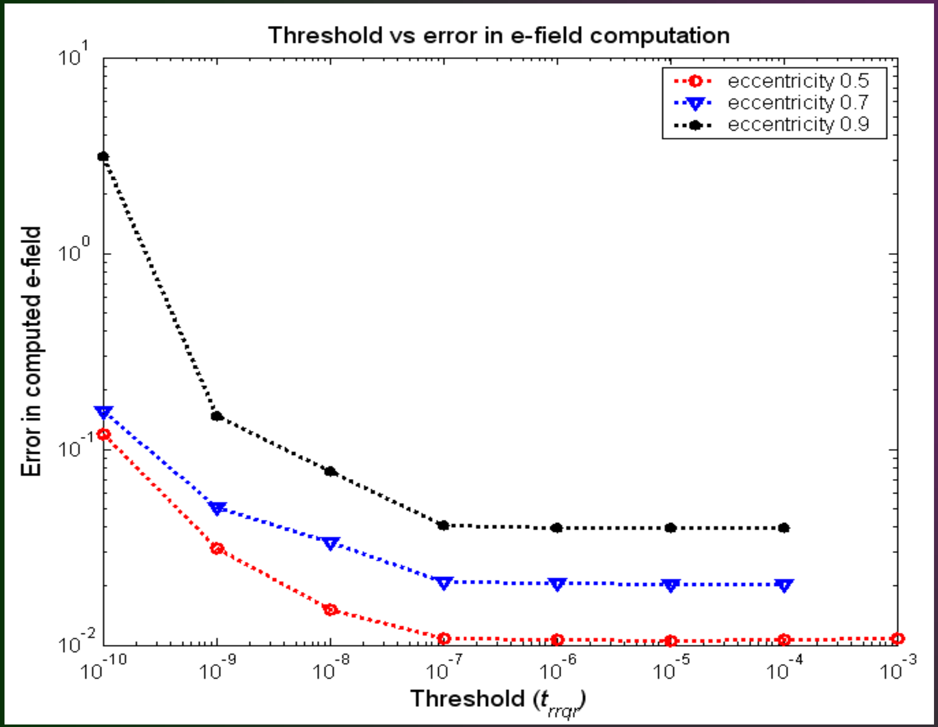
Calculated E-field amplitude after multipole reduction with threshold $t_{svd} = 10^{-3}$

No of deleted multipoles 53

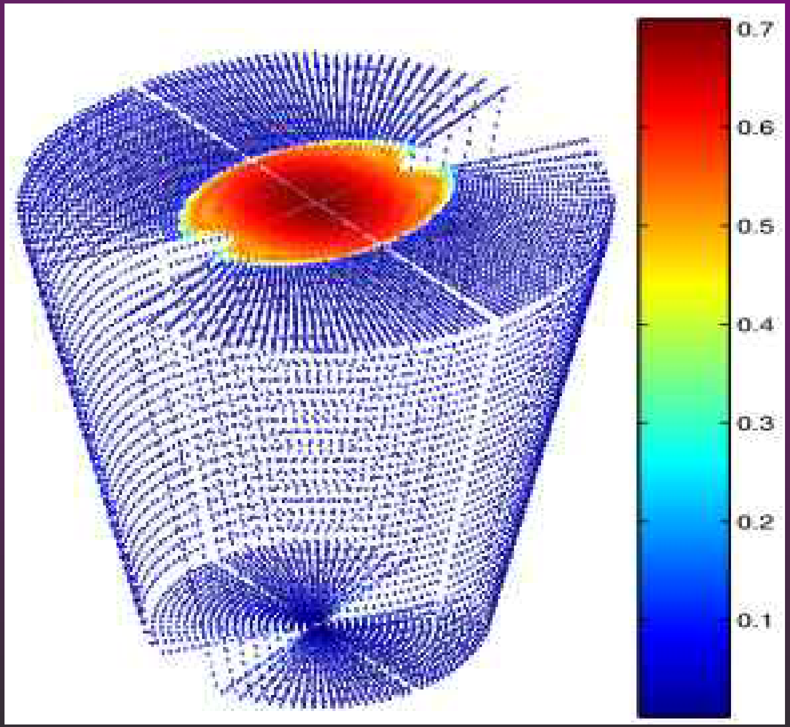
The LAPACK routines have been used for the SVD

5. Simulation results (contd.)

RRQR approach:



Error in the electric field calculation for different thresholds (t_{rrqr})

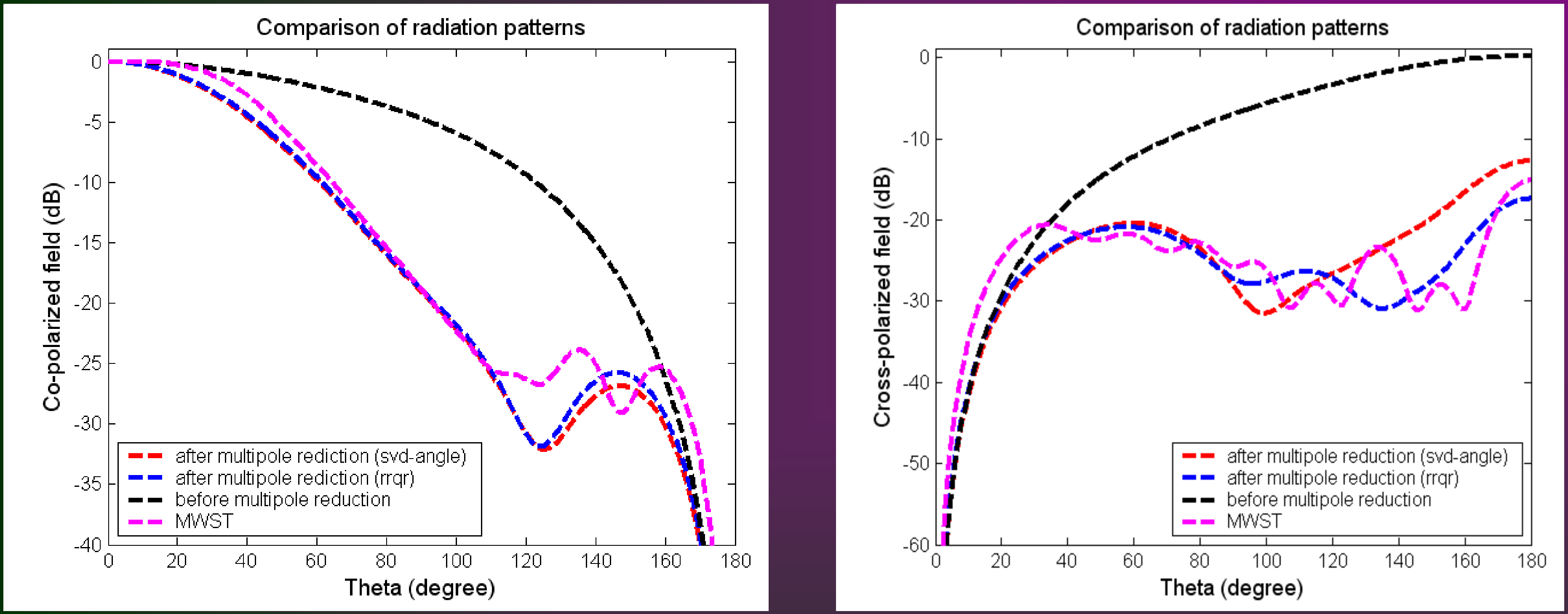


Calculated E-field amplitude after multipole reduction with threshold $t_{rrqr} = 10^{-6}$

No of deleted multipoles 51

The ZGEQPX routines have been used for the RRQR decomposition

5. Simulation results (contd.)



Comparison of the radiation patterns for the elliptical aperture before and after multipole reduction (at 4.5 GHz)

6. Conclusions

- The multiple multipole method has been applied to characterize radiating apertures.
- Two methods, one involving Singular value decomposition and the other one involving the rank revealing QR factorization have been applied to identify the redundant multipoles in Multiple Multipole Method.
- Applying these methods to characterize a simple radiating aperture, a substantial improvement has been achieved for the electromagnetic field calculation.
- These methods are general and can be used for automatic multipole setting for any structure.