# Coherent beam-beam effects and Landau damping

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# LHC main parameters

- LHC is a Large Hadron Collider
- Proton per bunch $\approx 1.1 \times 10^{11}$
- Energy = 7000 GeV
- Number of bunches per beam = 2808
- Bunch length $\sigma_s$ = 7.5 cm
- Tunes = 64.31/59.32
- Nominal $Q_s$ = 0.00212
- Nominal luminosity $\mathcal{L} = 1.0 \times 10^{34}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$
- No radiation damping

# LHC performance limits

- LHC intensity (and luminosity) is limited by beam-beam effects:
  - Head-on and long range interactions
  - PACMAN effects
  - Coherent beam-beam effects

# Why coherent effects?

- LHC is a strong-strong machine. i.e.:

- Approximately equal beams
- Strong beams, i.e. beams with high intensity
- Strong-strong interaction, i.e. the two beams colliding influence and change mutually due to their interaction and the force that they exercise on each other

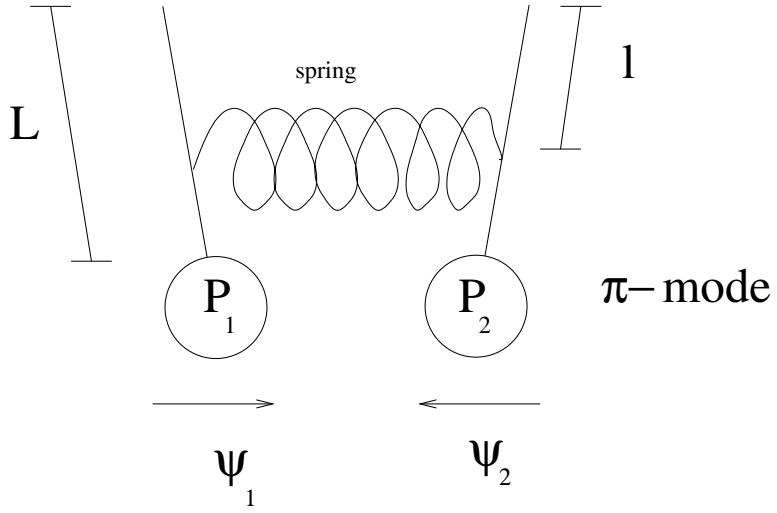
# Examples of other hadron colliders

- SPS was a proton-antiproton machine (weak-strong)
- Tevatron is a proton-antiproton machine (weak-strong)
- RHIC is a proton-proton machine and there the $\pi$ - mode has been already observed (strong-strong)

# We look at coherent dipole modes

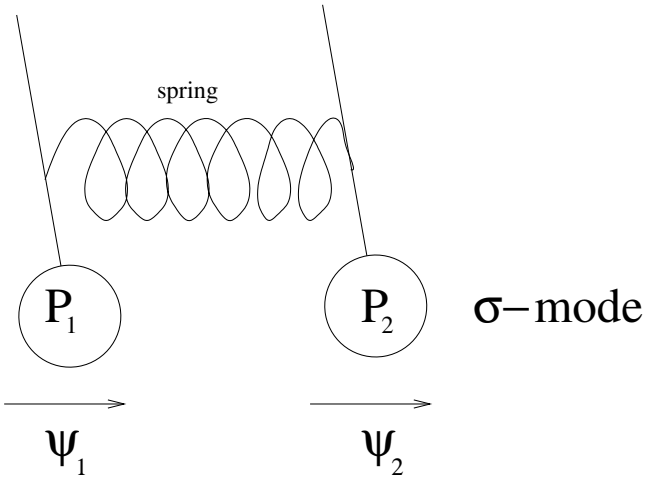
- Two principal coherent modes appear:
- $\pi$ - mode, where the centroids oscillate with equal frequencies but in opposite phase
- $\sigma$ - mode, where the centroids of the bunches oscillate with equal frequencies in phase

# Coherent modes, mechanical model




$\pi-$mode

$$\omega = \omega_0 + k$$



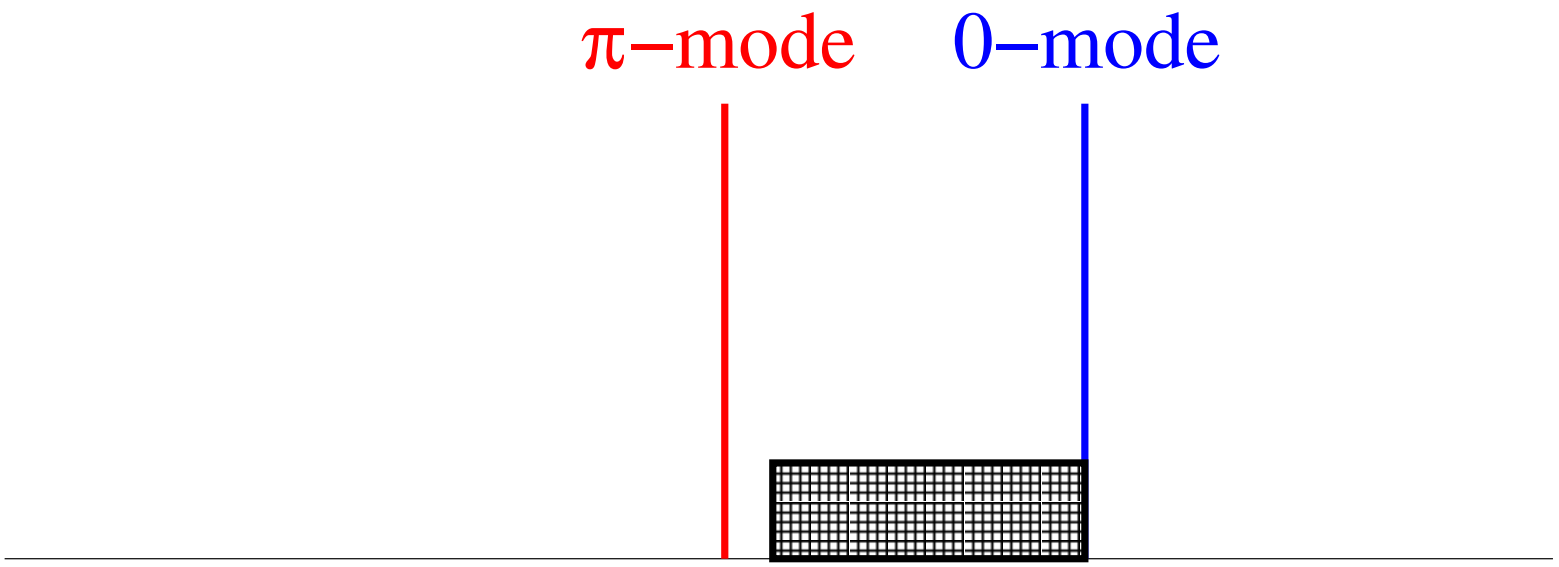

$\sigma-$mode

$$\omega = \omega_0$$

# Yuri Alexahin's prediction

- Incoherent spectrum from 0 to - $\xi$
- $\pi$ - mode outside incoherent spectrum
- $\Rightarrow$ No Landau damping
- This is true for intensity ratio > 0.6 (i.e. only for strong-strong)
- We want to study by a simulation

# Coherent modes and incoherent spectrum

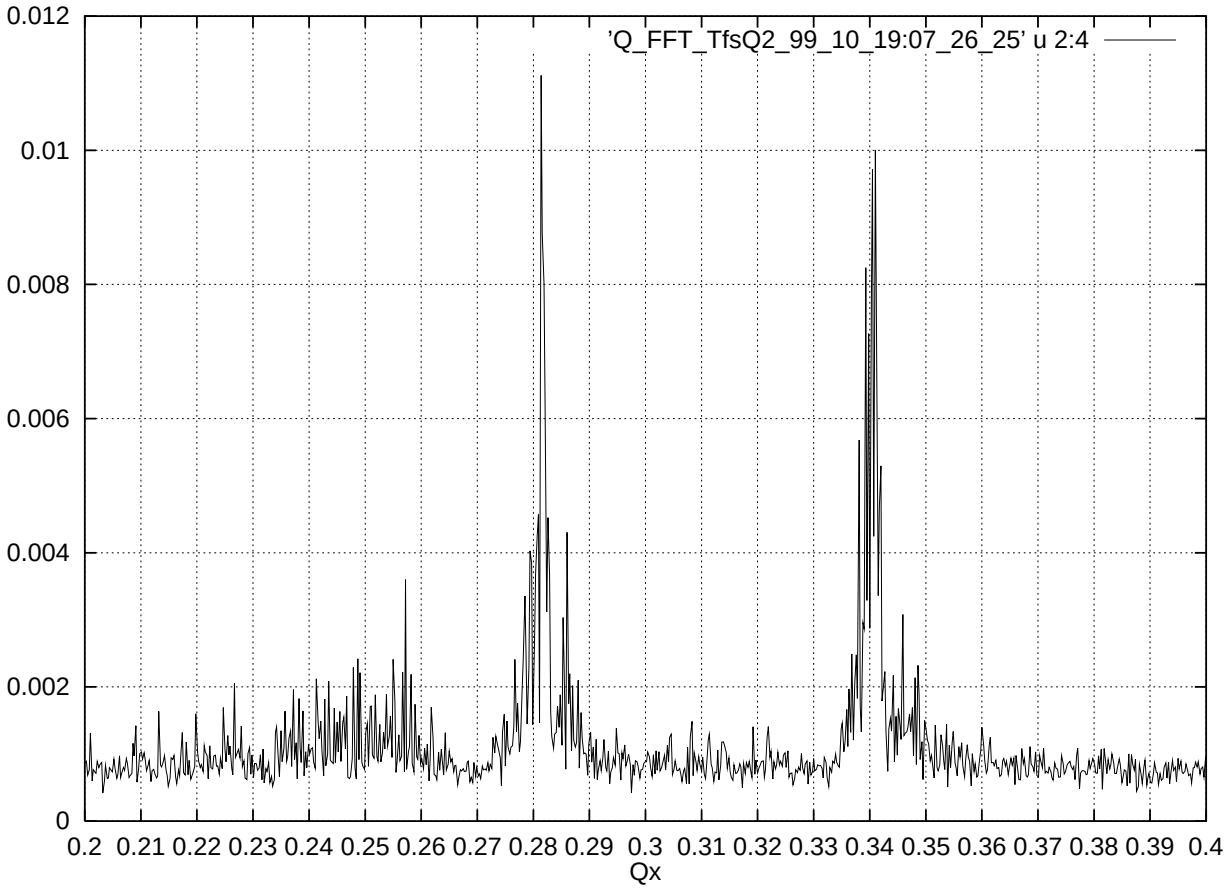



As shown in this schematic picture, the π - mode is outside the incoherent spectrum, and hence is not damped.

# Coherent effects = collective behaviour

- To have coherent effects the particles of each beam have to behave very similarly
- The system requires a very high degree of symmetry

# LEP measurements



We want to:

- Simulate the motion and compare to what the theory says
- Demonstrate that modes found at RHIC can appear also in the LHC
- Study the implications and possible tools in order to improve the running of the machine

# Simulation program (1)

- Our program should reproduce the beam dynamics
- Strong-strong simulation
- $10^4$ macro-particles per bunch, one bunch per beam
- one collision point with crossing angle (no long range beam-beam effects)
- Soft Gaussian approximation
- At every turn calculate c.m.s. of bunch 1 and 2 $(x_1,x_2)$ and emittance $\left(\varepsilon_{x,y}^{1,2}\right)$ and store them

# Simulation program (2)

- Look at dipole modes $\Rightarrow$

  Analyse the motion of the bunch centroids:

  $x_1$, $x_2$, $x_1 + x_2$ (suppress $\pi$ - mode),

  $x_1 - x_2$ (suppress $\sigma$- mode)
- Track up to $2^{18}$ turns (approximately 20 sec of LHC)
- No radiation damping and excitation
- No other non linearities

# Variables used

- $x$, $y$ are the transverse variables
- $x'$, $y'$ are the momenta for the $x$ and $y$ directions respectively
- They are normalized:
  $x = X/\sigma_{0X} \quad x' = \beta X'/\sigma_{0X} \quad y = Y/\sigma_{0Y} \quad y' = \beta Y'/\sigma_{0Y}$
  where $\sigma_{0X} = \sigma_{0Y} = \sigma$ are the nominal horizontal and vertical r.m.s., and $\beta = \beta_x^* = \beta_y^* = \beta$-functions at the interaction point
- The initial distribution is Gaussian for everything

# One turn map (2D)

Between interaction points we use a linear map

$$\begin{pmatrix} z \\ z' \end{pmatrix}_{n+1} = \begin{pmatrix} \cos(\Delta\mu_z) & \sin(\Delta\mu_z) \\ -\sin(\Delta\mu_z) & \cos(\Delta\mu_z) \end{pmatrix} \begin{pmatrix} z \\ z' \end{pmatrix}_n \qquad (z = x, y)$$

$\Delta\mu_z = 2\pi Q_z$ = phase advance between interaction

$\beta^* = 1$ and $\alpha = 0$

# Models for calculation of fields

We use several models to calculate the fields

- Gaussian approximation
- Mesh codes
- HFMM (Hybrid Fast Multipole Method)

# Beam-beam kick

- The beam-beam kick is (for Gaussian profiles):

$$\Delta y' = -\frac{2N_b^* r_p y}{\gamma}\int_0^\infty \frac{\exp\left\{-\frac{x^2}{2\sigma_x^{*2}+q}-\frac{y^2}{2\sigma_y^{*2}+q}\right\}}{(2\sigma_y^{*2}+q)^{3/2}(2\sigma_x^{*2}+q)^{1/2}}dq$$

and:

$$\Delta x' = -\frac{2N_b^* r_p x}{\gamma}\int_0^\infty \frac{\exp\left\{-\frac{x^2}{2\sigma_x^{*2}+q}-\frac{y^2}{2\sigma_y^{*2}+q}\right\}}{(2\sigma_y^{*2}+q)^{1/2}(2\sigma_x^{*2}+q)^{3/2}}dq$$

with:

$$r_p = \frac{q^2}{m_p c^2}$$

# Evaluation of this formula

- The beams are round
- We can simplify it:

$$\Delta y' = \frac{2N_b^* r_p y}{\gamma} \cdot \frac{1}{x^2 + y^2} \cdot \left(1 - \exp\left(-\frac{x^2 + y^2}{2\sigma_y^{*2}}\right)\right)$$

$$\Delta x' = \frac{2N_b^* r_p x}{\gamma} \cdot \frac{1}{x^2 + y^2} \cdot \left(1 - \exp\left(-\frac{x^2 + y^2}{2\sigma_x^{*2}}\right)\right)$$

With:

$$\sigma_x^* = \sigma_y^*$$

# Beam parameters

- Beam size r.m.s. $(\sigma)$ , and c.m.s. $(< z >)$ evaluated every turn
- Kick calculated assuming a Gaussian profile (Gaussian approximation)
- If $\sigma_x^* \approx \sigma_y^*$ use simplified round beam formula

- If $\sigma_x^*$ and $\sigma_y^*$ are too different (10%) use Basetti-Erskine formula

# Yokoya factor (1)

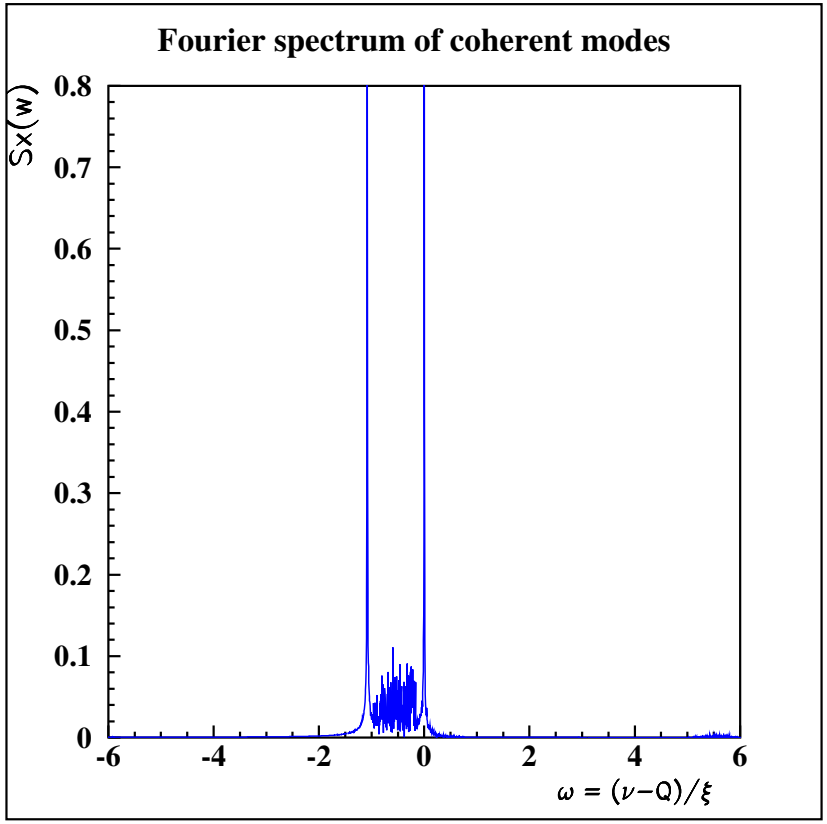
- When $\xi$ is sufficiently small, the incoherent tune shift of a particle with small betatron amplitude is given by $\xi$
- Single particles occupy the tune range from $[0 - 1] \cdot \xi$ (incoherent spectrum)
- The tune shift of the coherent oscillation ($\pi$ - mode) is given by $\Delta Q_\pi = Y \cdot \xi$, where $Y$ is the Yokoya factor and $\xi$ is the beam-beam parameter.

# Yokoya factor (2)

- In the perturbation theory (Yokoya, Alexahin) Y = 1.214 for round beams
- In a Gaussian approximation the Yokoya factor slightly too small: Y = 1.1
- In the HFMM model the Yokoya factor is:

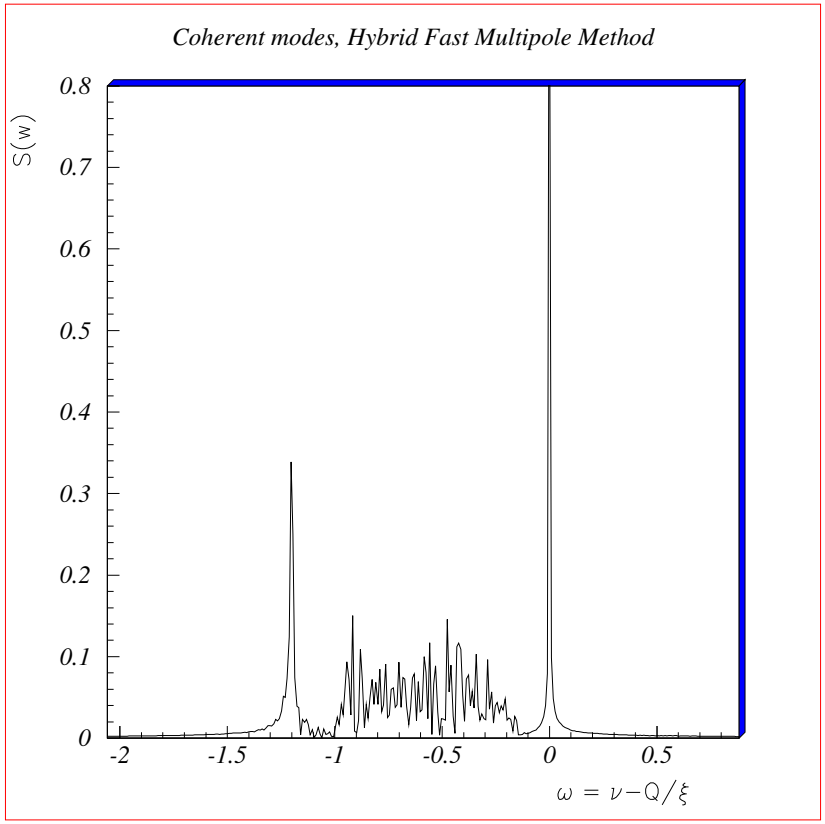
  Y = 1.214

# FFT Spectrum from simulation



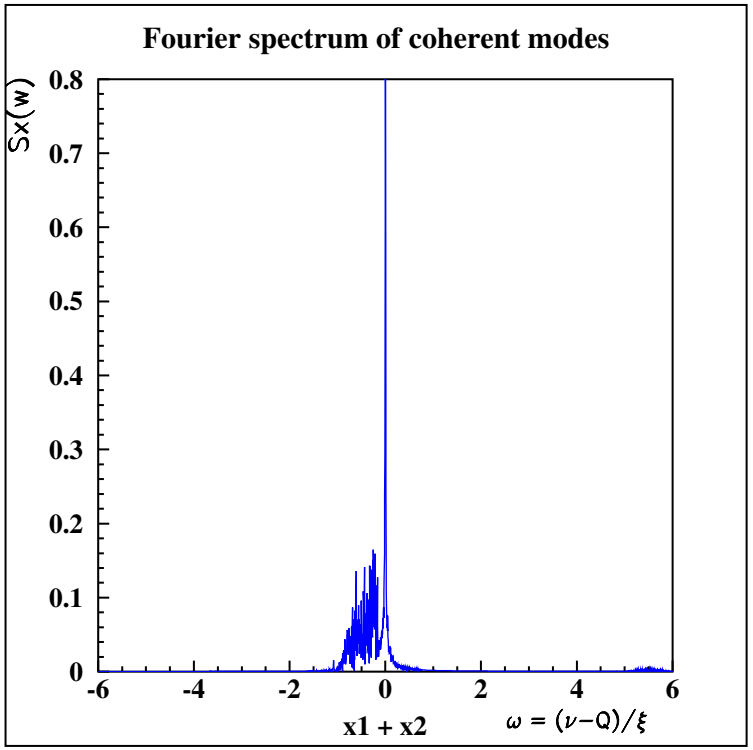

Gaussian approximation; $Y = 1.1$

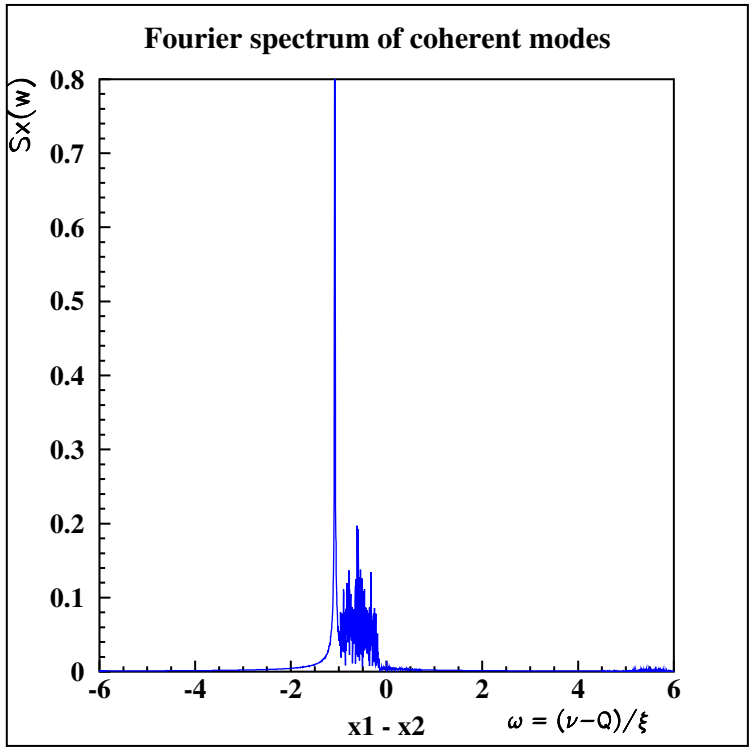
# Example from HFMM




$Y = 1.214$

# σ - mode and π - mode



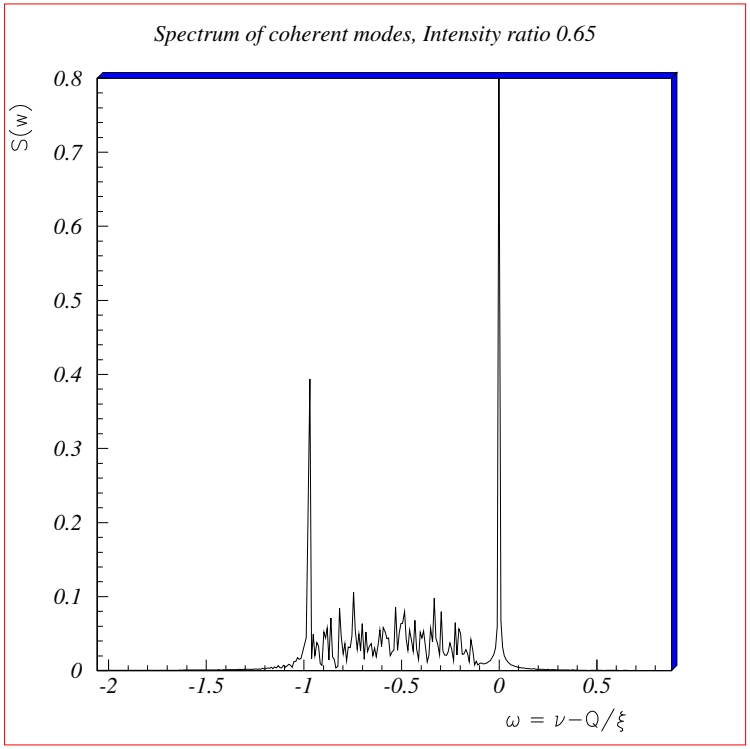

$x_1 + x_2$ *(left) and* $x_1 - x_2$ *(right)*

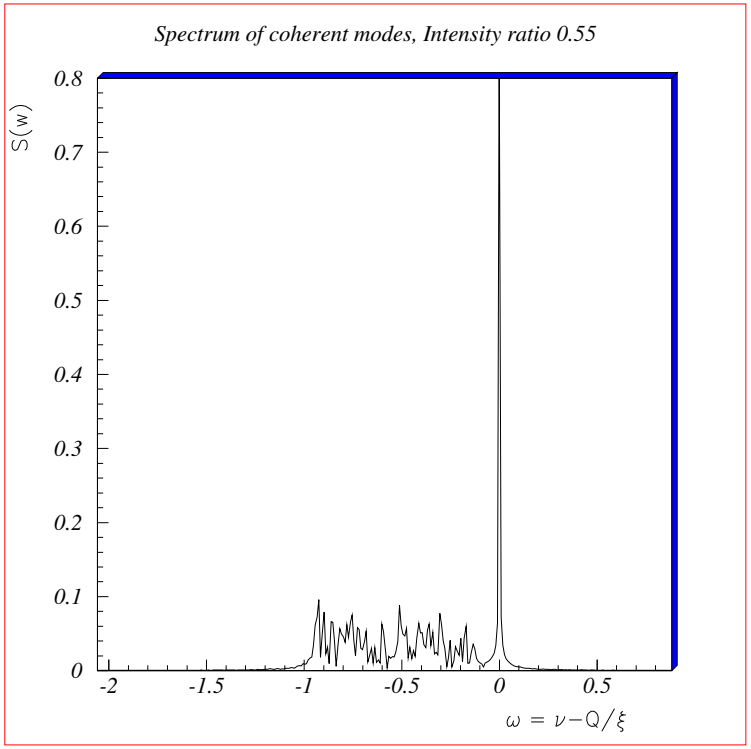
# Symmetry requirements

Since the coherent modes need a high degree of symmetry to sustain, we could break this symmetry to provoke the damping of the $\pi$ -mode:

- Use different intensities for the two beams
- Use different r.m.s. for the two beams
- Use different tunes

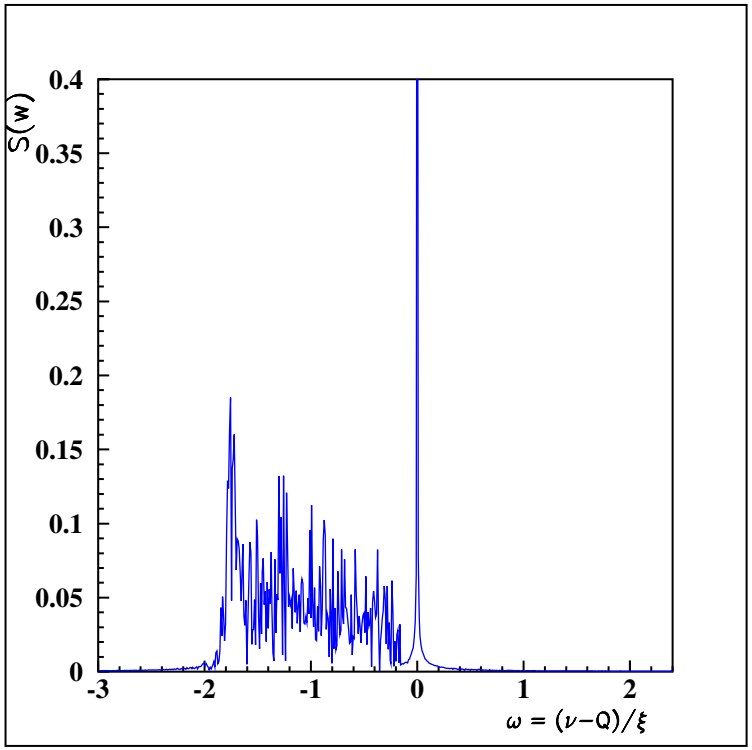
# Different intensity

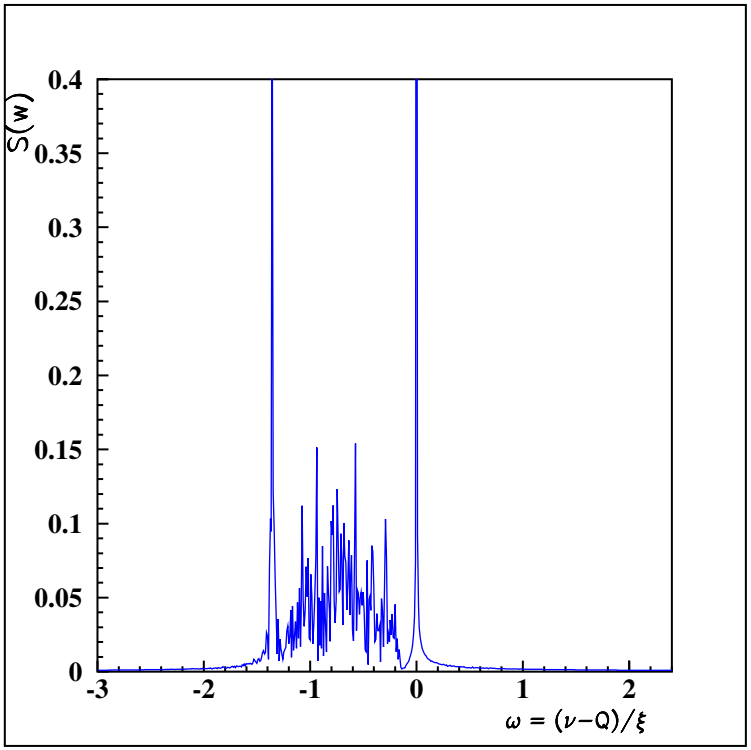



Intensity ratio of 0.65 (left) and 0.55 (right)

# Different beam size



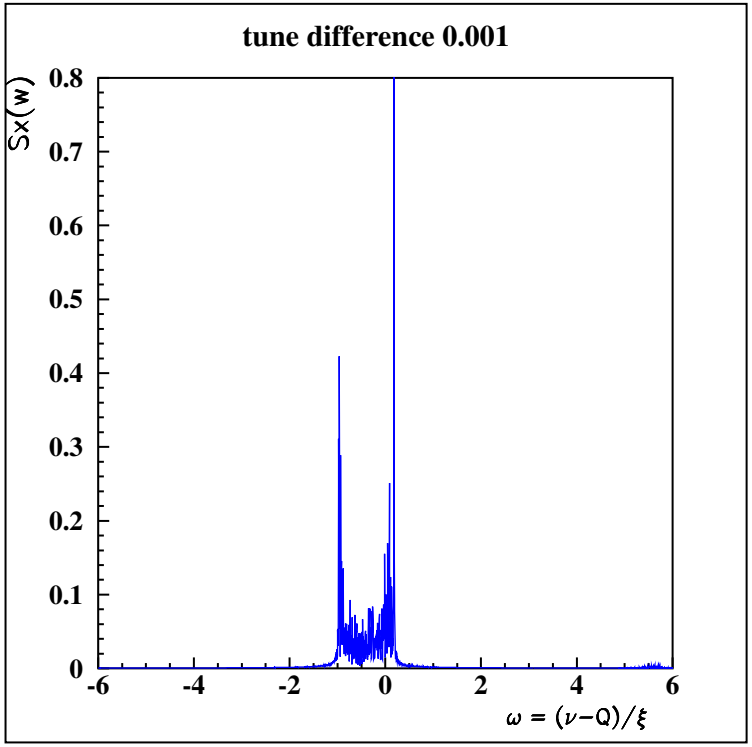



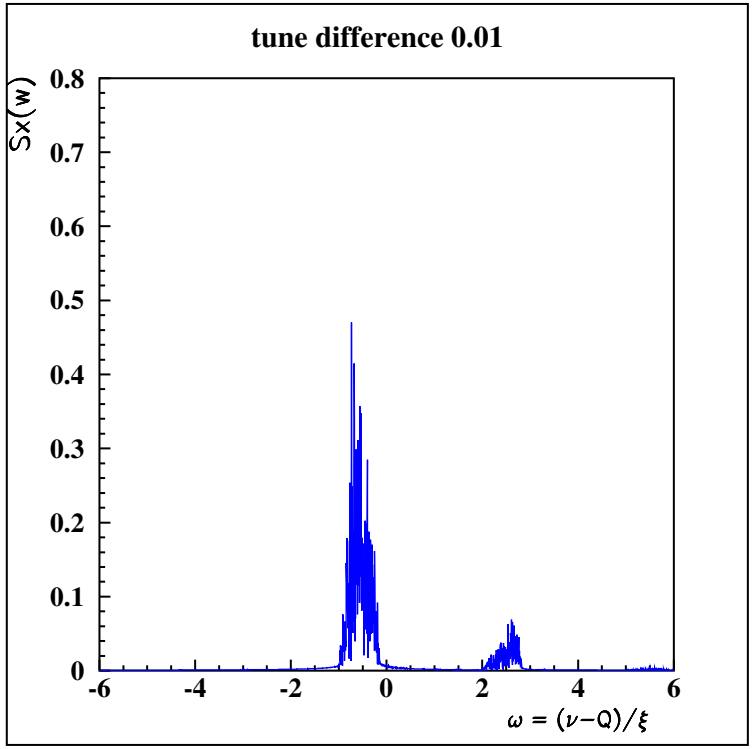

Beam size ratio of 0.7 (left) and 0.9 (right)

# Different tunes

- If the two tunes are different enough the beams are decoupled
- They oscillate on their own but with the other beam acting as a fixed focusing element $\Rightarrow$ smaller tune shift ($\approx \frac{\xi}{2}$)
- We have simulated the case of different tune

# Tune difference




$\Delta Q = 0.001 \approx 1/3\ \xi$

$\Delta Q = 0.01 \approx 3\ \xi$

- For a small difference of the tunes the modes are still present ($\Delta Q \approx 1\text{-}2 \cdot \xi$)
- That means that one can still maintain a coherent motion
- If the difference becomes too large the coherent motion and the modes disappear

# Up to now

- The motion has been studied in two dimensions
- To have coherent motion we need a very high degree of symmetry
- As predicted by the theory two coherent modes appear
- Between them there is a incoherent spectrum
- The $\pi$- mode is outside the incoherent spectrum, i.e. it can not be Landau damped
- Now we want to study the motion in 3D and the effect of the synchrotron motion to try and damp the $\pi$- mode

# Effect of synchrotron motion

- Introduces an additional degree of freedom
- Expect lower degree of coherence $\Rightarrow$ lower Y factor
- In case of coupling with transverse motion (e.g. crossing angle):
  - synchro-betatron side-bands
  - “another” incoherent spectrum
  - can we have Landau damping by overlap with side-bands?
- Study with full simulation

# One turn map (3D)

Between interaction points we use a linear map

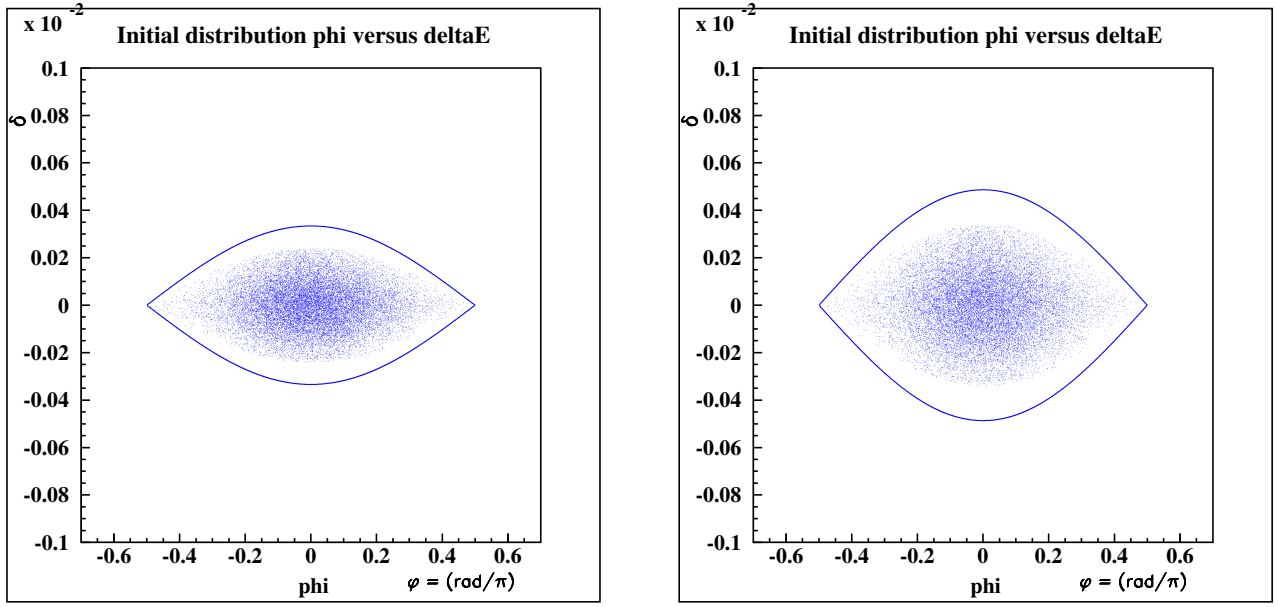
$$\begin{pmatrix} z \\ z' \end{pmatrix}_{n+1} = \begin{pmatrix} \cos(\Delta\mu_z) & \sin(\Delta\mu_z) \\ -\sin(\Delta\mu_z) & \cos(\Delta\mu_z) \end{pmatrix} \begin{pmatrix} z \\ z' \end{pmatrix}_n \qquad (z = x, y)$$

The longitudinal variables are updates once per turn according to:

$$\begin{pmatrix} \delta_{n+1} \\ \phi_{n+1} \end{pmatrix} = \begin{pmatrix} \delta_n - \dfrac{eV}{E} \cdot \sin(\phi_n) \\ \phi_n + 2\pi h \alpha_m \cdot \delta_{n+1} \end{pmatrix}$$

$$s = \frac{\phi \cdot c}{h \cdot f_0 \cdot 2\pi} \qquad \text{(distance to bunch centre)}$$

# Initial distribution in longitudinal phase space

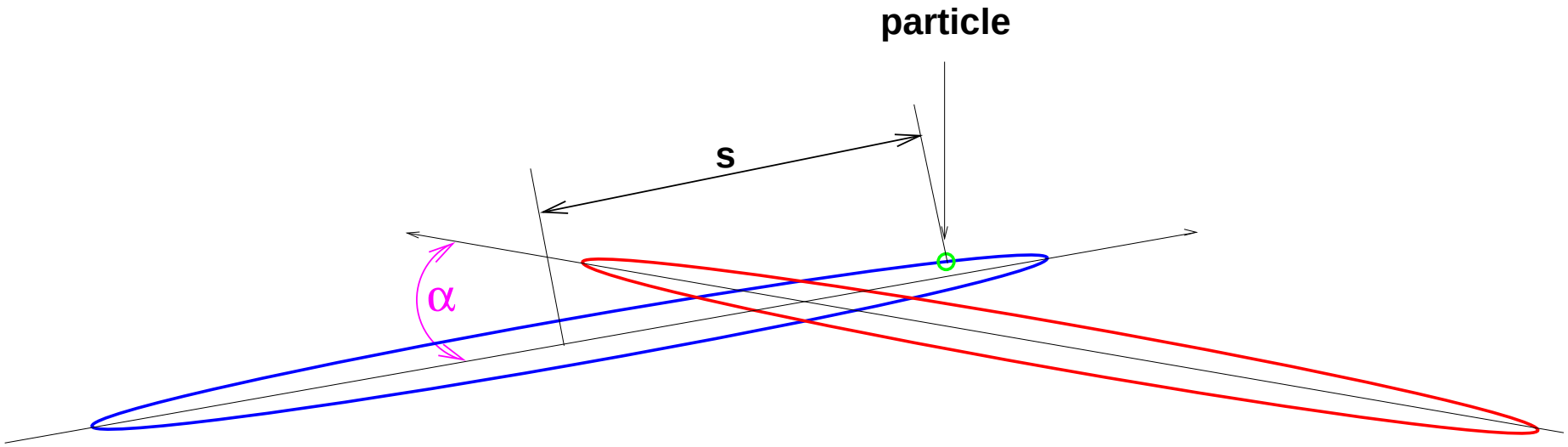



- Make sure particles are well inside the bucket.
- RF Voltage V = 16 and 32 MV
- Check during tracking if they fall outside
- Check that correct $Q_s$ is reproduced

# Stability tests of the program

- We have made several tests to verify the values for the parameters:
  - Number of particles
  - Number of turns
  - Initial offset
  - Different intensity

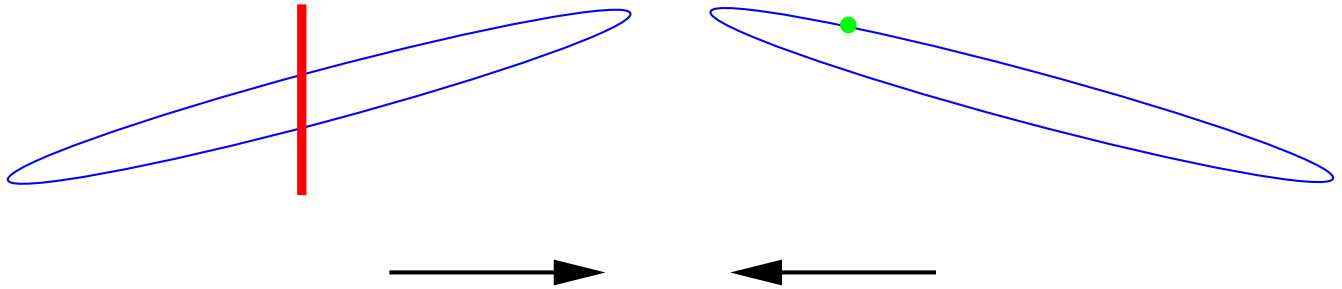
# Coupling with crossing angle



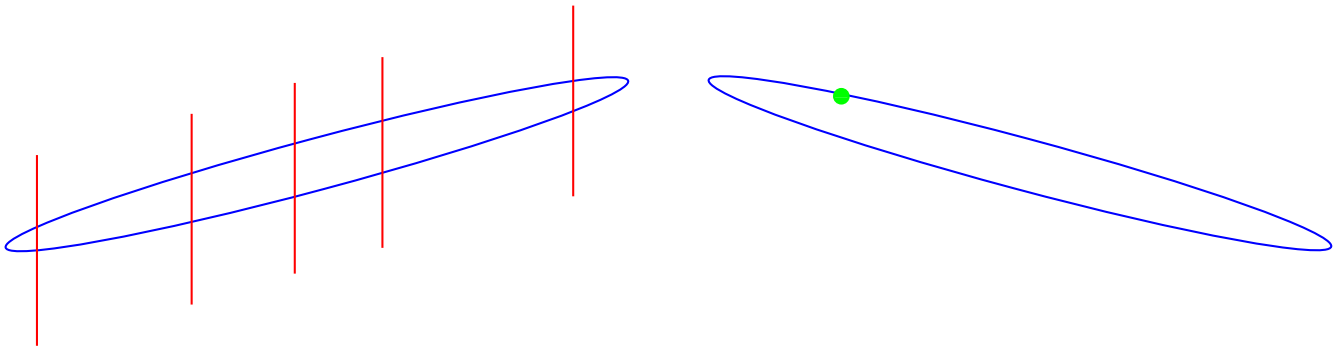

- Beam-beam kick depends on the longitudinal position
- Crossing angle couples longitudinal and transverse motion

# Options for simulation of crossing angle

A)



B)



- Slicing into small portions
- Simplified algorithm (adopted from Piwinski, DESY 77/18)
- Can be used when bunch length effects are not important (e.g. hourglass effect)

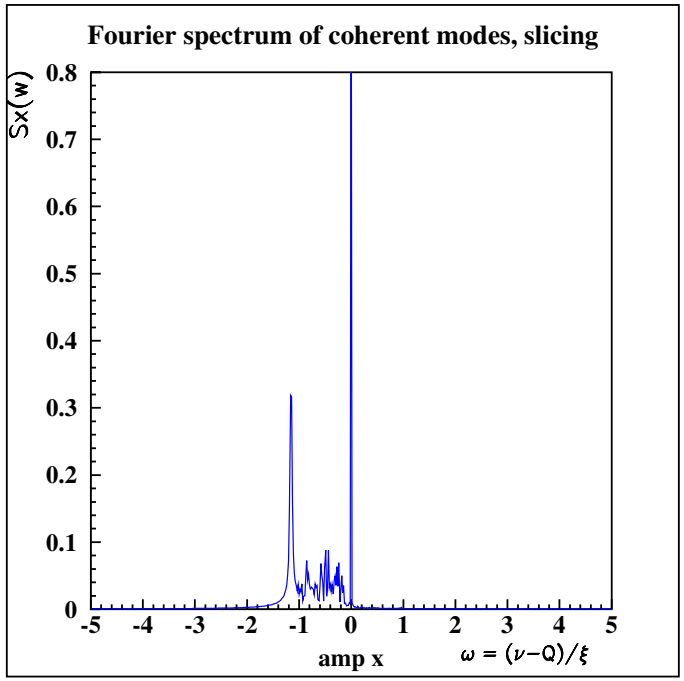
# Formalisms

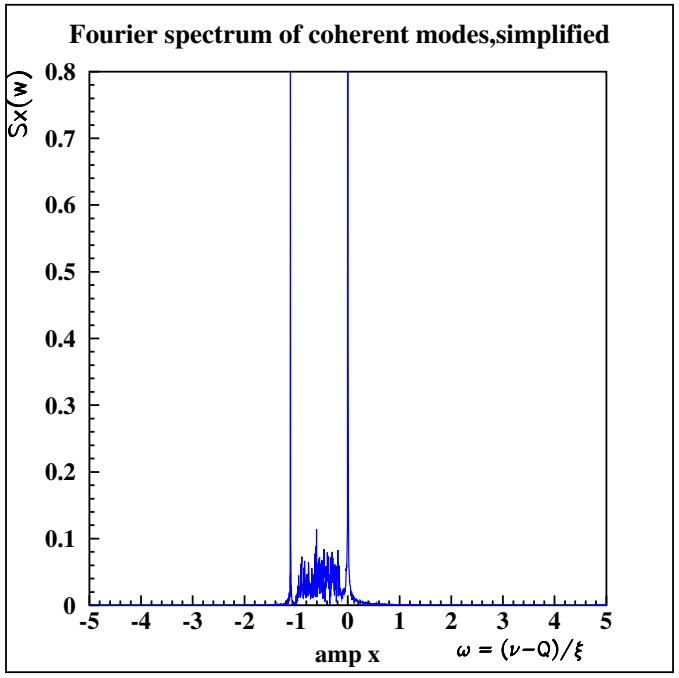
- Simplified formalism (introduced by Piwinski):
  - Replace $x$ by $x + \alpha_x \cdot s$ for the calculation of $\Delta x'$ , $\Delta x'$ depends on $s$
  - Energy change is then $\Delta\delta = \alpha_x \cdot \Delta x'$
- Cut beam into longitudinal slices (e.g. Hirata, Anderson, Herr, Jones)
  - Lorentz transformation to make them collide head-on
  - Used for cross check, it is 20 to 50 times slower

# Chosen formalism

- In this simulation program we use the simplified formalism
- Why: much faster
- We can use it because the bunch length effects are small (i.e. for $\frac{\sigma_s}{\beta^*} = 0.14 << 1$ )
- We are mainly interested to have synchro-betatron coupling (to have side-bands)

# Comparison of both methods (1)




- Without RF, no crossing angle
- Identical position of modes within errors:
- Y= –1.155 ± 0.020 (left, slicing method) and
  Y= –1.143 ± 0.003 (right, Piwinski formalism)

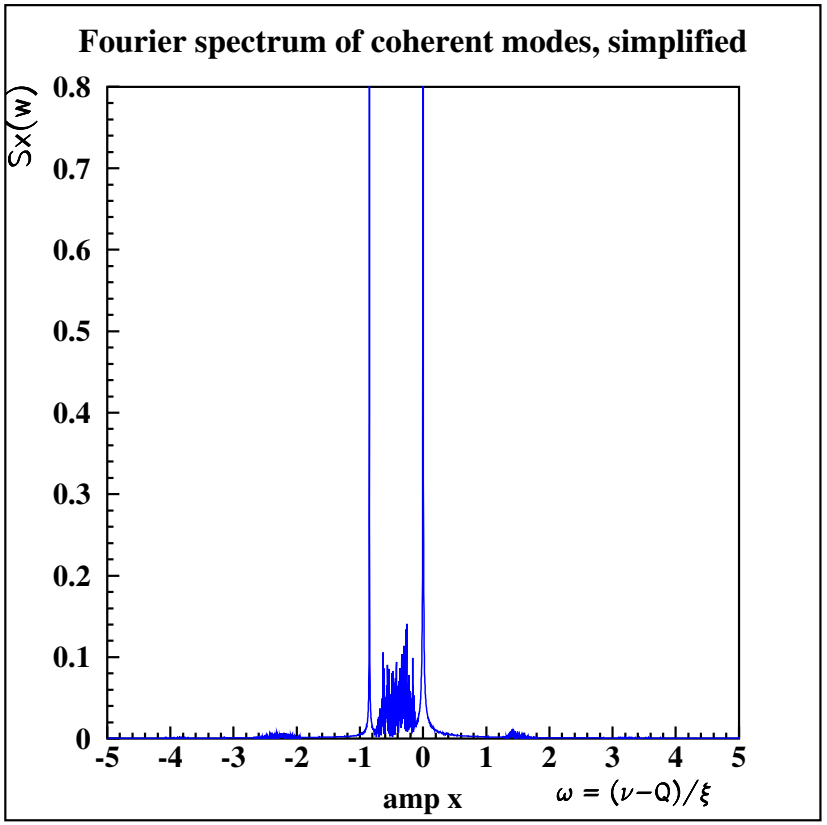
# Modes position

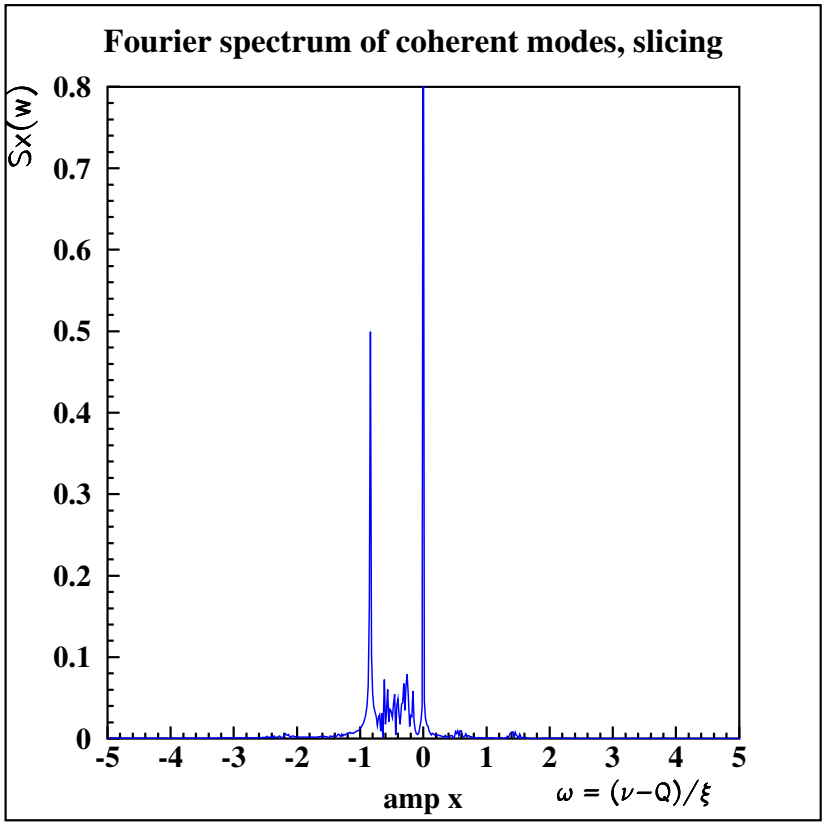
- We have found that two coherent modes appear
- Between these we find the incoherent continuum
- The position of the π - mode (horizontal and vertical) obtained with simplified method is

  (-1.143 ± 0.003/-1.153±0.003)
- The position of the π - mode ( horizontal and vertical) obtained with the slicing method is

  (-1.155 ± 0.020/-1.162±0.030)

# Comparison of both methods (2)



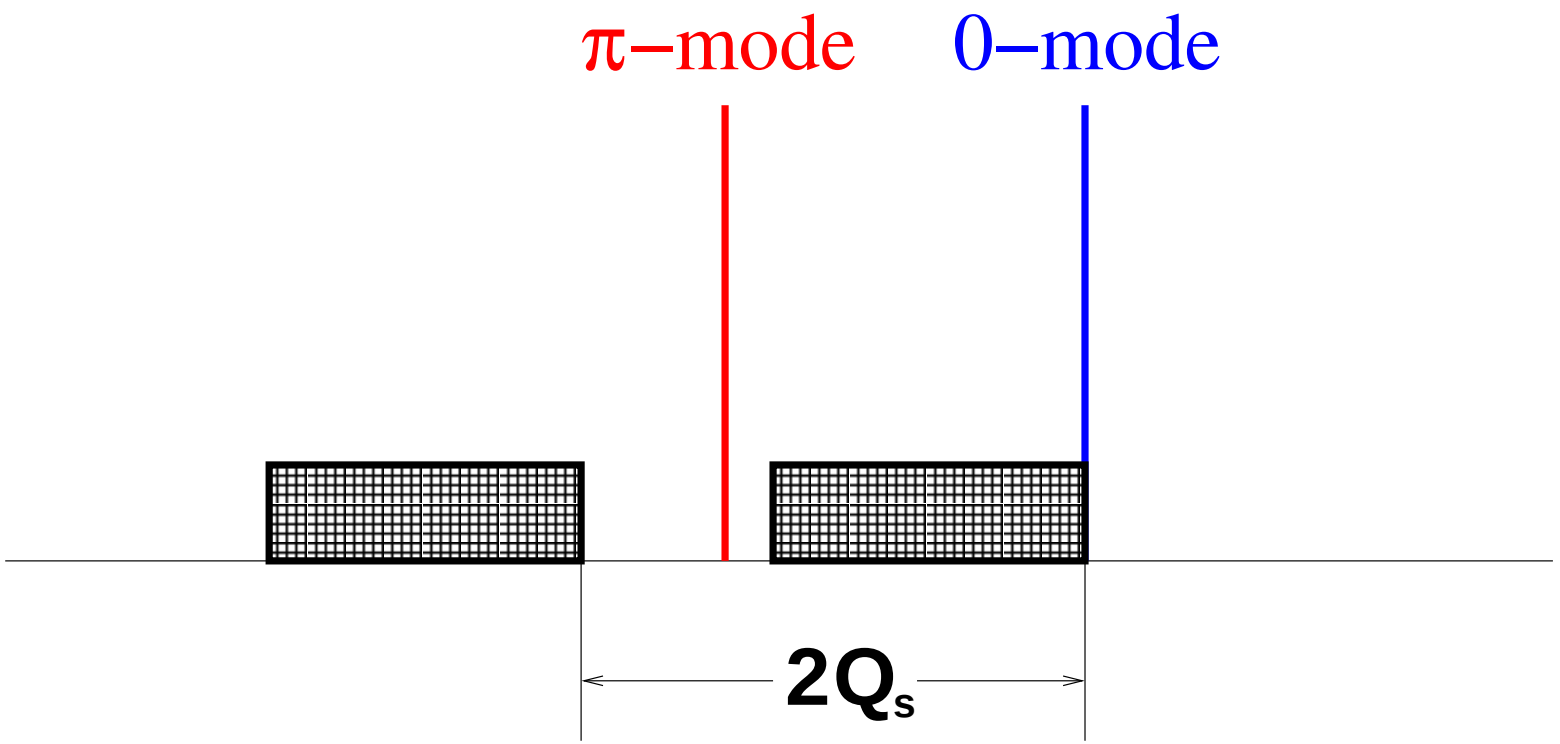

Simplified method

$Y = -0.849 \pm 0.003$

Slicing method

$Y = -0.838 \pm 0.010$

# Coherent modes and incoherent spectrum



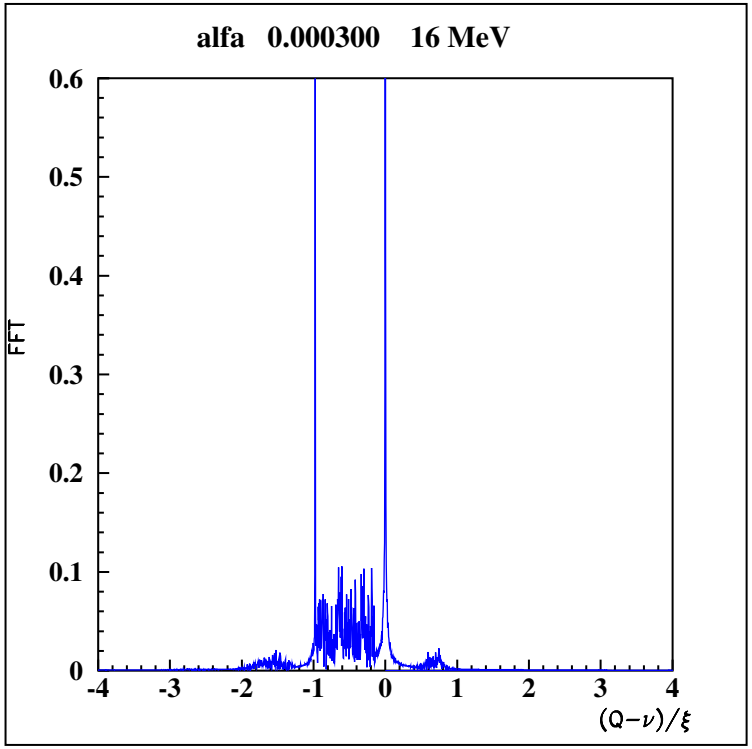
# Different crossing angles

- We have investigated the effects of different crossing angles
- It reduces the incoherent and the coherent tune shift for geometrical reasons
- For a horizontal crossing angle of

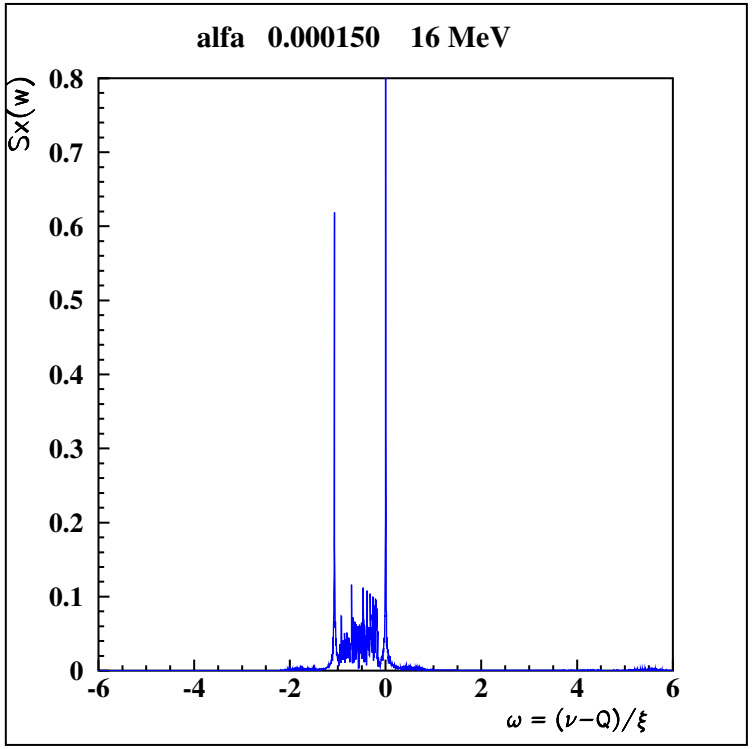
  $\pm 150$ μrad $Y_x$ is $-0.941 \pm 0.020$

  ($Y_x$ normalized to $\xi$ for head-on)
- $Y_x$ is about 20% lower

# Comparison using different crossing angles (1)




$\alpha = \pm 300$ μrad




$\alpha = \pm 150$ μrad

# Comparison using different crossing angles (2)

- For ± 300 μrad the synchrotron side-bands are clearly visible:

  - non-linear betatron-synchrotron coupling

  - the amplitude change with α

# Comparison using different crossing angles (3)

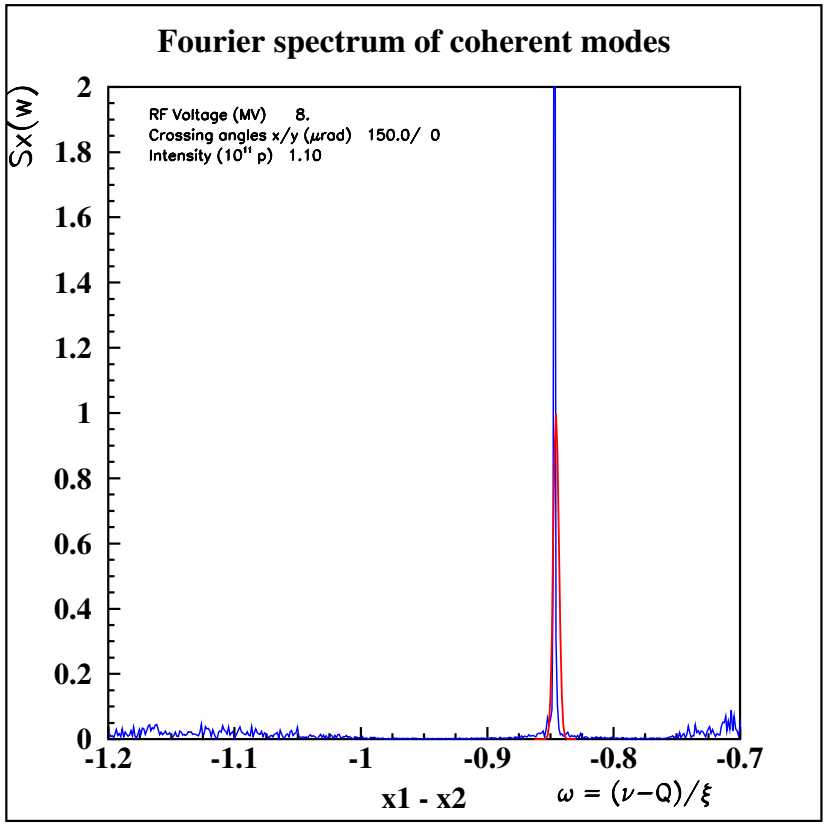
- The coupling is stronger when the crossing angle is bigger
- The additional degree of freedom reduces the coherence
- The Yokoya factor is reduced: it becomes more difficult to organize and to maintain a collective behaviour

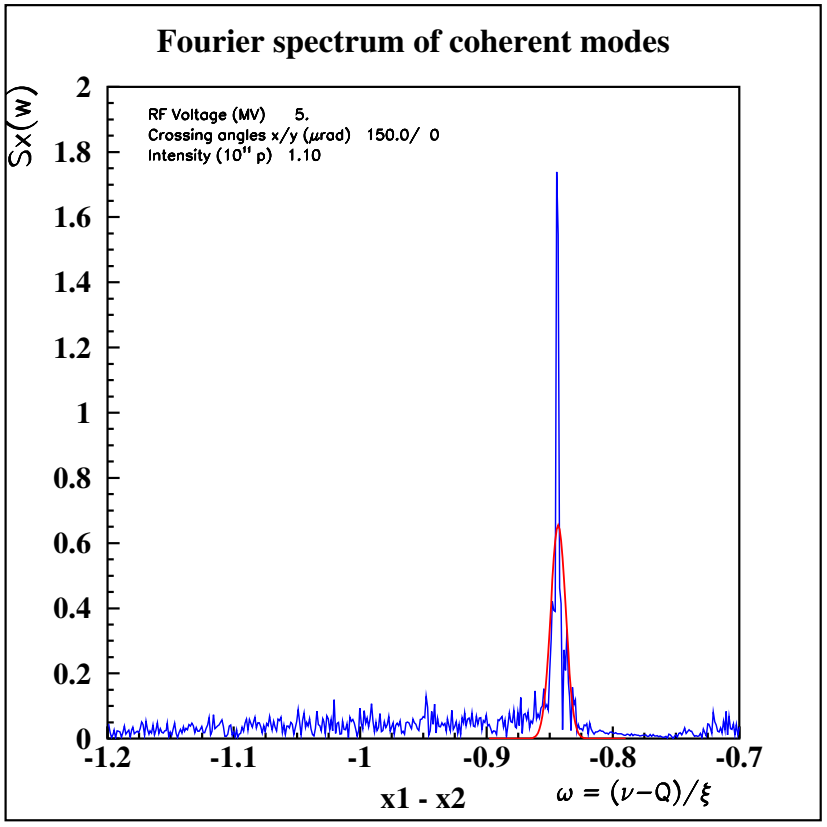
# Expectations

- For the nominal RF Voltage the side-bands are well separated from the

  $\pi$ - mode
- Reducing the RF Voltage in steps the side-bands move closer to the modes
- For certain values of RF Voltage the side-bands move through the $\pi$ - mode

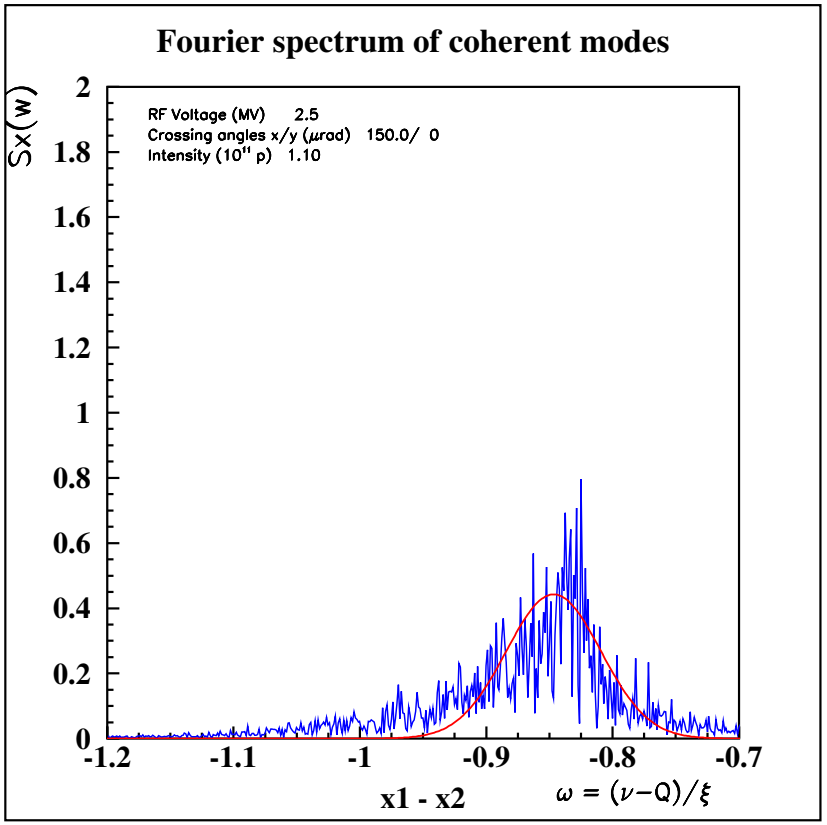
# Comparison for different RF Voltages (1)

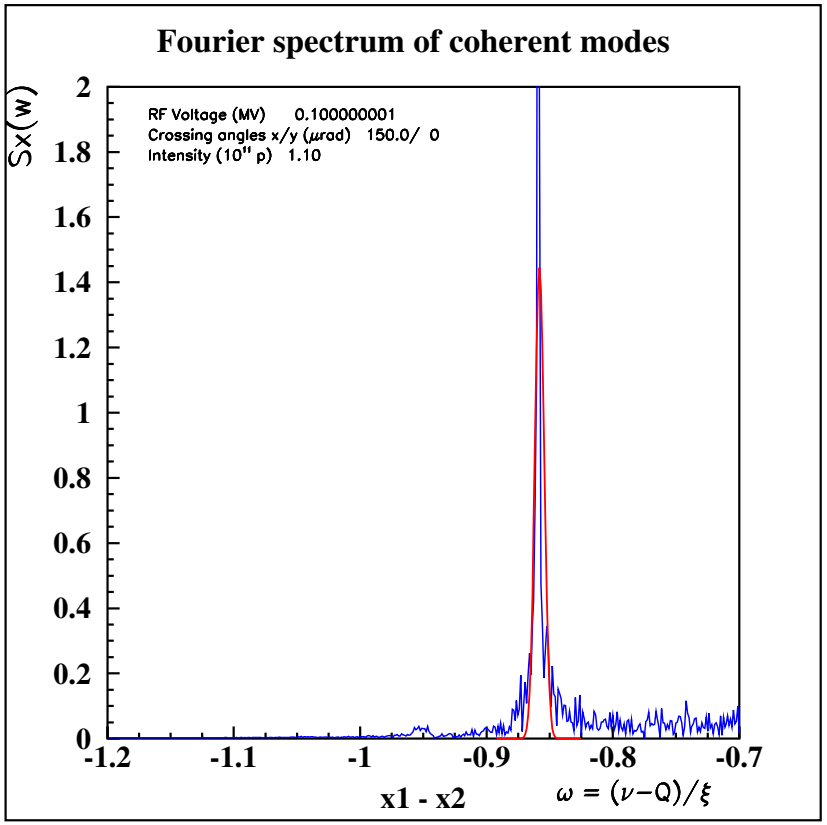



Comparison of the frequency spectrum between the results obtained using 8 MV (left) and 5 MV (right)

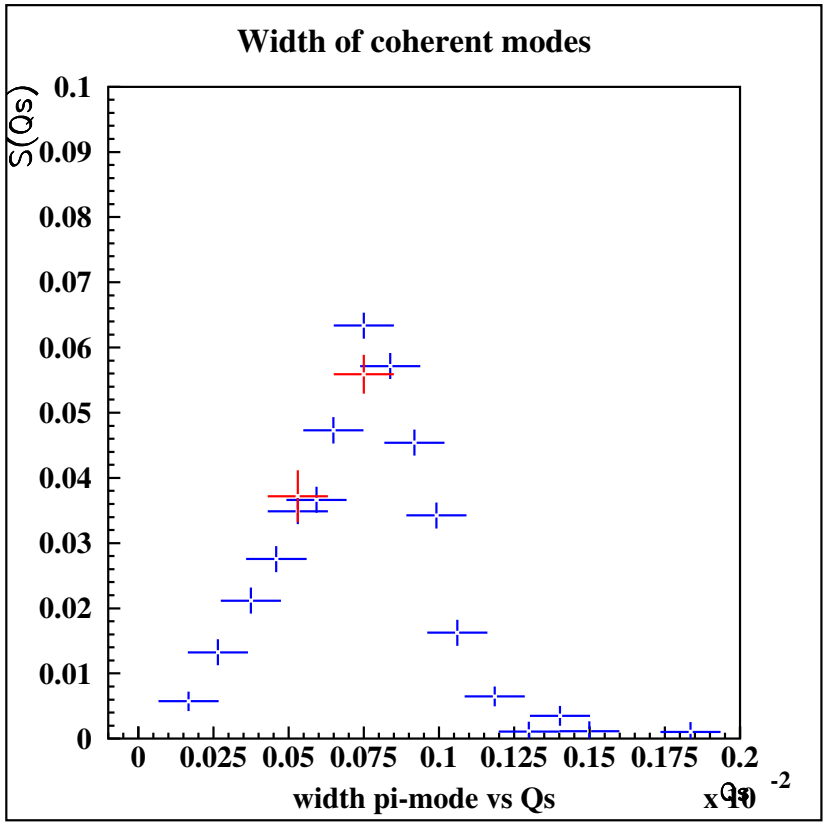
# Comparison for different RF Voltages (2)

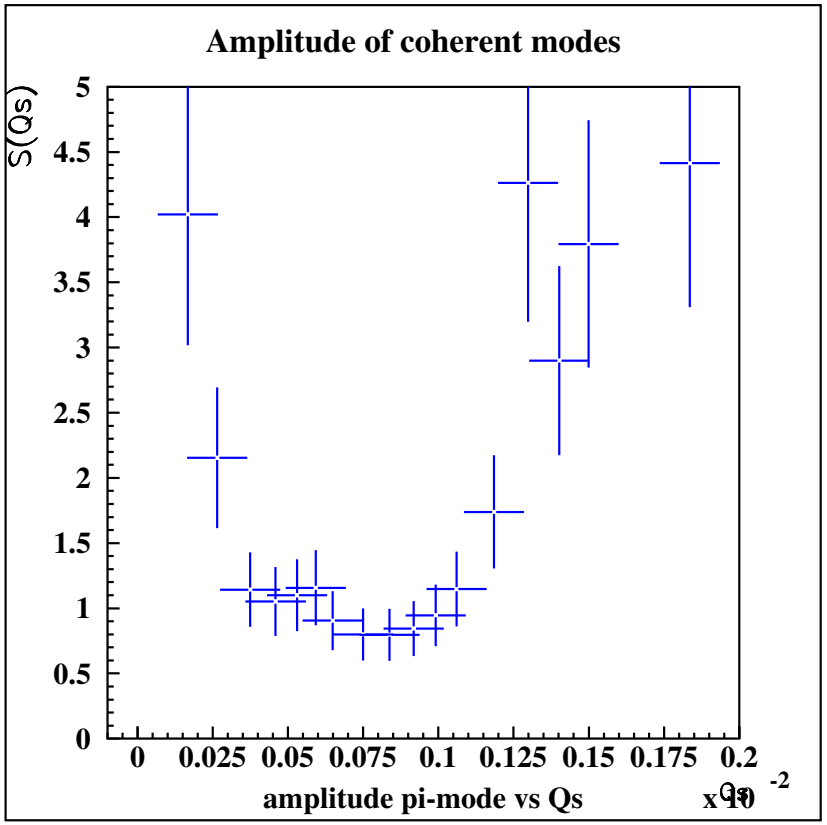



Comparison of the frequency spectrum between the results obtained using 2.5 MV (left) and 0.1 MV (right)

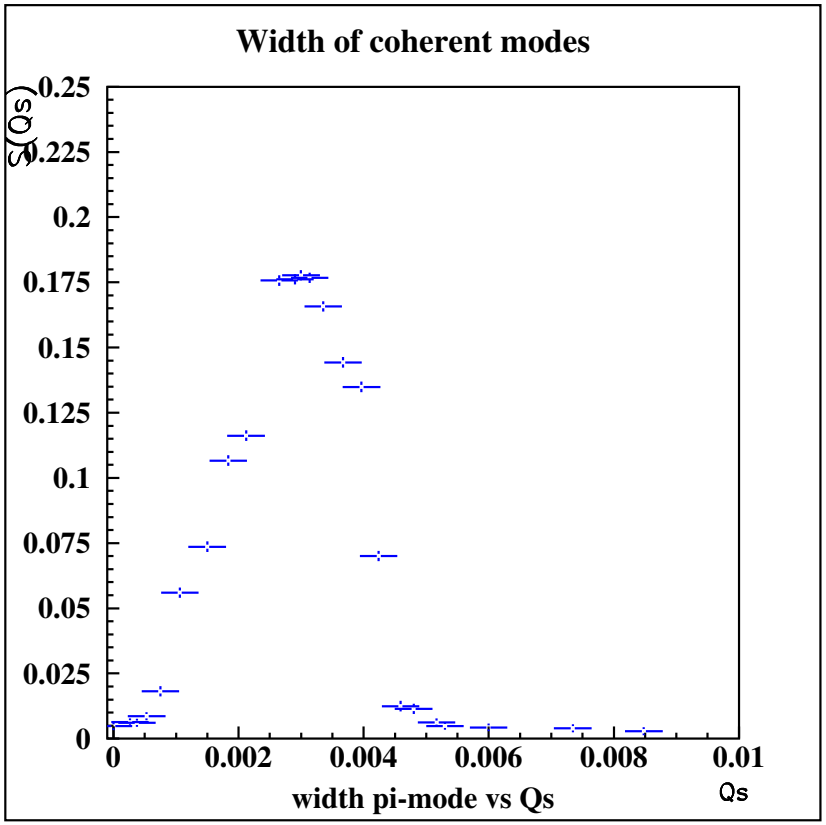
# Widening of the π - mode (1)

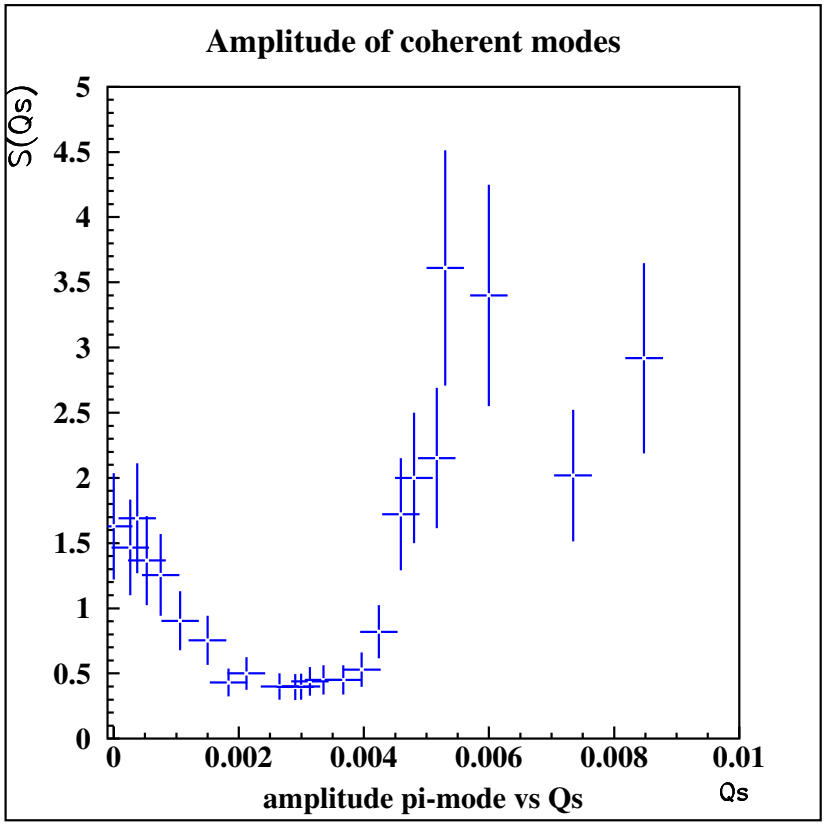



Width and amplitude of coherent π - mode as function of synchrotron tune.

# Widening of the π - mode (2)




Width of coherent π - mode as function of synchrotron tune.
Beam intensity increased to $4.4\times10^{11}$

# Comparison for different RF Voltages (3)

- For 2.5 MV the overlap is maximal
- The mode is strongly damped
- Strong reduction of the mode amplitude
- Very strong increase of the width of the originally discrete mode

# Comparison for different RF Voltages (4)

- The widening of the π - mode and the exactly corresponding reduced amplitude proves the damping is by overlap with synchrotron side-bands
- The best overlap is for $Q_s$=0.0075

  (i.e. 2.5 MV)

# Conclusion (1)

- The theory predicts the presence of two coherent modes and one incoherent spectrum between them
- Moreover it says the $\pi$ - mode is outside the incoherent spectrum $\Rightarrow$ this mode can not be Landau damped
- We have introduced an additional dimension and shown that side-bands appear
- Simulation gives very good agreement with theory

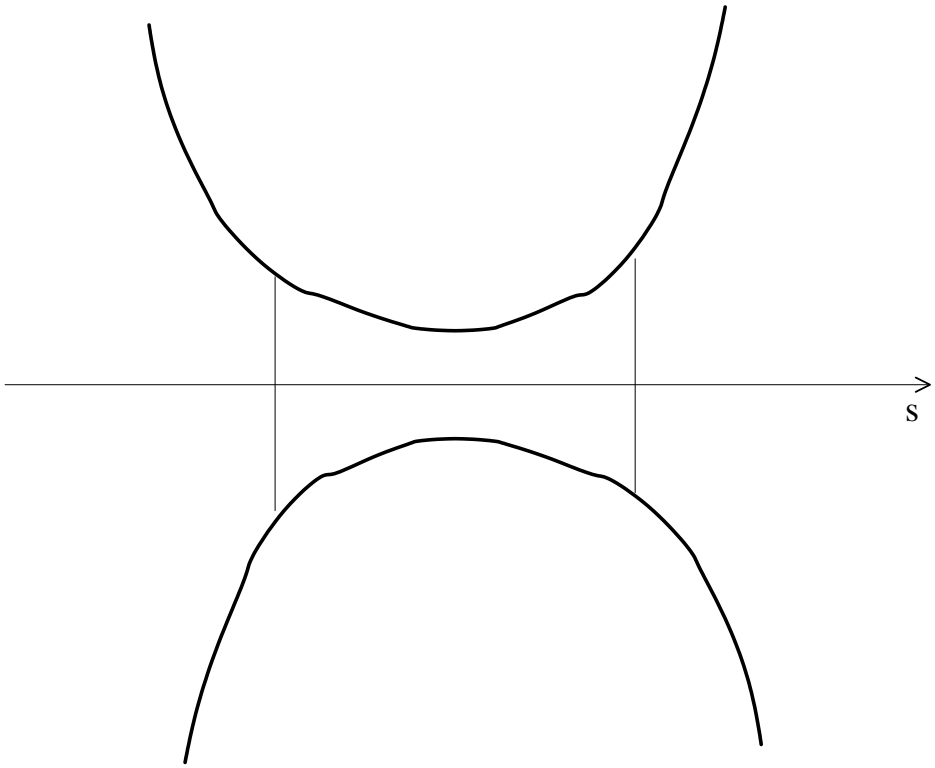
# Conclusions (2)

- We have shown that the additional dimension has a stabilizing effect
- The tune shift is reduced and closer to the incoherent spectrum
- The synchrotron side-bands can overlap with modes and provide Landau damping of the $\pi$ - mode

# Conclusions (3)

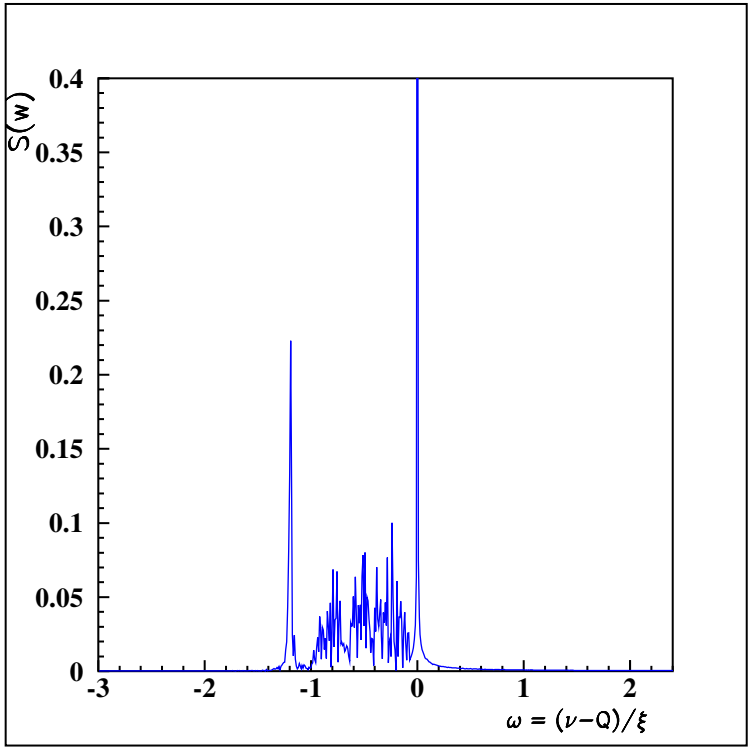
- We have demonstrated this damping in the simulation
- For nominal LHC parameters with 16 MV of the RF Voltage the modes do not overlap with side-bands but should make it easier to suppress coherent effects

# Hourglass



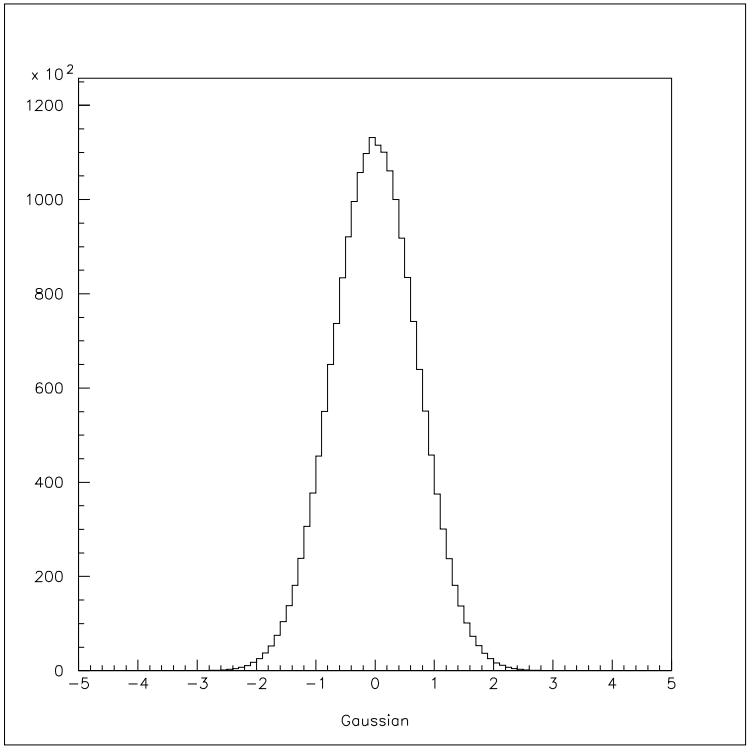

The beam size function $\left(\sigma\ (s) = \sqrt{\varepsilon\ \beta\ (s)}\right)$ looks like an hourglass, but in our simulation, we suppose that it is a constant for $s < \sigma_s$ .

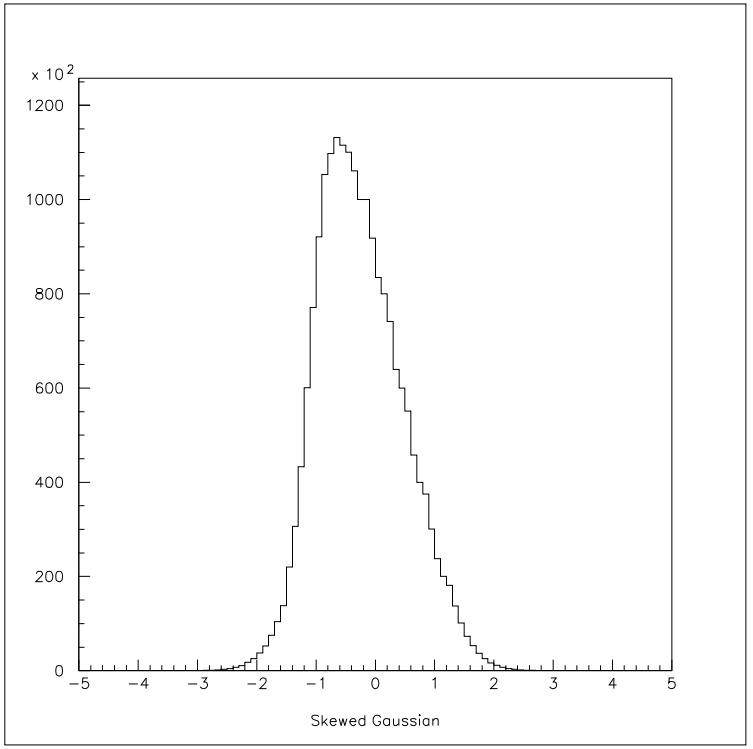
# Different intensity and different r.m.s




Intensity ratio of 0.5 and r.m.s. ratio of 0.7

# Yokoya factor (3)




r.m.s. = $\sigma$ r.m.s. = 1.08 $\sigma$

The Gaussian of our approximation is tilted (right) $\Rightarrow$

the calculated r.m.s. is bigger than the Yokoya factor is smaller

Rita Paparella, CERN, Switzerland

Good afternoon,
my name is Rita Paparella and I am from the
Politecnico from Torino. The last 9 month
I was technical student in the Accelerator
Physics group at CERN and I have contributed to
the work on coherent beam-beam effects in the LHC.
In this talk I would like to give an overview
and summarize the main results of my work.

First I would like to show a few of the main
LHC parameters which are important for our
studies .....

In this plot I show the frequency spectrum
as a function of the difference between the
measured and initial tunes, normalized by
the beam-beam parameters. The position of a
mode in this picture would directly give
the Yokoya factor